PROFESSOR: --the back of the room. This course is being taped. It may appear in the future on EdX or MITx. It will appear on OpenCourseWare. So that means that by just sitting in the classroom, you're choosing to participate in this adventure of having the course videotaped. If you want to hide, you can sit in the back row. If you want to be part of the-- if you want to be famous, sit in the front row.

So I handed out the syllabus. And I'll just go through some of the things on here. So as a prerequisite-- so at MIT, we have three quantum field theory courses, 1, 2, 3. As a prerequisite, I've listed quantum field theory two. I don't want to stop people that are taking quantum field theory 3 this term from also registering for this course. But some of the things that you'd usually see in quantum field theory 3 are kind of important background things for this course.

So what I've done is I've posted on the website for the course my lecture notes from when I taught quantum field theory 3 last year. So you can look at that. And I'll assign some reading for that as background material for this. Some things actually were taught in quantum field theory 2 last term, some things like, for example, the beta function of QCD for normalization group. So those things are going to be important prerequisites for this course. And I'll assign some background reading so that you can remind yourself about those things.

If you have any concerns about your preparation, please feel free to talk to me. In terms of grading, this is pretty low key. We have five problem sets that will be roughly every two weeks. We won't start the first problem set until next week. And then there'll be a presentation at the end of the class. There's no exams, no tests.

The purpose of the presentation is basically, effective field theory is a very broad topic and there's no chance we'll be able to cover every possible thing about effective field theory in this course. In particular, a lot of applications we'll have to be brief on or we won't touch on at all.

So if you look at the syllabus on page three, there's a list of possible presentation topics. Each one of you will pick-- each one of you that registers for the course and wants to give a presentation will give a presentation at the end on one of these topics, prepare it, and stand up and lecture to the class. And the idea is that basically you will teach the class for half an hour or so about one of these topics, and then that way we will broaden the scope of things that we'll be able to cover. Even this long list is not complete. And you can also pick a topic that's not on this list if you talk to me about it.

So a little later I'll also give you some suggested references for reading about this. I just, at this point, give you the list. And we'll talk about where you might read about these things later on. I'll give you a more detailed handout. So even if you just look at this list, you kind of see the breadth of-- the idea of effective field theory, how broadly it impacts physics, and particle physics, in particular.

So you know, everything from finite temperature QCD, finite density, effective field theories in inflation, popular these days. Effective field theories of cold atoms, are also very popular these days. Non-relativistic QCD production of quarkonia, both in medium, out of medium. Relativistic super fluids. Conformal effective field theories. The list goes on and on. Lattice effective field theories. So it's a very broad topic.

And what I'm going to do in this course is I'm going to start out with the first half of the course teaching you about the ideas of effective field theory, some of the basic ingredients that go into all effective field theories and some more technical things that only show up in some effective field theories, but that are kind of providing you with lessons if you had to ever construct your own effective field theory. That's the idea of the first half of the course.

The second half of the course, I'm going to focus on a particular effective field theory-- soft-colinear effective field theory, which is an effective field theory that's close to my heart that's exciting these days because of its applications to the jet physics in the LHC. So we'll talk about that as the second half of the course.

So as I say on the outline, there's not really a textbook for this course. I suggest some text that may be useful, but they're not any one of them will be useful for more than 20% of the material, many of them for less than 10% of the material. So I don't necessarily recommend that you go out and buy all of them-- or buy any of them. I will try to post chapters of things on the website when it's required reading. And these books will be available in the reading room, so you can do it that way, if you like. If you're really serious about phenomenology, of course, you should own all these books.

So on the course outline, I also listed office hours. And I will adjust those when I find out all your schedules. And I also listed that [INAUDIBLE] who's sitting in the back, is a 10% TA for the course. He'll be doing some grading for us. He won't have office hours.

AUDIENCE: Thank you.

PROFESSOR: So I know that some of you have registered for the course. And some of you are here as listeners. If you are here is a listener, I have an add form and I want you to add it as a listener, just so that we have a record of you, partly because that affects the ability to offer this course in the future, also because we're going to use online tools and we may even use some online MITx tools if I find out that they're very useful for the course.

So I want you to be in the system. And I want you to be able to access that information if we end up going that way. So I think that's it. So as far as preamble, does anybody have any questions? OK.

All right. So let's start. So let's start with the big picture. I like to say that the big picture is that there's interesting physics at all scales. What effective field theory lets you do is it lets you tease out this interesting physics at all scales. So in particular, you can focus on a particular scale and find the interesting physics there using the tools of effective field theory.

Now that's a little different than how we teach physics. If you think about drawing a diagram from your freshman year to now many of you in graduate studies about how you learned physics, it was not by kind of focusing in on individual particular scales, but more from a kind of bottom-up point of view.

So I would draw a picture of how we teach physics in the following way. So you start out as a freshman. You learn classical mechanics. You learn E&M, classical E&M. You learn Newtonian gravity.

And then you build on these things. These things are your starting point and then you build on them. So you learn quantum mechanics and that builds on classical mechanics. You learn special relativity, which is building on both classical mechanics and electromagnetism.

At some point, you learn general relativity and quantum field theory. And quantum field theory is synthesizing quantum mechanics and special relativity, and then these two here.

