IAIN STEWART: OK, so let me remind you where we were before spring break. So first of all, there's a posting with the project list. Feel free to pick a project as soon as you're ready. Send me your choice. Problem set number 3 due today. Problem set number 4 is posted this morning. It will be due in two weeks, plus two days so, two weeks from now there's actually a holiday, so it will be due on that Thursday.

Last time, we were talking about two-nucleon effective field theory. So this was a nonrelativistic field theory. Both of these terms here in the action were the same size. They were the same relevance. And we talked about that last time. So in HQET, it would just be the first term, but in this case of two nucleons, we actually need the kinetic term, as well, and both of these terms are equally important. And that basically means that in this type of system the time derivatives and spatial derivatives are counted differently.

But you can organize the field theory using equations of motion so that these are just local operators with spatial derivatives. So you basically deal with the time derivatives once and for all, and you could always use equations of motion to eliminate them there. And so you just have really this one time derivative.

This theory is particularly simple if we neglect the relativistic corrections, and that's because it's not-- because it's a nonrelativistic field theory, we just have these bubble diagrams. And they're just contact interaction, so they all decouple from each other, so at N-loop order we can just write down what the loops are.

We talked about the fact that, in terms of power counting things, where it could be a little subtle, and that was related also to a power divergence that occurs in this diagram, and we could track that power divergence by using particular schemes that gave this extra mu r term, and I won't repeat all that story. But that led us to adding a finite counterterm to this bubble diagram to track the power law divergent term, and that allowed us to come up with a power accounting that would take into account systems where the scattering length is large, or in particular, the scattering length times momentum is greater than 1, and in these two-nucleon systems that's exactly the scenario that we're in.

So what we then had was a power counting that assigned the very first operator to scale like 1 over a power of momentum. So not like 1 over the power of a large scale, but actually 1 over a power of something small, the momentum but we're interested in studying. So it's enhanced in the power counting that we wanted to adopt.

And that meant basically that you had to treat this one coupling, c 0, nonperturbatively. You summed it up to all orders, so the term here comes from adding up all the diagrams with any number of bubbles, and that all give this first term. And then the rest of the couplings are treated perturbatively, the higher derivative term. So a term, c 2, which comes with grad squared, and that gives this extra p squared. And the numerator here gets treated perturbatively, and there's this term, and then there's higher terms.

And that's how we could organize this theory, and this sums up all these powers of a times p, these denominators that I'm showing you there. So what I want to do for the first part of today is just really continue with this effective field theory and talk about a few more of its features. First I want to start with symmetries, and then we'll talk about something called the deuteron. Yeah?

AUDIENCE: I have a question about the relevance of the kinetic term.

IAIN STEWART: Yeah.

AUDIENCE: So other than getting pinch singularity in your integrals, is there, I don't know, a more power counting-friendly kind of argument for [INAUDIBLE]?

IAIN STEWART: Yeah, so you can think about it like-- really, what we're doing here is nonrelativistic quantum mechanics. And if you think about nonrelativistic quantum mechanics, you think about, like, the virial theorem. You're used to thinking about the fact that kinetic energy and potential energy are equally relevant, right?

So from a physics standpoint, if I didn't want to talk about diagrams or I just wanted to make a physics argument for why that would be the right thing to do, I could simply tell you, I want nonrelativistic quantum mechanics at lowest order. This is a system with two heavy particles, so that's which the physics should be. If you start with the wrong theory, then you run into problems, and that's kind of what we saw if we started with an HQET-like reasoning. Then we see this pinch singularity, which is resolved by the kinetic term.

AUDIENCE: OK, then, why is it that nonrelativistic quantum mechanics has the form that it does, then?

IAIN STEWART: Why does nonrelativistic quantum mechanics have the form that it does? [CHUCKLES]

AUDIENCE: I know, I just--

IAIN STEWART: Yeah, so think about it from the following point of view. So when we just had this term, what was the particle doing? When we just had the partial t term, it's like a source. We had a single source sitting there, and we were interacting with it with some light stuff. That was HQET. Here we have two sources, and they have to interact with each other.

And the problem is that two sources interacting with each other, really, the kinetic energy becomes a relevant thing. It's not-- I mean, it's actually a little more subtle than that, if you really want to go into it in a gauge theory. There's sort of situations where these two heavy sources act like two heavy sources, like HQET, and there's situations where they wiggle.

