MITOCW | 8. Heavy Quark Effective Theory (HQET)

PROFESSOR: So last time we were talking about heavy quark effective theory, and we phrased several motivations for talking about heavy quark effective theory related to things that we hadn't seen in effective theory yet. We also said that our discussion here is more general than just talking about heavy quarks. Heavy quarks is our example, but we're really thinking about having some very heavy source. So you can think of it as either a heavy particle that you want to leave in the theory--

So let me rewind. We talked about heavy particles already, and what we did is we integrated them out. We got rid of them. But what if you have a heavy particle, and you actually want to study it? That's related to this discussion. That's one way that this is important. If you have a heavy particle that you want to study at low energies.

Or you could just have a source, a static source of some quantum number, in this case, color. And this would be the right way of thinking about it. So those are kind of some of the motivations, and then there was also some more technical motivations of things that we wanted to see with this theory.

So we started by saying it's a top-down approach, so we should be able to just take QCD and take the limit and expand. But it's a little more tricky than it was so far with, like, light quark masses for chiral perturbation theory because the mass is in the numerator, and we want to take the limit that it goes to infinity, OK? So we put a question mark beside this equation, and we started instead thinking about expanding the propagator.

So if we write the propagator out for a heavy quark, you've got mass terms in the numerator and the denominator. And when you expand it out, after using the equation of motion and expanding with p equals mQ v plus k. So we introduced a vector, v. v squared was 1. And then we introduce some fluctuations, k.

So there's m's hiding in this p as well as that p. The quadratic term in the denominator cancels. There's a linear term that survives. There's linear terms in the numerator. The linear terms, once I just look at what they give, they give this first contribution. And then there's some higher-order terms. Then we drop them.

OK? So for the propagator we had no problem taking the heavy quark limit, and we got out this expression. We also then thought about the vertex. It's the gluon coupling to a quark, heavy quark. And if this is the result in QCD, this 1 plus v slash over 2 is a projector. But that means if you square it, you get back the same thing.

And we said, well, if there's two propagators on either side of this thing, I can think about it putting a projector on either side of this thing, and that gives you this result with just a v mu. OK? So the Feynman rule for this guy can be simplified by using the fact that on either side of it are sitting these projectors.

And if you take those two facts into account, we can actually encode them in a Lagrangian. We could just write it down, OK? We want a Lagrangian that gives that propagator, the lowest-order propagator, and that lowest-order vertex. And that's what this is. So the v dot D has in it a v dot A. v dot A is giving this guy. So that's the v dot A inside there.

And then 1 over v dot k, that's the v dot partial, right? So i v dot D equals i v dot partial. I don't know which sign I'm using here, but I think it's the minus sign.

AUDIENCE: I think you're using plus.

PROFESSOR: Using plus?

AUDIENCE: No, you're using minus.

PROFESSOR: Using minus. I'll switch throughout the course. In each chapter I'll use a different sign convention to keep you on your toes. [CHUCKLES] In this chapter, I will stick with this sign convention, since the source that we're using for the reading is using this sign convention. OK? So this is the v dot A, and this is the v dot k that we have over there.

Now, I've encoded-- the way I encoded the projection relation, as I said, let's work in terms of a four-component field. So our field, Qv, still has four components, but it satisfies this relation. And that actually projects it down onto two components, and we'll talk more about what the physics of this is in a minute.

This is very convenient when talking about heavy quarks because you often want to couple heavy quarks to light quarks, and if the light quarks are four-component spinors, it's easy to couple them to the heavy quarks if you have heavy quark fields which are four-component spinors. So you could work in terms of a two-component Lagrangian that doesn't have this projection relation, but for technical reasons it's better to work in terms of the four-component one. Makes things simpler.

So that's in some sense an indirect derivation of the leading-order HQET Lagrangian. Is there any questions about that?

AUDIENCE: What if you don't have the propagator on the other side?

PROFESSOR: What if I don't have the propagator on the other side? Yeah, so we'll talk about the spinors in a minute, but it's the same story for the spinors, that they have a projection in relation, which is exactly the same projection relation. So whether it's a line sticking out or whether it's attached to something else, it's the same.

OK, so let's now go and derive the same thing directly. And the trick we're going to do is the following. We need to cancel out this m in the numerator, and the place that m hides is in the p in this formula here. So if we think about some field that's fluctuating in spacetime, we can pull out from that field the analog of that big piece, p, if we pull out a phase factor that looks like that.

So let me just write this-- so far this is just some redefinition of these objects on the right-hand side. And I'm setting up two objects here because I want-- I'm going to break the field into these two pieces that obey different projection relations.

OK? So if I sum these two up, the v slashes cancel. Qv plus-- These are just two components of the-- I've just split it by two orthogonal projectors, two pieces. And I've pulled out this piece here. And the reason that I pulled out this piece here is the analog of what I did here with the p, where I divided it into a big piece and a small piece. This is going to take care of pulling out that big piece.

OK, so if we want to take the limit here in this equation. So far we've written out what the Q's will be. We also have to write out what the D slash is. So it's convenient to do the following with the D slash. Break it into two pieces-- a piece along v and a piece orthogonal to v.