So you think about learning physics from the bottom up in this picture. You learn these things first. And then you learn these things. And you sort of keep synthesizing, keep putting things together. We're going to be doing in this course the exact opposite of that. We're going to be taking one of these blobs-- in particular, this one-- and we're going to be looking deep inside it trying to make more and more specific field theories.

So if you like, if you want to draw it as a blob, we'll be taking quantum field theory and we'll be looking for derivatives of it-- or perhaps derivatives of derivatives of it, like this-- and figuring out how to take a very general theory, like quantum field theory for the standard model, and finding things that are more specific and in some ways more powerful than just having the original theory we started with-- more powerful in the sense of being able to do calculations.

So why do we want to do that? There's a couple of different reasons-- or why do we do that, since, as I've tried to convince you with the outline of the course and some of the presentation topics, that this is something that happens all over the place.

So as you go up in this chart, it actually becomes-- even though you have a more general theory and it becomes more beautiful and you can write down a synthesis of physics in fewer lines, it also becomes harder to compute things. So just as an example, if you just wanted to compute the energy spectrum of hydrogen, and you know very well that you can do that in quantum mechanics. And particularly it's a classic example, and fairly easy.

If you try to do that in quantum field theory, it's much harder, because quantum field theory, in some sense, has too much for that problem. So that's one example. Another is the elliptical orbits of the planets, which are easier in Newtonian gravity than in general relativity. And these are just two examples. There's many more where the field-- one of these blobs may be too general for actually tackling the problem that you want to deal with.

And by focusing in, which is what effective field theory allows you to do, you can get more ability to compute more accurately and in a simpler fashion. So what we want when we think about effective field theory is we want the simplest framework that captures the essential physics. We don't want to carry along for the ride a whole bunch of superfluous things that are not important for the problem we're trying to deal with.

But we also don't want to give up anything. So we're very demanding. So even if we're giving up something in our leading order description, we want to retain the ability to correct that leading order description-- order by order in some expansions-- so that we can make it as precise as we desire.

So I'll say that we can correct it, in principle, to arbitrary precision. So if you like, what we're doing is we're taking quantum field theory and we're expanding it. And the lowest order determine that description is an effective field theory. And that effective field theory may have different fields. It may have different symmetries. And it will certainly have ability to calculate in a different fashion than the original theory.

But we will keep higher order terms in that expansion. And therefore, we'll be able to correct it to arbitrary precision just by expanding to higher and higher order. So examples of this that even are familiar here-- non relativistic expansion, getting back to non relativistic quantum mechanics, or doing a post-Newtonian expansion in general relativity to go back towards Newtonian gravity. You don't have to stop at the first term. You can keep higher-order terms.

And in that way, you could calculate, for example, energy levels in hydrogen using a non-relativistic framework that encodes all the ingredients of quantum field theory. Or you could use-- you could look at the orbit of planets and relativistic corrections and general relativistic corrections by expanding this theory. So those are two examples.

So most of the examples that I've listed on the project list are the type of taking quantum field theory for the standard model. Some of them involve gravity, but most of them involve the standard model and expanding it and focusing in on particular degrees of freedom. So let's say that we've picked a physical system and we want to describe it. What are the things that we should do in order to develop an effective field theory? What are the steps?

I should say that I'm going to post my lecture notes. So if you don't want to take notes, you don't have to. If you'd like to take notes, feel free. But I will scan and post my notes. So the first thing you need to do is figure out what the relevant degrees of freedom are. What are the things that actually matter for the problem you want to study?

Sometimes that is easier than other times. Sometimes it's completely obvious. You want to study some low energy properties of the standard model. You get rid of the heavy particles. You keep the light ones. Fairly straightforward. Other times it may be tricky to actually determine what the relevant degrees of freedom are. And people in the field may even argue about what they are. So we'll talk about examples of both types here throughout the course. So it sounds trivial, but it may not be.

You also want to think about the symmetries. Sometimes that guides you in thinking about the relevant degrees of freedom. Sometimes these things go hand in hand. But that's certainly an important ingredient in developing the effective field theory. And you also have to be careful here, because sometimes you might have a theory that has no symmetry, or doesn't have an apparent symmetry.

But when you start expanding-- which is what I've argued you're going to be doing-- when you start expanding, you may have a symmetry suddenly appear. And we'll actually talk about several examples of that happening throughout the course, as well. So your effective field theory may have more symmetry than the theory you started with. Because you've neglected something, you could have more symmetry.

And the other important thing is to figure out what you're expanding in. And at the same time, what the leading order description of the theory is. What is the lowest order Lagrangian? And basically, these are the things you have to do to get started. If this is true, actually, independent of kind of what theory you're talking about, if you're doing quantum field theory, what this first one means is figuring out what fields you're going to be using.

The symmetry is basically guiding you about the interactions. If you have gauge symmetry, then of course you're going to write down something that respects that symmetry. That's going to tell you something about the interaction terms. And then finally, this expansion parameters goes under the rubric of what is called power counting.

And if you take those three things together and you figure out the leading order description, then you have an effective field theory. You write down he Lagrangian for it. You should try to use some color.