And the situations where they wiggle are exactly the things that the Schrodinger equation is giving you. If you like-- yeah, so to say it precisely, the potential between the two heavy particles, for the purposes of figuring out that potential, you can treat them as static, as fixed. But then, once you have that potential, these bubble diagrams are kind of the way in which that potential gets iterated into the thing that you would get from solving the Schrodinger equation.

So solving the Schrodinger equation, you're effectively treating the potential to all orders in this series, exactly this bubble series. And when you solve that you need to include the kinetic, this grad squared over 2m, for the same reason that it's a relevant thing to include in quantum mechanics. OK, good. Good project choice. I urge you-- no. [CHUCKLES]

OK, so let's talk about some of the symmetries of this theory, which are kind of interesting. So the first one is that this theory actually has a nonrelativistic conformal invariance.

So what is a nonrelativistic conformal invariance? So this is an extension of the Galilean group, rather than the Poincaré group. So we have the usual translations. We have rotations. We don't have Lorentz boosts, but we do Galilean boosts for a nonrelativistic system, like this. So what would those be?

So under a Galilean boost, time is not changed and the spatial coordinates are changed. And then there's additional generators that you have in a conformal invariance. One is a scale transformation, one generator. And the way that the scale transformation works is as follows. You rescale the coordinates by some-- let's call it e to the s. But you have to rescale time differently, by e to the 2s.

And the reason that you have to do that is exactly because these two terms should be treated the same. So you have to rescale time twice as much as you rescale the spatial derivatives, if you like, so that's what gives rise to the 2 there. And that just goes to the heart of sort of counting time and space differently.

And then, finally, there is a conformal generator. And there's actually only one generator in this nonrelativistic case. And basically what this generator corresponds to is a shift of inverse time and then a corresponding change to the coordinates. And if you take all these things together, it's actually something called the Schrodinger group.

And the theory we've been talking about actually has this nonrelativistic conformal symmetry, if a goes to infinity. So I'm not going to spend a lot of time talking about this, but just to give you some of the ideas of what is going on. If you look back last time at our solutions for this coupling constant, and you take a goes to infinity, that's a perfectly smooth limit. You can do that.

And then the coupling, c 0 of mu, has no free parameters. It's just given by 4 pi over m mu. And that corresponds to sitting at this-- what I call-- what we last time saw was a fixed point of the beta function. And at that fixed point, there's enhanced symmetry, which is this conformal symmetry.

And basically what you can show is that, if you scale here mu just like it was a momentum-- so mu goes to e to the s mu-- then the Lagrangian is scale-invariant. My notation up there would be minus s.

OK? And that is basically kind of the same thing as making it into a relevant coupling. Once it's got no free parameters, then you can sort of see that, if I have a certain scaling that makes the free theory Schrodinger-invariant, which is-- so the Schrodinger group is the symmetry group of the free Schrodinger equation, so just this term and this term. Then the question is, what interactions can you add that would preserve that symmetry?

And I'm just telling you that, in terms of just the scale transformation, you could-- if you are able to scale the coupling in some way, in particular to make it a relevant interaction, then that will make it a scale-invariant theory. And so the ability to scale this coupling because you scale this mu allows it to be a scale-invariant theory. It's a little harder to see that it's conformal, but you can work that out, too.

Scale-invariant theories tend to be conformal-invariant, and the same is true of this nonrelativistic conformal invariance. So if you sort of add up the bubbles that we were talking about, then you find in general in amplitude, which is this thing that has a square root-- but if I'm not particular to what frame I'm in now, if I just write it for a general frame-- before we write it for the center of mass frame, now let me write it for a general frame. Then it would look like this.

And that thing goes like 1 over p, so it has-- and again, e's scale differently than p's, so this thing has the right scaling for a scale-invariant amplitude. And it turns out, if you do work out the conformal transformation, it's also conformal-invariant. And I'm not going to go through that, but I've posted a reference to show you how it works. And basically what this leads to is a cross-section, which is 4 pi over p squared, which is the scale-invariant version of a cross-section, where the cross-section units are just set by p, so there's no other scale in the problem.

OK? So that's kind of interesting, and one can make some predictions with that. This is actually something that's also investigated these days, both from a string theory point of view as well as in cold atoms, so that's why conformal symmetries are always fun to play with.