So the definition here of D transverse is that it's D minus the piece along v, so if you like that v dot D transverse is 0. That's what I mean. With these relations up here, there's also another way of writing them that I should write down, as well, which is a little simpler.

If I rearrange these formulas, I can also write as v slash on Qv is Qv, and v slash on Bv is minus Bv. So there's a 1 on this side, and if I come at it with the half and put things together that's what I get.

OK, so let's put those things together into here, and then we'll figure out if we can take the limit. And the crucial thing, really, for being able to take the limit is to pull out this phase, and we'll talk about what physically is happening there in a minute, once we've done the algebra.

OK, so just plugging those pieces together, we have that. Pull the phase through. So pull this negative phase through these terms. With this v dot D, when it hits the v dot x, we get some extra contribution. And I can group it together with this mQ term because what it does is it brings down an mQ. The D transverse gives no derivative, no contribution, when it hits that phase because of this relation.

So pulling the phase through, the only term we pick up is this minus 1 here. And I just grouped it together-- sorry. The only term we pick up is this v slash here, and that comes from this v dot D hitting that term.

All right? So now we put these pieces together, multiply things out, and use the fact that I've divided up the field into these two pieces that have different-- I know how v slash acts on them, so I can get rid of all the v slashes by just using the formula. When v minus 1 acts on Qv, I get 0, OK?

So for that term, for the term that has two Qv's, I just have the v slash i v dot D. You can also prove that when you have the Qv and the Qv bar that this term actually is 0, and to prove that you basically use the projection relation and the fact that if I have 1 plus v slash over 2 D transverse, that that can be written as D transverse 1 minus v slash over 2. Again, using this formula and the fact that gamma matrices anticommute. And then 1 minus v slash over 2 kills Qv, and so a term like this one between these fields is absent, and the only term with two Qv's is that one.

Likewise, with Bv's it's the same story. I don't get a D transverse term. But here, the mQ term does survive, and it doubles, so the v slash gives another minus 1, so there's 2mQ sitting there. And then there's the transverse terms, and they come in the cross terms. So Qv Bv. If I have this, then this projection relation won't get rid of it because, if I have a 1 plus v slash over 2 on the right, it becomes the correct projector for the other field on the left, and then vice versa.

OK? So there's four terms here, and we've simplified things as much as we can. So now let's think about what these fields are describing. In particular, let's think about what would happen if we only had external Qv fields.

So if we consider only external Qv fields, then we can think about what happens as mQ goes to infinity. And if you have a term-- so as m goes to infinity, nothing happens to this term. We have to figure out what happens to those terms over there.

But this guy here, it's clear what happens. We get a field that has a mass, mQ, and that particles are just getting heavier and heavier, and they're decoupling. And when you get particles that get heavy, they decouple. So Bv decouples in this limit. So diagrammatically, think about it like this.

You have a Qv. It could switch into a Bv with one of these D slash vertices, and then back to a Qv, and this would give some kind of diagram from this Lagrangian, which has external Qv fields. But the Bv field has a 1 over m-type propagator, and so this goes like 1 over m, and hence goes to 0. And that's one way of talking about decoupling, that intermediate particles of that type are giving diagrams that are suppressed by 1 over m. OK?

And you see here, sitting right here, you see our Lagrangian that we want, which is the Qv i v dot D Bv. So if we just threw away the Bv fields, we would get the Lagrangian that we want.

So physically what's going on with these two fields is that Qv corresponds to the particles, OK? We have a heavy quark. It's got particles that are of that mass and that flavor that are heavy, and it has antiparticles, too. Qv corresponds to the particles, and Bv corresponds to the antiparticles. So-- of this flavor.

And by making the phase redefinition that we did, we chose to focus on on-shell fluctuations that are close to the particles. If we'd chosen the opposite phase, plus i, then we could expand about the antiparticles, and things would have worked out the opposite way. So I'll come back to that a little bit more in a second.

So let's have several discussion points here. So what we did is we made a field redefinition, which we were always allowed to do.

AUDIENCE: [INAUDIBLE]

PROFESSOR: Yeah?

AUDIENCE: Isn't Qv like a [INAUDIBLE] eigenstate of positive helicity? And the other one--

PROFESSOR: We're talking about heavy particles, so helicity is not such a good thing to--

AUDIENCE: [INAUDIBLE]

PROFESSOR: Yeah, I'll talk more about it in a sec. Yeah. Yeah, give me a minute, and then if you still have a question after I talk about it, then ask. So first of all, what we did here was tree level.

OK? So when we make this type of redefinition and we go through this algebra, it's all tree level. Not thinking about any loops here. There will be corrections to this story once we start talking about loops, and there will be corrections to this story, namely that we just get the same L HQET once we want to take into account one of our m corrections, and will come back to both of those points. So both of those things to be discussed later.

What we do do with L HQET-- as we described correctly, the couplings to gluons at leading order. And often what people are interested in with k mu here is k mu of order lambda QCD, separating the scale lambda QCD that has to do with the confinement into a meson, like a B meson, from the scale of the heavy quark.

OK, so the antiparticles are being removed, which is point number 2. So if you want to ask what is it that we're integrating out in this theory, we're not integrating out the particle, because we want to study it. We're integrating out the antiparticle.