So if you thought about regular quantum field theory-- for example, for the standard model-- you'd also do these first two things. You figure out what the fields are. You figure out what the interactions are. But you don't think about too much about this question, about the power counting. And actually, more broadly, in the field of effective field theory, this idea here of power counting is very important. It's as important as something like gauge symmetry. It's really a fundamental thing about the whole framework that you're doing.

So in an effective field theory, power counting is just as important as figuring out things like symmetries. And in particular, just to make it sort of clear that it's important, I'll compare it to gauge symmetry.

It's really a fundamental ingredient in what you're doing and in the whole theory, because the power counting being consistent is actually necessary for the whole field theory-- effective field theory-- to make sense. OK, so what's the key principle? Well, there's a key principle of quantum field theory that we're using when we design effective field theories. And that is that we're insensitive to physics at higher energy scales.

So going back to Wilson, if we're interested in describing the physics at some scale m squared, we don't need to know the details of physics at higher energy. So at scales lambda squared that are much bigger than m squared, we don't really need to know the details of the dynamics that are going on there. We don't need to know the field content, necessarily, at those scales, or anything else about the dynamics.

And that is, in some sense, a key idea that makes the whole idea of effective field theory possible. I want you to make sure that you don't think of this too narrowly. In the way that I've written, it's actually a little bit too narrow, because I said that-- I've described it in terms of mass scale.

So I've kind of intuited in your mind something like a Z' boson or a W' boson, which is a heavy particle from some perspective. And maybe they treat that as a heavy particle and I'm interested-- and let me say it differently. Say I'm interested in a light particle, like the bottom quark. Then I can get rid of physics at a heavy scale, like the w mass.

Or if I think about how new physics impacts precision electroweak data and that new physics is heavy, then I can think about effective operators. That's kind of the intuition I've given you with the sentence that I've written here. But it's actually not-- that's not quite general enough.

And the reason it's not quite general enough is that this mentality doesn't always apply just strictly to math scales. If I say this-- m squared much less than lambda squared-- then you immediately think that you're expanding an m squared over lambda squared. And that, of classic effective field theory, that's exactly what you do. You expand in light scales divided by heavy scales. They're mass scales. They're invariant masses. They're Lorentz invariant quantities.

That's the most common thing that you do. We're going to be doing much more than that in this course. We'll be doing-- we'll be talking about cases where the power counting is in situations that are not simply m squared much less than lambda squared, but other things-- dimensionless parameters. So sometimes it will become more complicated. But still, this guiding principle that there's physics that's far away, in some sense, from the physics that you're interested in, that it can be removed from the theory when you're just talking about it, in that sense, this is more general.

It's really not just a strictly one-dimensional thing, as it would be if I describe it in terms of mass scales, but a more general principle. And I think that will become clearer when we actually do examples where it's not simply mass scales. We will, of course, start out by talking about mass scale, since that's the classic thing that most people think of when they think of effective field theory.

So can anyone give me an example-- just to try to get the class engaged here-- can anyone give me an example of an effective field theory that doesn't involve expanding strictly in mass scales, involves some other type of expansion?

AUDIENCE: [INAUDIBLE]

PROFESSOR: Well, OK. HQT you can say is just an expansion in lambda QCD over MB or MC, so not quite.

AUDIENCE: [INAUDIBLE]

PROFESSOR: [INAUDIBLE] Well, you've studied it. But correct answer. Any other thoughts? So another example would be something like non-relativistic QED. If you do non-relativistic QED, you're actually expanding in the velocity, not strictly in the ratio of mass scales, although you would think, for example, non-relativistic QED would be for a heavy electron, which is true-- a heavy, massive particle. That's not what the power counting expansion is in. It'll actually be in the velocity being much less than the speed of light.

And that actually plays an important role in designing the effective field theory, determining what the leading operators are. So even something as simple as going to higher order corrections in hydrogen involves thinking about-- thinking beyond this simple statement here.

Well, since we're on the topic of hydrogen, let me go into a little more detail there. So just to flesh this out a bit and to talk about some of the things that you have to be careful about, I'll phrase an example of this statement as the fact that we don't need to know about bottom quarks to describe hydrogen.

Well, that's good. When you took quantum mechanics as an undergraduate, you didn't have bottom quarks in your description. So if you did need them, you would have missed something. What did you have in your description? Well, in a quantum field theory language, you have this diagram. You had an electron and a proton with photon exchange.

And you also, when you thought about the binding energy, if we work in units where h bar and c is 1, which I will always do, then the binding energy is 1/2 ME alpha squared. And if you ask about the bottom quarks, the reason that you didn't need them is because they were suppressed. They weren't negligible-- completely negligible-- at least, at the level of how accurately we can measure this thing. Well, they're pretty small. They're at the 10 to the minus 8 level.

And that's because they come in suppressed by the mass of the electron squared over the mass of the bottom quarks squared. So how would you think of them coming in? Well you'd think of them coming in through some diagram, for example, where the bottom quark couples to the photon through a vacuum polarization like that. And this diagram, indeed, will give you corrections of this type.

Now, it's a bit more subtle than that. And that's because a diagram like this also has other contributions besides just these ones that I mentioned here. So the basic picture is, indeed, correct that we can neglect the bottom quark because it's giving small corrections. But there is one subtlety. And that has to do with the fact that we have to decide what we mean by this coupling.