There's also another set of symmetries that this theory has in this limit, which is also interesting. And that is, like HQET, there's an enhanced spin symmetry. This time, the enhanced spin symmetry comes about by combining spin and isospin. And this is a symmetry that's known as Wigner's SU(4) symmetry.

So Wigner-- I don't know how many years ago-- noticed-- long, long ago, Wigner noticed that nuclei tend to have a symmetry, which is a combination of spin and isospin, into a full SU(4). And we can understand actually partly where that comes from this effective field theory. So what is this SU(4)?

Think of it as just transforming the nucleon field under a way where you could mix up the spin and the isospin. So if sigma is for the spin, tau is for the isospin, then some combined transformation where I can mix up the mu and nus-- we just have some set of generators where I label them by mu and nu. Sigma mu here is 1, comma, sigma vector, tau nu, 1, comma, tau vector for the isospin.

If you just take the 1 from both of them, that's baryon number, and that would turn the SU(4) into a U(4), so you can remove that generator if you like. So there's a U(1), as well, but the nontrivial part, where you take either this or that or both is an SU(4). So we can think about this symmetry with our theory if we write it as follows.

So take the interaction terms that we've been talking about and write them in a slightly different basis. Just take these two-to-two scattering terms, which had no derivatives. And I'll write them in a slightly different basis. Before, we wrote them in sort of the physical basis, where we had 3S1 and 1S0. That's what we did last time. But there's other possible bases, and I can write them in this basis, singlet triplet basis.

And the advantage of this basis is this thing here is just SU(4) symmetric. And if you want to write down in relation to the previous basis, you can work that out. I'll just tell you what it would be by writing down what the coefficients here are in terms of the ones we had before.

So if we go back to the basis where we had coefficients for the 3S1 channel and the 1S0 channel, then this c 0 s becomes this combination, and c 0 t is this combination. Now, in nature-- so one way you could have SU(4) symmetry would just be for the scattering lengths, the physical parameter, c 0 1S0 and c 0 3S1, to be equal.

But in nature, they're not equal at all. In fact, their scattering lengths have different signs. There's no sense in which they're equal. So what is the sense in which we have this symmetry? The sense in which we have this symmetry is the fact that if the scattering lengths are both infinity, then both couplings are just this. So the fact that the scattering lengths have different signs but are both large makes them close to this fixed point, which then gives this symmetry.

So because of what I just said for a goes to infinity, c 0 s, which is sort of equal-- some symmetric combination of these two that's become equal-- would just become 4 pi over m mu, and this is for a 1S0 and a 3S1, both going to infinity. This goes to minus infinity. And then the triplet one, if we just work out what it is from our formulas for those other couplings--

We can just work out the combination from formulas we had last time, and we see that as the a's go to infinity, this coupling vanishes. OK? So when we're in the limit where both of the couplings become equal, or if we use this basis, where this coupling becomes 0, we just have this N dagger N squared operator. Then we have this SU(4) symmetry.

One of the problems on this new problem set is to think about to what degree you still have that symmetry when you think about higher body operators, three-body and four-body. And there's actually some very nice little group theory arguments where you can basically figure out to what degree you have that symmetry in the higher-body operators, and it turns out that it's also present there.

And that is one way of thinking about why Wigner's symmetries exist in nature. It's not that we're in any sense having scattering lengths that are similar in these two channels, but since they're both large we're close to the same-- we're close to these fixed points, which have this symmetry. OK? So it's like HQET, in a sense, where we didn't have heavy quark spin symmetry when the masses are finite, but if the charm mass and the B mass are both infinite, then there's this new symmetry that pops up. It's kind of like that.

All right, so rather than talk about the symmetries in any more detail than that, what I wanted to do is actually do something a little different with this theory. So there's lots of things we could discuss here. But the thing I want to discuss is actually treating a bound state in a field theory. This field theory has a very nice property that we can calculate everything.

So we can actually see how you're supposed to treat bound states exactly because we can actually just look for the pole that corresponds to that bound state in this theory. That's what we're going to do. So usually if you have a bound state, it's either something that's nonperturbative that you have trouble dealing with analytically, like in QCD, or Coulombic-- even then, the calculations get kind of hairy, although you can deal with it there.

But here, the calculations are very simple, so we can really follow all the steps of properly dealing with a bound state. And the bound state has something to do with nature. It's a state of nature called the deuteron. So the deuteron is a bound state of a neutron and a proton. It has isospin 0, spin 1, so it's in this 3S1 state that we talked about.