It's easiest to see that if you go to the rest frame. So taking v rest frame, which is just 1, 0, 0, 0, just timelike component, then our 1 plus v slash becomes a-- so if you have 1 plus v slash, then that's just 1 plus gamma 0 over 2. And if you want to talk about particles and antiparticles, the best representation to use is the Dirac representation because then the particles--

So let's take a spinor in the Dirac representation. Could be part of this field. Could just be a spinor for an external state. In the Dirac representation, what happens is that you have the guys that are the particles in the upper two components and the guys that are the antiparticles in the lower two components. So this is particles. This is antiparticles.

This guy is always projecting in the rest frame into the particles and antiparticles. It's just that if you read in other representations you'd have more trouble seeing that, but in the Dirac representation we just have a 0, OK? So this is one way of thinking about what this projector is doing. It's getting rid of the antiparticles.

So the way you should think about this physically is that, if you're studying some process, you're studying heavy particles close to their mass shell. What we're doing is we're measuring fluctuations near the heavy mass, mQ. We're perturbing about mQ. If you're perturbing about mQ, the antiparticles are very far away because the splitting between the particles and the antiparticles is 2mQ. So that's why they get integrated out.

So that's why, in this language of being near mQ, the antiparticles are just very far away, and we don't want them in our theory. One way of talking about that is pair creation. So if we draw some type of Time-Ordered Perturbation Theory diagram for pair creation-- so it's not a Feynman diagram, but a TOPT diagram, just so we can talk about a definite intermediate state at a definite time, then this intermediate state here has three heavy particles, and it has--

If this guy has mQ and this guy has mQ, this guy has 3mQ. So this intermediate state is off-shell by 2mQ. Time-ordered perturbation theory diagram. So I can talk about whether something is definitely a particle or definitely an antiparticle.

And another way of seeing this 2mQ that I'm talking about is, like, just to do vacuum polarization, where you have 2mQ. You need 2mQ of energy to fluctuate into a heavy quark pair. So we have a very light gluon, and it just doesn't have the energy to do that, so pair creation is not part of this theory, OK? So now you can ask your question if you still have a question. [CHUCKLES] Yeah.

All right? So the thing we're getting rid of is the antiparticle. Now, because we're getting really antiparticle, we no longer have an annihilation process. So unlike QCD, where you can have particles and antiparticles annihilate, here we have actually a conservation rule for the number of heavy particles.

We're talking here about quarks. The number of heavy quarks is preserved, and that's an extra symmetry that this theory has that QCD didn't have. OK? So the conservation law for the number of heavy quarks is u1.

More generally, we can extend this to something called heavy quark symmetry. So let's-- since we notice that there's this conservation law, let's think about what the biggest group we can come up with is that describes the symmetry of this L HQET.

So this u1 that preserves the heavy quarks is a flavor symmetry. And depending on how many heavy particles we have, it could be bigger than a u1. So actually, it's a uNh for Nh heavy quarks. So you could have some heavy sources that live under some group. And if you have N types of those sources, you'd have a uN symmetry.

And the root of this is that we got rid of mQ. mQ was the thing that would break this symmetry, but it disappeared. And once it's gone, we don't care. We don't know whether the thing was a charm quark or a B quark or a top quark. We don't see the flavor of the quark anymore because the mass term was what was tracking that flavor, and now it's gone, and we just end up with this theory that doesn't remember about the flavor, so we have a flavor symmetry.

OK? So that's kind of cool as an example of having an effective theory where there's an emergent symmetry, symmetry that wasn't so apparent in the theory you started with but is apparent once you get to the effective theory.

The other example is spin symmetry. There's actually an su2 spin symmetry because the Lagrangian didn't involve any D slashes anymore. It just became scalar, the v dot D. So if we think about our heavy part-- so we think about a four-component spinor. Two degrees of freedom have to do with particle versus antiparticle. The other two have to do with spin, a half. And we don't care about the spin, either, because our Lagrangian is completely scalar.

OK? We just had Qv i v dot D Qv, and there was no-- we didn't mention-- the v dot D doesn't have any D slash. It doesn't have any spin matrices in it, OK? So that's an su2 symmetry. So if we want to talk about that again, it's useful to think about it in the rest frame.

In the rest frame, the heavy quark spin transformation. Need some Dirac representation, which is gamma phi, gamma 0, gamma i. It's just the Pauli matrices, sigma i. And you could think of making a transformation related to the spin. So the infinitesimal version of that transformation would look like this.

Start with Qv. Do a rotation. Ask, does this change the Lagrangian? And the statement is that it changed the Lagrangian that you get, which is i v dot D commutator i epsilon dot sQ. But i v dot D is just this scalar quantity and just commutes through this epsilon dot sQ, OK? So that's the formal statement that there's a spin symmetry.

Since we're using this kind of projected formalism, you could ask the question, well, maybe this spin symmetry is mixing up the heavy quark components with the other components, these zero components, but you can check that it doesn't. It's actually acting within the subcategory, which is just describing the heavy quark. So you can also check it.