OK. So from your previous courses in quantum field theory, when you learned about running couplings, you learned the diagrams like that one contribute to running couplings. And so the B quark, therefore, can affect the coupling if you worked in, for example, the MS bar scheme, since it contributes to the running of the coupling.

And in particular, you know for the electromagnetic coupling, if you ask about what that coupling is, it has a different value because it runs-- if you evaluate it at a scale like the W mass, then it's like 1/128, versus if you evaluate it at a very low energy, the electron mass or below, then it's the classic 1/137.036.

OK. So there's some change. And the bottom quark is part of what contributes to that change. Of course, other particles are contributing, too. So if we want to say this statement about bottom quarks and we want to state the conclusions more precisely, then we would do it this way. We would say if alpha is a parameter of the standard model and we imagine that we fix it at a high energy-- so we could imagine that we fixed it by doing [INAUDIBLE] on physics in an E plus E minus collider.

But some process that's a high energy process, we determine, say, for example, this value. If we take that attitude as how we define the parameter, then the parameter that actually matters for hydrogen, which is this parameter at the low scale, does depend on the bottom quark, because how we get from the high scale to the low scale depends on the fact that the bottom quark exists.

But we could also take a different attitude. And that is we could take a low energy attitude. So we could say, let's forget about doing high energy physics. Let's just do low energy physics and extract alpha of 0 from some low energy atomic experiments. And if that's the way that we define the parameter, then the value can be used in other experiments. And we never have to know anything about MB.

So we didn't really have to know about the high energy-- about the higher energy theory unless we actually were doing some experiments up there. Is there any questions about that? Good. So if we want to write an equation for that, what it means is that when you integrate out particles like the B quark, remove them from your theory, stop considering them, that it's not simply the case that you generate higher order terms in the series.

You can also affect what you mean by the leading order term in the sense of changing what you mean by the coupling. So if I write it in terms of Lagrangian, I would say that the Lagrangian for hydrogen, if we include the B quark, well, it's got our proton, electron, and photon. And let's keep the B quark. Alpha and MB are parameters.

If we drop the B quark because it's giving small effects, we just have these fields-- proton, electron, and photon. We get a different coupling in practice-- in principle. So you think of this as being a higher energy coupling and over there alpha prime being the low energy coupling. There's a lot of other things, actually, that if you think about hydrogen for a minute, there's a lot of other expansions that you've done. Hydrogen is a very fertile ground for effective field theory.

So let's do that. Let's make a little list of what we dropped when we thought about hydrogen. How much did we lie to you when we first taught you the hydrogen atom in a quantum mechanics course? Well, we didn't teach you about quarks. Why didn't we teach you about quarks?

And the reason we didn't teach you about quarks is because if you think about the typical momentum transfer in hydrogen, three momentum transfer, it's [INAUDIBLE] the mass of the electron times the fine structure constant. And that's much less than the proton size. So the typical photons that are involved in binding together the hydrogen atom just have much lower energy and they can't see inside the proton. They just see it as one overall object. And so we don't need to know about the quarks inside the proton.

So that was an expansion. It's also an insensitive to the proton mass itself. So the proton we keep as an object, but the mass of the-- again, the momentum transfer, ME alpha, is much less than the mass of the proton, which is of order of GeV. And so we expand in our treatment of the proton, as well.

And basically what this means is that the proton acts like a static charge. But proton mass wasn't showing up in our lowest order description of the energy here. It would show up in higher order corrections that we neglect. And again, it's because we're expanding. And that actually affects, when we design an effective field theory for this situation, how we would treat the proton, what type of Lagrangian we would write down for it.

And that will be one of our topics is to figure out how we treat heavy particles, like a proton, in this case. Another expansion that we did is we used the fact that the momentum transfer is much less than the electron mass, not just the proton mass. And that meant that the theory is non-relativistic and that's why we did non-relativistic quantum mechanics. If we wanted to do it as a quantum field theory, we would do a non-relativistic quantum field theory.

So already in something as simple as hydrogen, we have here three expansions plus many more, thinking about the particles that we neglected in the description.

AUDIENCE: So the second point [INAUDIBLE] and the alpha, how do you know that's not just the ratio of ME by MP rather than ME alpha?

PROFESSOR: So--

[INTERPOSING VOICES]

AUDIENCE: --electron was 1 [? GeV. ?]

PROFESSOR: That's right. So--

AUDIENCE: We would care about it.

PROFESSOR: Absolutely. So in some sense, I could have written ME here, and that would have been fine. If you take these together, then of course you could take a ratio and you'd get ME over M proton. The reason I wrote ME alpha is I was really thinking about the momentum, sort of the non-static properties, the dynamics. And the momentum of the photons, the largest energy is this. That's why I was thinking about it. But you're right that I should write ME over M proton, as well. As any other comments, questions?

So another point that I just want to briefly comment, which has to be true if everything I'm telling you is right, which has to be true because it's what we taught you, is that this whole description is true even though there's ultraviolet divergences. When you start doing quantum field theory, even if you do quantum field theory, in this case, for the hydrogen atom, you run into ultraviolet divergences where things are blowing up. Actually, even this diagram has ultraviolet divergences.