And when you want to look for a bound state in field theory, what you do-- you may have heard of this from the lattice point of view. What you do is you write down some interpolating field that has overlap with that state. So what would that be for us? That would be this operator that we called p i 3S1, which had the quantum numbers for i equals 0 and s equals 1. And we can build an interpolating field out of our N transpose N.

And actually, any interpolating field that we write down here is equally good. The lattice QCD people like to tune what they write down in order to improve the overlap with the state. But for our purposes the simplest possible thing will be enough. And then you can ask the question, is the deuteron in the theory?

And to answer that question, you can just look for the pole that would correspond to that state. And the advantage of this theory is that we can just look for that pole analytically. So let's do that. So I'll write down a Green's function for the two-particle state, which is just the time-order product of two of these interpolating fields.

And then ask the question, if I look at that time-order product, is there a pole that would correspond to this state, i.e., somewhere is there a pole which would behave as follows? Some kind of z factor, and then, in this case, because of the way we've scaled energies, we would just be looking for a pole that's when you're in the energy-- at some negative energy corresponding to the binding energy of that state.

So remember that we pulled out the mass of the nucleon. That wasn't in our Lagrangian. So really, if you think about what the sort of energy of this state is, it's two nucleons, so it's got the energy of those two nucleons, but then there's a little bit less because it's a bound state. And this is the amount by how much less. And so the energy for the pole is negative. E bar is less than 0 because it's much less than the two nucleon mass.

So this E bar is the two nuclear center of mass energy, which in the conventions we were using last time-- we had energy E over 2 for each of the nucleons, so the sum would be E. There's the kinetic term, as well. So what are the diagrams that we would consider to do this?

Well, at lowest order it's just our friends the bubble sum. And in fact, when you go to higher orders, we also had bubbles there. So to answer this question, we insert the field. We create two nucleons. And then they can interact with c 0's, and then we annihilate them with our interpolating field. So these are just our c 0 bubbles.

OK? And those are very easy to compute. It's just the geometric series that we already computed. And instead of having two nucleons in and two nucleons out, we just have this deuteron state in and deuteron state out.

OK? So the Green's function starts with one of these and then just iterates into a geometric series, where sigma I could define as two-particle irreducible c 0 graphs, so sort of taking out the c 0 iterations, so the sigma 1 that I'm talking about here is just this bubble all by itself. And that's something that we computed before and in these renormalization schemes that we like. Just comes out to be this.

So there's an m out front, m nucleon. It's not-- that's an m, not a mu. And then in terms of this E bar, it just comes out to be that. So you could think of making a positive energy by taking E bar to minus E bound, which is less than 0. And then the square root would be square root of m E B. And it's convenient to define that to be something, so I'll call it gamma B. So that's some quantity greater than 0.

And with this little notation, this square root of minus E bar, minus i 0 is equal to minus i p. And we're looking for whether that would be sort of going over to this gamma B. Sorry, so this is equal to gamma B.

So gamma B is just like a momentum version of the bound state energy absorbing the extra i. OK? This is what it would have been previously, but now it's gamma B. So we know what G was. We already computed it earlier. G was just proportional to 1 over a plus i p.

And now if I just make that replacement, which is why I wanted to derive that, because I wanted to just see how I would replace the i p, it becomes 1 over a minus gamma B. So gamma B is a kinematic variable. It's like a momentum. But it's the right kinematic variable to be talking about for the bound state. It's a positive thing. And you can see there's a pole.

OK? So there's the pole corresponding to the deuteron state. It just corresponds to gamma equals 1 over a of the 3S1, which in nature is like 36 m e V is greater than 0. If you looked at the other channel, the 1S0 channel, then the scattering linked with negative, so then both terms would be negative, and there's no pole.

OK? So for the 3S1 channel, we have a pole. That's the physical deuteron state. For the other channel, there is no pole. And the binding energy that you would get from this is gamma B squared over m, our conventions, which, if you calculate it, is 1.4 m e V, and that's not ridiculously different than the real binding energy of the deuteron, which was 2.2 m e V. OK? It's not-- it's the right order of magnitude.

All right, so that's the deuteron. It exists in this theory, and we could perturbatively correct towards this correct binding energy. OK? So this deuteron really exists in the field theory, and that means we can calculate properties of the deuteron with this field theory. We can calculate scattering processes that involve this bound state, and it's no more difficult than doing calculations that we would normally do with free fields.