One way of phrasing that is that you can check that this is true. So that the rotated field still satisfies the projection relation. So really, this is a rotation that's within the two-component subspace that are describing the physical degrees of freedom. OK? So there's this enhanced symmetry group of this theory.

And we can put it together because nothing stops us from doing this, or doing this, or doing a bit of both, so we can build one big group. So all together, you have a u2Nh, and that's heavy quark symmetry. A Qv field is a fundamental in that group.

So [INAUDIBLE] spinors look like this, with some number of components, and this guy could be-- for example, maybe this is a B quark with spin up. And then some other component might be a charm quark with spin up or a B quark with spin down. That's what heavy quark symmetry is. OK? So any questions about that?

So we'll talk a bit more about how you actually make predictions related to this heavy quark symmetry. As you saw, it's kind of emergent, and so you should build some tools for seeing what kind of impact that has on observables, and we'll spend a little bit of time doing that today. But before we do that, let's continue our list.

So when we made this field redefinition, we pulled out this phase, and that phase depended on v. And we also stuck a label v on the field. So we had this v mu, which appeared on our field, OK? And that was just a reminder that we pulled out a phase, and the phase depending on our choice of v, and we could make actually different choices.

This v mu is actually something quite useful because it isn't changed when you talk about low-energy QCD interactions. So it's a conserved quantity. So in our notation of dividing up p into mQv plus k, this changes.

So if you tickle with very soft gluons the heavy quark, you can't change this term, but you can change that term. So the v term is not affected by the gluon interactions. It's a conserved quantity, and hence having it as a label on our field tells us what v we're talking about. Basically, what v signifies is you could talk about a heavy quark in its rest frame, and then you could talk about tickling it. Or it could be moving with some constant velocity, and you could talk about that moving frame tickling it. And any such choice is equally valid, and that's what v is encoding. OK?

But we can formulate tickling a heavy quark in any given frame that we'd like, and v is encoding that. Things are kind of natural to see in the rest frame, but we don't have to work in the rest frame, and that's what v is encoding. OK? So that's kind of different than we've seen before.

What's the power counting? Power counting is not so difficult here. It's just powers of 1 over mQ. So our leading-order Lagrangian had no mQ's and then we could formulate corrections to that leading order, and they would be suppressed by powers of 1 over mQ. So let's think about that in a little more detail.

So if you take the field Q of x and you write the mode expansion for the field-- but there's this term, and then there's a term that involves the antiparticles.

What we did with the Q of x field-- that's the full theory field-- is we pulled out this phase. And the phase that we pulled out was just something that depended on a fixed quantity, m v, so it would come outside that integral. So we pulled out-- so you can think about really literally sticking p into this formula and then pulling out that phase. And then that leaves, in Q v, just the e to the minus i k dot x. OK?

So if you think about derivatives on Qv of x, then they are scaling like k. And this guy here has no mQ's in it. So there's no mQ's in the derivatives. So if we build operators out of this Qv field, if we let derivatives act on that field, there's no factors of the heavy quark mass hiding anywhere in those derivatives.

That's the magic of making this kind of phase redefinition. We've made the mQ's explicit by pulling that out so that we could get something remaining that doesn't have any mQ's. Since we want to count mQ's what we want to do is we want to make all the mQ's very, very explicit. We don't want any to be hiding anywhere, and that's what we were doing before with this field redefinition, was making all the mQ's explicit. This is one example of why that's important.

AUDIENCE: [INAUDIBLE]

PROFESSOR: Yeah. So you should-- if you think about the action-- yeah, if you think about the action and you thought about making a gauge transformation, you could worry about making a gauge transformation that would inject too much momentum into your field so that it no longer described low-energy degrees of freedom. But usually people don't talk about that so much in HQET, so we'll save that discussion for [INAUDIBLE], but we will have it.

So the coordinate, x-- one way of thinking about this equation is that the coordinate x that's in Qv really corresponds to variations that are the low-energy variations. So our field is doing what we want. It's describing low-energy fluctuations. And that's another way of thinking about the fact that there's no mQ's in this derivative.

So if we look at subleading operators and-- if we look at subleading Lagrangians and external operators, all the powers of m Q are going to be explicit, and that makes doing the power counting quite simple because we just look at them and count.

So as long as we're sure that they're all explicit and none are hiding anywhere, then it's easy to count. There will be logs of mQ hiding in Wilson coefficients, but we're not counting logs. If we're just counting powers, then what I said is sufficient, OK? So is there any questions about that?

So there's actually one mQ that's hiding, and we should be a little bit careful about it. So if we talk about states, there's actually an mQ hiding in there. So let's think about states of some particle that's relativistically normalized. It could be a heavy meson. Let me just call it H. And the usual normalization convention for a relativistic particle is this one for the states.

So this guy, if you look at the dimensions of the right-hand side here, you've got minus 3 plus 1 is minus 2, so each of these is dimension minus 1. So that's one thing we can [INAUDIBLE]. But if you think about what this Ep is, this Ep has the M still hiding in it. So this Ep is really the physical on-shell energy, so it's mQ squared. If this was a heavy hadron, for example, it'd be mH squared plus the 3-momentum squared. And mH, for a heavy hadron, has an mQ hiding in it.