So before you regulate the theory, the bottom quark group is infinity. Once you regulate the theory, it's well-defined. And you can make everything well-defined. But you may worry that this diagram seems to be contributing an infinite amount rather than a finite amount. And the whole story goes through even in the context of having ultraviolet divergences.

That better be true, because, for example, if we had graviton loops, they would also lead to ultraviolet divergences. And so we're neglecting gravity. So it better that these ideas of effective field theory are not changed by having ultraviolet divergences. And we'll encounter that in our discussion later on.

OK. So that gives you a bit of a sense for how these ideas of effective field theory, you've been using them all along, whether or not you knew it. And we will, in this course, flesh out how we figure out some of these corrections, how we would actually compute them, how we would actually figure out how to even the leading order-- what the leading order description of a theory in cases where we may not know it or someone else hasn't figured it out yet. Those are the type of things we're after.

Now, if you talk about the categories of effective field theories, if you look at the list that I handed out to you or the list of things we're going to do in the course, then there's really, in general, two ways that effective field theories are used.

So the two ways are from the top down or from the bottom up. So we'll start with the top down. So in the top down situation, you know what the high energy theory is. I'm going to keep using this language of masses, where I have a high energy and low energy theory. And we'll think about it more generally when we come to examples where we need to think about it more generally.

So in this top-down case, we have a high energy theory-- say, the standard model-- and that theory is understood in the sense that we can write down the Lagrangian for it. But we're not satisfied with that. We find it useful to have a simpler theory to do some low energy physics, or even to do some high energy physics, where not all the degrees of freedom of this high energy theory are relevant.

So we're in a situation where we have some theory, which we'll call theory one, which is this high energy theory that we understand it that we can think about doing calculations in it. But we want to go over to some other theory, which I'll call theory two, which has less degrees of freedom. And we're making expansions. And that's a low energy theory. This is the high theory. This is the low theory.

So that's what we are in this situation of what we call top down, coming from the top, from high energy, down. So what do we do? Well, in this case, it's kind of nice, because we can actually use the fact that we know this theory one and can do calculations in that theory one to even think about constructing theory two.

So what we can do is we could just start calculating things in theory one and integrate out-- i.e. remove-- the heavier particles. And in doing that, we can do what's called matching onto the low energy theory. That means we can use this ability to do calculations in the high energy theory to find what the operators are of the low energy theory, just by direct calculation.

And also if there's new low energy constants that show up, we can calculate the values of those constants by using information and connecting them to the high energy theory. So in this case, we're able to use calculations really to construct the low energy theory. So just schematically, I start with some high energy Lagrangian and I go over to some low energy Lagrangians where there's an infinite series, which I've indexed by n. And that index is to denote higher order terms that are less relevant in whatever expansion you're doing.

So just to be general, I'll say it's an expansion in decreasing relevance of the terms. So if you're in this situation, then these two theories are, in some sense, describing common things. The high energy theory describes more than the low energy theory, because you've removed something in constructing a low energy theory. But the two theories have to at least agree where they overlap. And so they have to agree on certain infrared observables, which I will also often denote as IR for infrared.

The place where they differ is in the ultraviolet. So they might have different ultraviolet divergences. Most often, they will have different ultraviolet divergences. They don't have to agree in the ultraviolet. And actually, you exploit that to do things with the effective theory that would be hard to do with the full theory. We'll talk about how we do that later on.

So the fact that they differ is actually not necessarily a negative thing. It can be a bonus. And finally, you have to ask about this sum over n. Well, this sum over n is infinite. Goes on forever. And so you have to ask the question, when should I stop?

And therefore you have to look to experiments and see how precise they are, or just to your own perseverance and figure out what level do you want, what precision do you want in your description? So what n do you want to stop at? Sometimes experiment tells you to only do the first two n's. Sometimes you have to decide, maybe I only want the first one. If you're doing it for the first time, I suggest you stop at the beginning and let someone else to do the corrections.

This idea of doing this can be important for separating physics of the high energy theory. One example of this is in QCD. If you take QCD for just about any process, there will be some parts of it which were perturbative and some parts of it that were non-perturbative. And by doing this kind of thing where you expand, you could construct a low energy theory that only has the non-perturbative scale in it and removes all the perturbative scales.

If you did that, then just doing this procedure would allow you to figure out, what is the non-perturbative physics and what is the perturbative physics? You'd separate out, in that case, into operators. You'd separate out the infrared physics. You'd have operators built out of the infrared fields. And those operators would describe the non-perturbative physics. And you'd have some new low energy constants, which would describe the perturbative physics. And some of the examples that we'll do will make use of that probably.

So that's kind of a motivation, actually, for some of the examples in the standard model. So if you think about, for example, integrating out heavy particles like the w or the z or the top quark, one of the motivations is sometimes what I just said, to separate out perturbative and non-perturbative physics. So that's one example.

Heavy quark effective theory is also an example of this top-down effective field theory. In heavy quark effective field theory, you have a field theory for the B quark or for the charm quark but you want to describe things like the B meson or the charm mesons. Objects have non-perturbative physics as well as perturbative-- have mostly non-perturbative physics.

And in order to do that, you want to actually integrate out the mass scale of the charm quark or the bottom quark. If you integrate out the mass of the charm quark and the bottom quark, you go over to something called heavy quark effective field theory. But it can be done in exactly this way that I described to you, where do you start with the theory with the full B quark relativistic description and you actually just expand and figure out what the heavy quark effective theory is.