And we just have to-- if we want to do something with the deuteron, we have to go to the right pole. That's basically what it boils down to. So in order to show you really how to do that, I'm going to give one example of calculating something nontrivial that involves a deuteron.

So we'll calculate a form factor for the deuteron. So this is a nice example, if you like, of really showing you how to use the LSZ formula, which you learned about it in a very sort of abstract way at some point, how to use it for bound notes. Because here we have a bound state exists in the theory, and if we want to derive properties of that bound state, we'd better properly use the LSZ formula for that bound state. Let's see how that works.

OK, so what we'd like to do, so we'd like to have-- we have some deuteron state, and we'd like to take the matrix element with the electromagnetic current. And if you work out what possible form factors you can have for this matrix element, there's actually three. And I'm not going to spend much time talking about the sort of different couplings with spin and stuff that correspond to these different cases, but I'll list them for you, at least, provide you some additional references if you want to dig deeper into this.

So there's a magnetic, electric, and quadrupole form factor. The charge of the deuteron tells us that this electric form factor at 2 squared equals 0 should be 1. The magnetic form factor is like the magnetic form factor for an electron in the sense that, if you go to q squared equals 0, then that's giving you the magnetic moment. But here it's the magnetic moment of the deuteron, not the magnetic moment of the electron.

That's a nontrivial number in this case, whereas here it's just 1 because charge is conserved. And if we wanted to think about putting electromagnetism into the nucleon theory, which we haven't so far, then we would turn our derivatives into covariant derivatives. So D mu nucleon-- we have to write down a charge matrix that couples-- that takes into account the proton has charge but the neutron doesn't. So this is that charge matrix for electromagnetism.

OK? So it's not difficult. So that's easy enough. Just change your derivatives into covariant derivatives for both D 0 and D partial and D vector. There's also some additional operators that you have to write down. So there's operators, for example, that are just contact interactions with the electric magnetic fields in principle, and the first such one that shows up, just by way of sort of telling you the range of possibilities, would be something like this, where you just have a sigma dot B magnetic field insertion. This is the magnetic field for electromagnetism.

And so if you want to think about the type of Feynman diagram you'd draw for this, you'd have four nucleons, and then you'd just have a photon coming out, right? And if you want to think about where that could possibly come from, there is one really easy thing to think about, would just be to have a pion exchange, and maybe it's pi minus. And the photon couples to the pi minus because it's got electric charge.

And when I integrate out the pion, then I just get something that looks like this. It could be any exchange. It could be a real minus, or anything would give an operator that looks like that. OK? So there's additional contact interactions involving electromagnetism, and you gauge the derivatives. Then you can put electromagnetism into the theory. It gets a little more nontrivial then.

And that's what we need to do if we want to talk about electromagnetism coupling for the deuteron. But I'm mostly interested here in how the LSZ works. So what does LSZ say? So LSZ says that if I have this state, p prime, J out, J mu electromagnetism, p i n-- i is like a spin index because we have a triplet under spin-- then this is how you should calculate it.

So q is like the-- well, I guess I'm in momentum space all of a sudden. Well, no. q is just p minus p prime. E and E bar are related to the p and the p prime also. And what I do is I have some z factor out front, which is the bound state z factor. LSZ tells me to be careful about that z factor. These guys here are just two-point functions, and that's because I have to truncate the lines. This is truncation by two-point functions, but they're two-point functions for the bound state, so two lines. And then this is the three-point functions.

OK? So for the three-point functions, what would the time-order product be for that? It's a time-order product of three things. So the two-point function we already defined. It was just the time-order product of the two deuteron fields. And the three-point function has an additional electromagnetic current, which we would formulate-- you know, all the couplings here are this way that I was talking about over there.

So in terms of diagrams, which are easier to think about in some ways than time-order products, we have the two-point function, which, if I just define all possible two-point functions as G-- and this is the thing that was our two-particle-- was our irreducible c 0 contributions, plus iterations of c0's, right? Since the c 0 is a nonperturbative coupling, we have to iterate, and this is what we already said you could define as the summing of that geometric series, and then just define it as something which is the full Green's function in terms of this irreducible piece in the usual sort of way.

And then, for this other G, I'll draw a blob, which I'll also call G, but with an additional photon hanging out. Those are all three-point functions. And in a kind of a similar notation, I can have kind of an irreducible piece, and then I can have iterations with c 0's.