OK, so what we have to do if we want to make everything explicit is use HQET states. And you can think of HQET states simply as the states that you get if you define your quantum field theory using L HQET. So this is the mQ goes to infinity, so it's just the leading-order term.

And if you define your states with this Lagrangian, you can still talk about a bound state like the B meson. Because the B quark didn't go away. It just became static. And all the dynamics of the light degrees of freedom that are making it into a hadron are still there, and they're described by a usual QCD Lagrangian. So you still have hadrons, so you still have a B meson, but it's not the same because you don't have any mQ's. They're all gone. So you can't possibly have this formula for the HQET definition of the state.

And so the way that you should think about this is that the relativistic version of the state is related to the HQET definition by a formula like this. So the HQET definition is this Hv, the relativistic is this Hp. And there's some normalization factor between them, and as well as there can be 1 over M type corrections. Actually, both things are there.

And if we talk about this H v field, where v is some external label telling us the velocity and k is describing some residual momentum of that state, the normalization convention would be as follows for HQET. So I've still left it to be 2. Sometimes people to remove this 2, as well.

So there's several things going on here. One is that v and v prime-- in order for these states to have overlap, we're saying that you should be talking about hadrons that are moving with the same velocity. Remember velocity is conserved, so if we had two things in different sectors, they just wouldn't overlap. That's [INAUDIBLE] of delta. This delta 3 of k minus prime is exactly the analog of this guy over here, but now it's with the k, which is the residual. It has no mQ's.

And I got rid of the mQ-- I got rid of the thing in the numerator that was pulling out this root mH. So there's no mQ's in that formula, and that's the right way of thinking. So you have to remember, if you have some formula for the cross-section that you derived with relativistic states, which all of the formulas you're used to are derived that way, you have to remember about that root of H when you're going to make predictions with HQET.

And the same thing actually is true of the spinors. There's a square root of the quark mass hiding in spinors, and you have to take care of that, as well. I'll leave that for your reading. OK? So up to these little subtleties here, that's all the mQ's that you have to keep track of, and once we've kept track of all those mQ's, then we know how to do the power counting in this theory.

What can we do with the theory? Well, one thing that we can do immediately is do some spectroscopy. So this will actually show us partly what kind of predictions we can make from symmetry. So there's light quarks and gluons, and they are still described, as I said in words but now I write on the board, by a full L QCD. Nothing changes. There was no heavy quark masses in that anyway, so nothing happens to that, at least for what we're talking about here. There could be some loop corrections, but--

So if we think about mQ going to infinity, we can ask what happens to these hadrons, which I've just told you you can still describe by the theory. So if you think about a Q and a light quark, which is a meson, it has the quantum numbers of the heavy quark. And remember that the number of heavy quarks is conserved in this limit.

And then it has the quantum numbers of the rest of the stuff, which could be an antiquark. It could be any number of Q q bar pairs because we don't know how many are there. We have nonperturbative interactions. And it could have any number of gluons. And generically, we call this stuff light degrees of freedom because that's what it is.

This is heavy. This is light. And it's just a bunch of light stuff. OK? So that's kind of how we can think of the state. It's a bunch of-- it's an infinite number of single particle states because it's a complicated QCD bound state. And what we want to do is we want to sort of enumerate what symmetry can tell us.

Well, since we have the-- we can think about total angular momentum, and that is actually a good quantum number. It was in QCD, and it remains true in HQET. The thing that's new is that the heavy quark spin is conserved. So SQ is conserved. And that we can use to say something more about the states.

And usually the way that this is done is by defining Sl to be J minus SQ. So if both J and SQ are good things to talk about, we can also talk about Sl, which is the difference. And if you want to think of quantum numbers here, then J squared, right, would be, in terms of quantum numbers, so J, J plus 1. And so we can do the same thing with Sl [INAUDIBLE] Sl, Sl plus 1. On a state with good quantum numbers of Sl.

So if we organize things in terms of Sl, then we get symmetry doublets for the mesons. So let's pick an Sl and some parity, make a little table. If we take a 1/2 minus, there's two mesons that fall in that camp, a B and a B star that have little-- just call it capital J. Let me call it little j.

j is 0 and 1 for these guys. So this is the scalar guy, and this is a vector guy. And the parity is negative, so it's a pseudoscalar. We can keep enumerating. There's a guy with opposite party. These guys turned out to be heavier. This is the lightest ground-state heavy mesons. 3/2 plus called B1 and B2 star. All these things have been observed.

And you could keep going, and you could do the same thing for baryons. So if you do baryons, then the spin of the light degrees of freedom are not half-integer, but integer. 0 plus is just the lambda B, and then the only possible j is to add 1/2 for the SQ. So what's happening is this is Sl, and then remember that the Lagrangian and the dynamics is independent of SQ.

So I can add and subtract the SQ from this Sl, and I get total j of 0 or 1, but the dynamics of these mesons doesn't care about the heavy quark spin, so that's why they come in a symmetry doublet. So the heavy quark spin symmetry is relating things in a doublet of given Sl. That's how you should--

AUDIENCE: Notion to star just always mean excited [INAUDIBLE] state?