So non relativistic QED and non-relativistic QCD are also examples here. And soft-collinear effective theory, one of our main subjects, is also an example of this type where we can just start in QCD, do an expansion, and get the effective field theory.

So what's the other category? So the other category is from the bottom up. And typically in this case, you're interested in using effective theory logic. But maybe you don't know the high energy theory. You don't really know anything about it. Maybe we've never probed it. That's one way in which bottom-up effective field theory shows up.

Or it could be that the high energy theory is known but actually doing the matching calculations to integrate out the degrees of freedom to do those calculations explicitly could be just very, very difficult. Maybe it would be non-perturbative, for example. So if the matching is too difficult, then you may also want to be thinking in this bottom-up framework where you really just start by thinking about the low energy theory without worrying about what the high energy theory was, or without thinking too hard about the high energy theory, and in particular, without doing calculations in the high energy theory in order to motivate the low energy theory.

You need to know some things about the high energy theory, like you may need to know that it's [? llorens ?] invariant, that it has certain gauge symmetry, that it's not totally crazy. But you don't need to know it at the level where you would actually carry out calculations with it in order to construct the low energy theory. Instead, you think about the lower energy theory from the bottom up, where you can just devise it based on the symmetries, based on your power counting, and based on identifying the degrees of freedom.

So construct the series simply by writing down the most general operators that we can think of consistent with whatever degrees of freedom we have, and of course, consistent with the symmetries that we're imposing. OK. So the picture is that you don't know or you want to remain agnostic about theory one, but you still are interested in constructing theory two.

If you do this, then unlike the other case, the couplings that you have when you write down these operators, they all are multiplied by some couplings if they're higher dimensional operators and they're not constrained by gauge symmetry. Then all of those couplings are unknown. But you can fit them to experiment.

So the effective theory may still be powerful because you can make more predictions than the number of parameters that you have, like for hydrogen, where we have very few parameters and the effective theory, but we can make lots of predictions from non-relativistic quantum mechanics. Or it could be in the case where you imagine is too difficult that maybe you have to carry out the matching numerically, like with a lot of QCD. And so that would be another possible way of determining couplings.

And again, the desired precision tells us when to stop. So it's important that we have a power counting for this theory. But that power counting is in some sense defined irrespective of what the full theory was so that we can stop, even in the bottom-up case.

So what are examples here? Well, the classic example of this type is chiral perturbation theory, when you're thinking about a field theory for kaons and pions. And doing the matching onto kaons and pions from QCD is a non-perturbative process, so you think about constructing the effective theory just from low energy and from symmetries, knowing the symmetry breaking pattern, in particular.

And you construct the chiral perturbation theory without thinking about doing the matching explicitly. So that's one example. Another example of this type is actually the standard model itself. If you think about the logic that we used when we construct the standard model, it was exactly this effective field theory logic. We said, what are the relevant degrees of freedom? Electron. Quarks. W bosons. Listed them.

We wrote down. We said, what are the important guiding principles? Gauge symmetry. And then we wrote down the most general Lagrangian that we could think of. That was the standard model. OK, so it's an example of a bottom-up effective field theory. We don't ask questions about what's higher up.

Let me write down the leading order Lagrangian. And we can actually construct higher order terms in the standard model expanding in the idea that there's physics above the scales of the standard model and write down higher dimension operators and have a real standard model that has an infinite series. So the standard model is an effective field theory that has an infinite number of operators. And we'll talk about that momentarily as well next time.

So that'll be the first example we actually treat, is the standard model as an effective field theory. Another example is quantum gravity. If you take Einstein gravity and you make it quantum and you allow yourself to expand-- i.e. you say you're only interested in low energy physics. So you allow yourself to write down an infinite number of operators. Then you can also renormalize that theory order by ordering those infinite number of operators.

And so it's also an example of something that you can treat from this effective field theory paradigm. That was the last topic that I would actually be cover in this-- if we had a couple extra weeks of lectures, I would get to this. But probably we won't. So that's a topic that somebody might pick for their project at the end. That's a good presentation.

OK, so important stuff. Is there any question about that? Sits well with everybody, makes them feel good inside? OK. So far when we've been talking about this sum over n, we've been really thinking about expansions in powers. Some mass scale divided by some other scale being much less than 1. OK, that's what we've meant by it.

But when you have two scales like this, m and lambda, you also get logarithms. So it's not always powers. There's also logs that show up. And this comment I meant about-- that I made about ultraviolet divergences in the low energy theory, it can actually help you to understand those logarithms. Let's get you over here.

So when you treat the renormalizaiton of the low energy theory, as you know from quantum filtering, you have different types of divergences, power divergences. But the logarithmic divergences, in particular, are things that are playing an important role, often, in quantum field theory. And in effective field theory, it's the same thing. Logarithms can be tied to the re-normalization of the low energy effective theory and allow us to sum infinite series of those logarithms.

So often just the power counting and the re-normalization of the low energy theory will actually allow us not only to calculate the logarithms, but to think about summing up infinite series of those logarithms. OK. So that was kind of just elaborating on a point I made earlier.