Et cetera. And if I sum up those irreducible diagrams, I can write the full Green's function in terms of the irreducible pieces, again, in a similar sort of notation. It's just gamma, which carries the indices, and then there's just a couple of-- now there's geometric series on both sides, so when I sum it up I get that.

OK, so this is giving you the ingredients that we need to plug into this formula. We need G inverse. This is G. The two-point-- we need the G mu i J, which we can write in terms of the irreducible guy, like that. Then the last thing we need is this bound state factor.

AUDIENCE: [INAUDIBLE]

IAIN STEWART: Yeah?

AUDIENCE: That cofactor has a contribution that you just erased?

IAIN STEWART: Yeah, what I really-- I'm kind of using a shorthand here with this t product for all the diagrams that come from what I was talking about over there. Yeah, I kind of wrote it as an electromagnetic current, but really what I mean by it is sort of all the ways that electromagnetism can appear in the effective theory.

So you can think of formulating a current in the effective theory by just sort of looking for one photon, lopping it off, and then writing down that operator, all the operators, and that's really what I mean by this J mu. Any other questions?

OK, so what's the final thing we need, which is the bound state z factor. So that's-- let me look back at this kind of form that we had for G. We said it's i z, but now let's denote the fact that z is really the principal function of E bar, and it's really the residue of the pole. And what z is, z that appears in the formula over there, it's z at the pole, so that's where I set E bar equal to minus B D in the numerator. So this is the residue of the pole.

And with a little bit of manipulation, we can write that as the following. We can relate it to a derivative at the pole. OK? So take the inverse, take a derivative to sort of get just this factor. Take an inverse again. You can extract off that residue, killing off some other pieces by going to the pole. OK? So I won't go through the derivation of that little manipulation, but it's a straightforward consequence.

And if you put in kind of the formula that we had over here for G, and if you do that, then you can write a formula in terms of sigma. And after that kind of manipulation, it's-- again, just some algebra to show that this is true.

So now we can put all the pieces together. We have this formula, we have this formula, we have this formula. Put them into the formula up there. And what we find is that we can write the kind of result of it using LSZ in a kind of very compact way, just in terms of irreducible things, c 0 irreducible things.

And it's not just the irreducible three-point functions. There is a contribution from the two-point function, which is in this case D sigma D E bar, where you take E bar and E bar prime, incoming and outgoing energies, to be the right energies for the bound states, which is minus the binding energy. OK?

So this factor came from here. There was a sigma squared in the numerator, but when I take the inverse of two of these guys, there's a sigma squared in the denominator, and those canceled. The inverse of these guys also had factors of 1 plus i c 0 sigma in the numerator, and that canceled these factors here, so the only thing left is the gamma and the D sigma D E bar. OK? So that's what LSZ says in this situation.

And it's actually important to take this factor into account. If you didn't take this factor into account, you wouldn't even preserve, for example, that the charge of the deuteron is 1. It's quite important. If you look at this at lowest order and you just look at the electromagnetic current for the electric case, and you can do that with J 0, looking at the D 0, and we can use our previous calculations to figure out what the two-point function is.

And when I go to the pole, it's just this factor, where gamma B would be set equal to exactly the binding energy. OK? So that's just from the trivial diagram. And then the first contribution that comes into the three-point function can be calculated. It's pretty straightforward. It gives a tan inverse, actually.

And that just comes from coupling to the electromagnetic proton that's inside the deuteron. So this is the proton, which has an electromagnetic coupling. This is the neutron. OK? And that gives this formula. You take the ratio. You get the lowest-order form factor. This is a systematically provable thing, so you can do NLO. You can do NNLO, et cetera, OK?

So we can really calculate properties of the deuteron and check whether they work experimentally, and this works extremely well experimentally. You actually don't see the violation of charge conservation in the form factor until you do NLO. Then you really see that you need this guy in order to make sure that the charge is conserved.

There would be some pieces that would show up that would seem like they were correcting the charge, violating electromagnetism, magnetic gauge variance until you-- but then when you put the z factor in, the z factor contribution in, everything is nice. And this theory has actually been used to do phenomenology, so just to give you some examples of that.

So you can do a process like neutron on proton produces a deuteron and a photon, which is something that shows up in Big Bang nucleosynthesis. And it's been calculated to N 4 LO in this effective field theory. [CHUCKLES] OK? And it provides the most accurate way of determining the process for Big Bang nucleosynthesis. You could reverse it, talk about deuteron breakup.