PROFESSOR: Star is just another-- star is just something you can put on the B to make it not look like a B. [CHUCKLE] You could put a twiddle-- I mean, generically, it started out as star being the vector. Star is like kind of a notation for vectors. But then, of course, here B0 star is a scalar, so-- [CHUCKLES]

AUDIENCE: OK.

PROFESSOR: Yeah. So we have the lambda B, and then here we have sigmas. That's telling us that we're already making some predictions about mesons, and baryons, for that matter, just from symmetry. Now, if you want to really make predictions, you'd like to look at a little more dynamics, and for that there's something that's kind of cool called using a covariant representation of fields. So I want to show you how that works.

So we're going to see how to make heavy quark symmetry predictions by building up some object that has good transformation properties. And this is in general something that's a more general lesson. If you have a symmetry, and you want to make some predictions that are just based on that symmetry, then you want to think about how things transform, and you can do tricks like the ones I'm going to show you.

So we're going to encode heavy quark symmetry in objects with nice transformation properties. The moral of this story is that it's much easier to take traces than to think about Clebsch-Gordan coefficients.

AUDIENCE: [INAUDIBLE]

PROFESSOR: Sure.

AUDIENCE: The way you've done it so far, when you throw in this v, should I be worried that you've broken [INAUDIBLE]?

PROFESSOR: Yeah. Angular momentum is conserved. It's the boosts that are broken. But we'll talk about-- we're going to come to that in a minute. Yeah, yeah. But the ang-- I mean, good question related to this discussion. The angular momentum is actually conserved. Yeah.

OK. But rather than try to make this a kind of derivation, I'm going to just tell you the answer, and then we'll talk about its properties. So I'm going to write down a field that annihilates the meson doublet for the ground-state mesons. Q is the heavy quark flavor.

pv mu star Q is something that annihilates a vector, B star-type meson. So Q could be charm, as well. And pv without the mu is annihilating the B meson if Q was B. This is the flavor, which you can think of as for this discussion bottom or charm. And so this guy is a bispinor. It's got the indices of Q q bar, but it's turned a vector field and a pseudoscalar field into a bispinor.

This thing here, this 1 plus v slash over 2, is projecting out the heavy quark degrees of freedom. This is a vector. So this is like something that, if you acted on a state, would get replaced by the polarization. Epsilon squared is minus 1, and v dot epsilon is 0, so that means that v dot this guy is also 0.

And this is pseudoscalar. So this is the vector meson. This is the pseudoscalar. So what are the transformation properties of this? So it's a bispinor. If you do a Lorentz, it transforms like a bispinor should under a Lorentz transformation.

This is a spinor Lorentz transformation, and v prime is the transformation of v, and x prime is the transformation of x. So that's how we want it to transform. We want something that has-- it's transforming like this Q q bar. We've got v slash on Hv equals Hv. That's one way of writing this 1 plus v slash over 2Hv is equal to Hv. And that says that there's no antiquark, so that's good.

We can also, just by studying the properties of this thing, work out that Hv B slash is actually minus Hv. That one is not something we built in. That's an outcome. So you could work this out, so I won't go through it. You just push it through the other side, and you can figure out that that's true. So we'll use it, but it's not something that directly-- it's an outcome of putting things together, not so much something that we built in.

If we talk about heavy quark and light quark symmetries, so SQ cross Sl, then it's 1/2, 1/2 under that. So if we work in the rest frame, which is this vr, which is 1, 0, then you have-- if you think about doing a symmetry transformation, you get a commutator, and basically what happens is that you would get-- you have two sets of-- you have a bispinor, so you have two parts of the spinor that are corresponding to the heavy quark and two parts that are corresponding to the light quark.

The heavy quark part is on the left, and then the light quark part is on the right. That's the spin transformations. And this is the right spin transformation for something that's spin 1/2, where the sigma 4 by 4 is the usual spin matrix.

Let me just say a bit more about heavy quark spin symmetry, which is part of our discussion there. So Hv is transforming. If you do a heavy quark spin symmetry transformation, it's transforming on the left. That's what I just was discussing. And if you do some transformation parameterized by some theta, which I just dot into the S over there, then this is the way we think about what the transformation gives.

So you could take this, and you could plug in our Hv field and then just do the Dirac algebra with the gamma and the gamma 5 and simplify this thing down and work out a transformation for delta pv and delta pv star. And what you'd find is that this guy in the rest frame here-- everything in the rest frame is that, and this guy, theta cross pvr star minus 1/2 theta pvr.

So if you talk about the transformations under heavy quark spin symmetry of the states, these are actually getting mixed up into each other. And that's what we already were saying before when we were talking about spectroscopy, that spin symmetry was relating the two. This is us now seeing that very explicitly, that if you make a heavy quark spin symmetry transformation, the vector guy-- so this guy should have a star-- and the scalar guy get mixed up.

And that's what spin symmetry allows you to do, is relate any statements about the v to statements about the v star. It's one thing it allows you to do. So the power of this Hvr thing is that it allows us to make heavy quark spin symmetry predictions easily because we've encoded the symmetry in this object. Kind of like our spurion analysis, we'll be able to use similar types of ideas to make spin symmetry predictions easy with this guy.