And again, I should say here that I've said this in the language of there being two masses, m1 and m2 and w over MB. But this is actually true more generally again. So I'd actually make a claim that there's not any log that you've seen in quantum field theory that shouldn't be possible to figure out an effective field theory that allows you to understand those logs and predict logarithms at higher orders in perturbation theory.

There's not an example in quantum field theory that I've met that hasn't fallen into that rubric where some effective field theory description allows you to understand the logarithms. OK. So that's kind of a bonus. It's not the guiding principle. It's not what we're doing when we're expanding in powers. But it's something that we get along for the ride.

And maybe it would be the motivation if you see some logarithms and some process and you want to understand them. Maybe we would say, well, I'd like to understand what effective field theory would give rise to a description where I could understand those logarithms from a re-normalization perspective. And sometimes that's very useful, because maybe those logarithms are phenomenologically important and you want to make predictions about higher order logarithms, or maybe there's controversy.

When I was a postdoc, there was some controversy about a term that was alpha to the 8 log cubed alpha in hydrogen energy levels. 8 powers of alpha, 3 powers of logs. There was four groups. Two of them had got one answer. Two of them had got another answer.

And using the ideas of effective field theory, we were able to figure out that one of those groups was right and the other was wrong very clearly, because you could connect these logarithms to an effective field theory. And then the whole consistency of that effective field theory really allows you to connect this logarithm to other logarithms and really to build a picture for what's going on with the physics that makes it totally clear what the answer must be.

OK. So just to give you an example from my own past. So let's now turn to this question of the standard model as an effective field theory. So we have sum over n and we're treating this from the bottom up. So we're going to just talk about what the degrees of freedom are and then think about constructing operators.

And part of the job has already been done for us, because I'm assuming you have a background in the standard model, at least at the level of knowing what the Lagrangian is. And if you haven't, then you should look at the quantum field theory three lecture notes.

So the L0 here is the standard model, as taught in quantum field theory three. So [INAUDIBLE] be interested in as the higher order terms. But let me nevertheless remind you of what the degrees of freedom were in the standard model. So you at least know what the players are when we go to talk about L1.

So it's a gauge theory. So we have color across Su 2 weak across the un of hypercharge. And so we have eight gluons here, three week bosons here, and one guy here. So just to introduce some notation for fields, I'll call these guys with an index capital A running from 1 to 8, these guys with an index lower a running from 1 to 3. And B here would be the analog of a photon field for U1 electromagnetism. But this is the U1 of hypercharge, so it's B mu.

So we have gauge bosons. We have fermions. Let me do the fermions over here. So an important thing and thinking about this as an effective field theory is to note what the mass scales are. So maybe I should do that already here. Protons are massless. That's one combination of the weak and U1 boson.

Gluons are massless. That's these guys. And then there's the mass of the W. 80.42 GB the mass of the Z, 91.19. And for the first time in me teaching this course, we also know what the mass of the Higgs is. So let me just-- that's not part of the gauge theory, but I'll just list it there, as well, since it doesn't fit in with the fermions.

So fermions-- so you can see that these scales here are kind of similar. For the fermions, there's a broad spectrum of scales. And that's why I wanted to put them all on one board. So quarks-- up quarks, down quarks, strange quarks-- they all come in left and right handed [INAUDIBLE] and the gauge couplings are different for left and right handed, for the electroweak and U1 parts of the gauge group.

So there are six different flavors and both right and left handed. What masses do we have? Up quarks and down quarks are rather light, about a couple of MeV. It's hard to measure the light ones. It's a little easier to measure. Everything's going to be in MeV. I'm going to stop writing MeV. Oh, that's not true. I switched to GeV. Sorry. Everything is going to be in GeV.

Oh, sorry. That's the top quark. OK. So there's a pretty wide range of scales here from an MeV to 100 GeV. That's just the quarks. And then we also have the leptons-- three types of charge leptons, again, with a fairly wide range of scales.

So now I'm switching back to MeV, just to keep you on your toes. Whoops. Then we have neutrinos. The left-handed ones we've studied much more than anything else. And in particular, what we know most about the left-handed guys is mass splittings from neutrino oscillations. And these are pretty small numbers.

We also know that overall, these things are quite light, from cosmological constraints and otherwise. And we don't really know about anything like a sterile neutrino but that we can put bounds on its mass. So even within the standard model, there's a lot of different scales. And if you think about it from an effective field theory point of view and you think about it from the top down, the first thing you'd get rid of would be the top quark. And then you'd get rid of the W and the Z and the Higgs.

And then you would proceed down. The next thing to go would be the bottom quark, et cetera. And you could think about constructing an effective field theory by integrating out one at a time, getting a new effective field theory every time you remove a degree of freedom. You could take the standard model and expand in that fashion.

That's not the sense in which we are thinking about it. That would be the top-down sense of taking the standard model and deriving something else. We're thinking of it here in a different context where we have all this stuff. And we're actually interested in thinking about physics at higher energy scales, beyond the scale of the weak bosons, beyond the scale of the top quark, the things we're trying to figure out at the LHC, scales we're trying to probe. That's the attitude in this bottom-up approach.