So let me give you an example of that. So an example is talking about heavy quark decay constants. And we can think about B mesons or D mesons. Correspondingly, B stars and D stars.

So think of the Q field here that I'm writing as a full QCD field, and the p field here with momentum p, it's either a B meson or a D meson. And just by Lorentz symmetry, much like one works out for the pion, one can write down a result that just falls from Lorentz invariance and parity for what the result for this coefficient is, because you also have to use time reversal.

And you could also write the p as mp times v if you want to make explicit that there's some velocity that this thing, that this p squares to the mass of that state. So this guy here is some dimension 1 constant. And if we just, in QCD, do the same thing for the other guy, which is the vector, we can do the same game.

Here we have to get a polarization on the right-hand side. There has to be linear in the polarization. That's the statement about a vector. And because of parity, it's not the gamma mu, gamma phi, but the gamma mu that comes in because we have spin 1 instead of spin 0. So there's an extra minus 1 from that. So instead of-- so we have a gamma mu here, and this is how we can parameterize the matrix element.

And this guy here, if we work out the dimensions, it would be dimension 2. So just in QCD, it looks like we have these two things, fp and fp star, that are different. But we know heavy quark symmetry relates the p and the p star, so they actually should be related. So you could ask the question, how are they related? Work it out. And if I asked you that question, the way to work it out, which I'll show you, is to use this Hv field.

OK, so there's two things we have to work out. We have to-- we're going to use the Hv field. I'll show you how. We also have to think about these currents that I've talked about here and writing them in terms of the Qv field, so let's do that first. So that's pretty easy. Just replace our field by the Qv field.

Now, if I just want to make a statement about heavy quark symmetry, then I can look at how this guy transforms under heavy quark symmetry because I know how Qv transforms under heavy quark symmetry. So [INAUDIBLE] D.

And so what I'd like to do is I'd like to encode in a general object-- much as we did in chiral perturbation theory, I'd like to encode in a general object that involves the Hv field something that has the same transformation properties as this current.

Much in the same way we could think about putting-- connecting currents that transformed in a certain way to a chiral theory, we're doing the same thing with heavy quarks. So let's pretend-- in order to do this, instead of thinking about covariance, we'd want to think about invariance. So pretend that the Dirac structure transforms such that we cancel that D. Then we have an invariant, and we can think about building invariance out of the Hv field. And it's very simple because if we think about that, gamma times Hv is invariant.

You only have to think about one Hv field because the number of heavy quarks is preserved. OK? So we know there's only one, and then the other thing that transforms is to gamma. And so that we can form an invariant, which is gamma H. So then if you look at this thing-- and then the sandwiching with this guy-- the indices of--

So this guy's got a vector index, and this guy's got a scalar index, but that's all encoded in our Hv field, right? It's got indices for those fields. So we're basically constructing things-- we're just basically constructing-- if we've already accounted for the indices here on the left-hand side of the equation, then we're just constructing things that are scalars.

So Lorentz covariance requires us to take a trace to get a scalar, and so we basically have trace. We can put anything we like here. And then we have gamma Hv, which is like the QCD stuff, and then the heavy quark part. So we just have to write down the most general x that we can think of. And there's not that much we can do because there's not that many vectors available to us.

So there's two things that we can do. We can write down something that just is a diagonal or something that's a v slash. But then we can simplify this guy because Hv v slash is minus Hv. See, I can take this, and this is a trace, so I can move it over to the other side of the trace, and if it's a v slash on the other side of the trace, then I can use this formula.

And so these two guys here actually just become, after taking that into account, one thing, which I'll just call a. So there's basically a unique thing that we can write down for the x. After we use all the symmetry properties of the Hv, we only have one thing. And so then you can just work out what that trace is.

Just take the trace. And you get a, and you get minus i v mu pv, or you get pv star mu, and this is what you get if you have gamma mu, gamma 5. And this is what you get if you have gamma mu, and if you have the other way-- so for gamma mu, you don't get any pseudoscalar, and for gamma mu gamma 5, you don't get any vector.

So the way that you should think about this is that in the matrix element and only in the matrix element, but between these ground-state mesons, I can really think that I've made some connection that looks like as follows. That Q gamma H v-- sorry, Qv is a over 2 times this trace of gamma Hv if I'm in the matrix element.

And then I know how to take matrix elements with these fields because they're the fields that actually are annihilating those states. So then Q bar gamma mu, gamma 5, capital Q. Now I'm using HQET states. They're actually annihilating HQET states. And I just use the formula that I've derived over there.

And I get these two results. So for HQET states, I have only one constant, a. And that's because heavy quark symmetry related the decay constants. And if I look at all the dimensions of the things on this side and count them up, I find that the dimensions here are such that this would be something that's lambda QCD to the 3/2.

So then I can try to go back and connect to my decay constants what this a is. And we know how to do that because I told you how to connect the states, and we talked about how to connect the currents, which is very simple. These states here are dimension minus 3/2, and that's basically what leads to the different power on the right-hand side.

So connect the states. That gives some extra factors of meson masses, and we find that fp is this universal constant, a over square root mp, and fp star is this a times the square root of mp star, and then we're getting correctly the dimension 2 or the dimension 1 quantities in terms of one universal a, OK?

So you can make predictions from this. So for example, you could take fB, and you could say, well, how big should fB be? Well, it should be, by dimensional analysis, now that we've figured out how to do dimensional analysis, it should be of this size. And that gives you something that's not too far off.

Or you could take fB over fD, and you could predict-- because actually we don't care whether it's a charm quark or a B quark. The a here didn't care about the flavor of the p and the p star state. So everything I said here could apply equally well to B states or D states. So if we make-- so the only place that it's coming in is when we have this explicit square root here. And just by power counting, we should say fB over fD is like mD over mB, which is about 0.6. OK?

So these are predictions that we can make based on symmetry. And then if you have a prediction based on symmetry, you could also talk about symmetry-violating corrections, and people do that kind of thing to make-- a is the lowest prediction, but you could talk about violating it with some higher-order a's, and those type of things are done.

The real place that this gives a lot of power, I have to mention, although we'll not talk about it in detail, is when you look at semileptonic decays. So if you look at the semileptonic decay of a transition of a B meson into a D, or a B meson into a D star, then you can again use all these symmetry tricks and again use this H v field-- and some of the reading will talk about this, these weak decays.

If you were to enumerate the number of form factors you'd have in QCD, you'd have six form factors. And once you go over to the heavy quark limit, there's just one. And it's actually normalized, too, because it's related to a conserved charge. [INAUDIBLE] spin symmetry to relate it.

You could use your heavy quark spin symmetry to actually relate it to the-- the operator that counts the number of B quarks. So these form factors, you don't know anything about their normalization. There's six of them. Take this limit, you get one, and it's normalized. It's kind of nice. It's called the Isgur-Wise function.

AUDIENCE: I have a quick question.

PROFESSOR: Yeah?

AUDIENCE: [INAUDIBLE] you said that that [INAUDIBLE].

PROFESSOR: Yeah.

AUDIENCE: [INAUDIBLE]

PROFESSOR: Why did I say that? So for example, imagine that I was to have done this for not the p and the p star, but for these other things that I was calling B0 and B1, or B1 and B2. Then when I would go through it, I would have not used the Hv field. I would have had to build a field that has those guys in it, which would have spin 1 and spin 2. And that would be a different matrix element. I can still do the whole same story, but really, this formula, this particular version of the current, is only true for these states, and for other states you'd have to use a different version. That's what I meant.

Any other questions? OK, so I want to just, in remaining two minutes, get started on something else that I'd like to talk about, which is related to the question that we left aside, and that is, what about alpha s corrections? And there's a couple of things here that are interesting. But basically, it boils down to--

We've already seen how to do matching and normalization group evolution, but now we have these extra labels, and so we can ask, what impact does having labels on that whole story of matching and normalization group evolution, what impact does it have? And we'll say that there are examples where it does have an impact. So--

HQET radiative corrections. So if you like, we're just thinking about the renormalization, as we should for any effective theory, think about what's the renormalization structure. And since it's a top-down, we can also think about matching. So we should think about renormalization of L. We should think about renormalization of external currents, like the ones we were talking about over there.

And if we match a full theory current, the way that you should think about what's happening from matching is that there'll be some Wilson coefficient that's generated by radiative corrections. So once we think about adding alpha s corrections to our story, there could be radiative corrections and Lagrangians-- we'll talk about that. But there also are nontrivial radiative corrections here in the currents, where they get multiplied by something that depends on virtual interactions having to do with antiparticles that we're integrating out, and that just affects these currents, the current relations, by multiplying them by some coefficients.

Yeah. So the first thing that actually differs, and it's because, again, there's no antiparticles-- that changes the nature of loop diagrams-- is the wavefunction normalization. So we have to go through that again. So if we do field renormalization, it's the same kind of story as it is in QCD. We have to consider this diagram, but we have to calculate it now in HQET. So let there be some external-- I'll call it p. I should call it k. Call it k.

There'll be some loop momentum, and we have q plus k. If we use dimensional regularization, then this loop integral looks like this. So using [? m ?] [? s ?] bar, there's some usual [? m ?] [? s bar-type ?] factors. And then there's the relativistic propagator, which I'm using Feynman gauge, so it's just q squared, and then there's the heavy quark propagator, which would look like that.

So the loop integral that I have to do here wouldn't have two relativistic propagators. It has one heavy quark propagator and one relativistic propagator, and it will give something different, and we'll talk about it in more detail next time, about evaluating this loop integral. So I wanted to do one loop integral for you. And basically the way I'm going to talk about these, the renormalizations and matching, is I'll do this loop integral for you so you have some idea what would be involved if I was asking you to do them yourself, which I don't think I'm going to do.

And then I'll just be quoting results for you and telling you kind of how the structure of the theory looks and where the corrections are going and talking about the important things that come out of the results, rather than dwelling on calculational details. But we will talk about one calculation because there's one trick that you should know if you ever encounter an integral of this type, since it's different than just using a Feynman parameter trick. And that difference comes about because this guy is linear in the loop momentum. This guy's quadratic. You have to know how to deal with that, so we'll talk about that next time.