OK. So the lowest order Lagrangian would be the gauge sector of the thermionic Lagrangian, the Higgs Lagrangian. And if we have right-handed neutrinos, we'd need a Lagrangian for them, too. So these are topics that come up in [INAUDIBLE]. I'm not even going to touch them at the moment. I can't give you a complete review, but just a taste, emphasizing things that are important.

So to give you a taste, I just write the other two down, which are the prettier parts, anyway. So we have field strengths for the kinetic terms for our gauge bosons. And the thermionic Lagrangian, I can write it as a sum over the left handed fields-- fermion, covariant derivative fermion. Add a sum over right handed fields. Fermion covariant derivative fermion where this covariant derivative is a covariant derivative with these gauge fields. So there's some gauge coupling, G1, for hypercharge, some gauge coupling, G2, for [INAUDIBLE] weak, and some gauge coupling, G, for QCD.

So what is the power counting? So we've just said what the degrees of freedom are and what kind of some of the guiding principles are-- the symmetries, the gauge symmetry. You learn much more about symmetries in quantum field theory three, so I'm not going to go into that. But those are basically the guiding principles in figuring out this L0.

What is it that we would do a power counting in here? So the power counting in this bottom up approach is related to what we left out. So we're expanding an epsilon here, where epsilon is mass scales in this standard model divided by things that we've left out of our description.

So in the numerator would be things like the top quark mass, the W mass, Z mass, Higgs mass, all the mass scales of the standard model. In the denominator, well, certainly something like M plank is left out of our description here. If we had some grand unified theory, that goes in the denominator. If we have supersymmetry and we broke it, that would go in the denominator.

So from this effective field theory point of view, any physics that we've left out of the standard model description is anything that generates a higher energy scale. That goes in the denominator. And this is what we expanded. So even not knowing something about what this physics is, we can come up with a universal description-- a universal L1-- that describes corrections beyond the standard model.

And what will be describing that physics is higher dimension operators-- operators beyond dimension four. But they'll be built out of standard model fields. So kind of from your teaching of quantum field theory, maybe perhaps the idea that these two things are connected may be clear to you, but it's something that we will actually cover mostly next class, actually.

Some part of the beginning of next class, we'll make this connection between the fact that we want to expand in that epsilon and the fact that we can do-- in doing so, we get higher dimensional operators. We'll make that absolutely clear. That'll come next time.

So in the remainder of today, let me just address one final point, and that is the idea of what it means to have a renormalizable field theory. So in our description of the standard model that I gave here, I mentioned symmetries. I mentioned degrees of freedom. I didn't mention renormalizability. So what does re-normalizable mean?

So the traditional definition of what renormalizable will mean would be the following. You would say a theory is renormalizable if at any order in perturbation theory in this quantum field theory the UV divergences can be absorbed-- so there's UV divergences from loop integrals. If they can always be absorbed into a finite number of parameters, then you'd say the theory is renormalizable.

But that's a traditional definition. And we will use a more general definition here. Certainly this was a guiding principle when people constructed the standard model. What is the effective field theory definition of this? It's a little more general because it brings in the idea of doing power counting.

So the effective field theory's definition allows for the possibility of having an infinite number of parameters. But at any order that you truncate the theory, there should only be a finite number. So it says that a theory must be renormalizable order by order in its expansion parameter.

Well, if there's more than one, it's expansion parameters. So even just this sentence alone tells you why power counting is such an important part of the effective theory, because the effective theory to make sense as a renormalizable quantum field theory needs to know about its expansion parameter. We're saying that it's renormalizable, that we can make sense of the theory, absorb all the infinities, only order by order in expansion parameters in general.

So it could be that you do some calculation, you counter some divergences. But if they're higher order in the expansion, then what you need-- and you have a power counting that tells you that-- they would be absorbable into some operators that you haven't even written down, you can just drop them. So this definition allows for an infinite number of parameters that are needed, for example, for normalizability, but only a finite number at some fixed order.

Now, if you take this logic that I just said to you, that you could think about things more generally, then you can ask, well, what was the point of thinking about the standard model, where the theory turned out to be renormalizable in the traditional sense? How does this fact, which is just a subset of this case, but an important one, how does it fit into this rubric from an effective field theory point of view?

And so the way that that fits in is as follows. It could turn out that your L0 in your expansion is renormalizable in the traditional sense rather than this more general sense. And if that's true, what it means is that you don't see the higher energy scales from your lowest order Lagrangian.

So we do not know directly about lambda new from just looking at L0. And that's what happens in the standard model. We don't really know precisely what the high energy scale should be just from studying the effect-- from studying the leading order Lagrangian. This will not always be the case.

Sometimes we'll be in a situation where when we study the effective theory at lowest order in the Lagrangian, we really find that even in order to make sense of that as a quantum field theory that we need to-- that there's a scale that gets generated and it's part of our expansion. And there's some terms that we calculate with L0 that end up being higher order in our expansion. And we can't renormalize the theory unless we actually include higher dimension operators. Chiral perturbation theory is an example of that type. And that's an example that we'll treat.

So it's not always the case in the standard model, whether it's renormalizable in a traditional sense. That's a special case, though it's an important one. So questions? OK, so hopefully this has been partly a review of some things that you've thought of before, but putting them together, perhaps in a nicer package. And we'll continue next time talking about the standard model as an effective field theory. What can be gained from that? How do we construct the operators? And we'll keep going from there.