You could do processes with neutrinos on deuterons. So you could have neutrino-deuteron scattering to proton-proton e minus, which is the charge current process at SNO or the neutral current process at SNO. All of these things are within the realm of things that you can talk about in this effective field theory, OK? So it has some uses.

Let me do one more process. So you could look at neutron-- nucleon-nucleon to nucleon-nucleon plus an axion. And there's sort of a lesson here. So if you look at how the axion couples to the nucleon, it's derivatively coupled, so you have a gradient of the axion field.

And there's two possible coupling that you can write down, just to consider the sort of lowest dimension in our power counting. And this is a process that actually is important for bounding axion physics, for example, axions in the sun, to worry about how they couple to nucleons.

And the thing that's actually important is, if you're thinking about this kind of process, you have to decide kinematically what region are you looking at. If you're interested in bounds from the sun, you're interested in a situation where the energy of the axion is of order of the energy of the nucleons, but the momentum of the axion is much less, so I'll call that k axion is much less than p nucleon, OK?

So from a power counting perspective, the energies are the same size, but the momentum of the axion, which is a relativistic particle and is comparable to the energy, is much less since the nucleon momentum is much bigger than its energy. Remember that the energy goes like p squared over m, so the momentum are bigger. And if that's the situation that you're in, then when you're doing a Lagrangian like this you have to be careful about implementing something like that.

And the way that it gets implemented is by a multipole expansion, which we'll have an opportunity to talk more about in the near future. But basically what that corresponds to is that you've got to make sure that you're sort of exchanging energy between these operators, but not momenta. That's what happens at lowest order.

So if you make this expansion in the Lagrangian, it corresponds to setting the spatial components of the axion field to 0, OK? So this is like doing an expansion around x vector equals 0, the same thing in position space is this expansion here. So this is-- implements the k much less than p expansion.

OK? So that would actually be the operator that you would use to couple axions to nucleon. Now, the interesting thing about this operator, which ties into what we started talking about today, is that once you take x equals 0, the things that you're left with here are related to conserved charges at the SU(4) symmetry.

So Q mu nu, which is an integral over all space of N dagger sigma mu tau nu N-- that's the charge of SU(4). And the reason that it's related to that is because, once I take x equals 0 here, once I do the integral d 4 x of the Lagrangian, the time-- this still depends on time but doesn't depend on space, so I can move the d 3 x through, just let it act on this, just let it act on that, and then I get Q mu nu. Well, Q0j and Q-- or Qj0 and Qj3, so this guy here. Color.

Qj3, Qj0. This guy here. Qj3. And that actually has nontrivial consequences because charges, if they're charges of a symmetry, are time-independent. So even though in principle I didn't make any assumptions about the time dependence here, and these operators could exchange energy, if I know that they are related to charges of the effective field theory, then charges are conserved. There's no time dependence.

So the charges of the field theory are time-dependent. And what that means then is that this is time-independent, and there's actually no energy exchange. So the axion can only sort of exchange energy-- the only way the axion can couple is with zero energy. And since it's derivatively coupled, it has zero energy, it has zero momentum, and there's no coupling.

So this limit that we talked about is a symmetry limit. The axion just doesn't couple. OK? So there we're predicting something falling through kind of the consequences of symmetry on this effective a field theory. We're predicting something about how axions couple to the nucleons.

So if you look sort of through the possible processes that you could have, you could have two nucleons in a 3S1 state try to produce an axion, and that actually vanishes irrespective of the scattering lengths because you're coupling here to spin, and that is always a good charge of the nonrelativistic theory.

But there is a way that you could have the process go, where you switch from a 1S0 state to a 3S1 state, and this guy vanishes if the scattering lengths go to infinity. So if you ignored what I just said and you ignored this here, and you just went ahead and calculated the Feynman diagrams, you would just find that the amplitude to this thing is proportional to this kind of factor that we saw before.

And since the scattering lengths are large, it's vanishing if the scattering lengths go to infinity. OK? So at lowest order, the effective theory, it's suppressed, this process, and that has implications for of course the phenomenology. You could also put in higher-dimension operators, like the c 2's, and then of course you can get a contribution, but that's higher order of the power counting, and that affects how the axion could be constrained in the sun. OK?

So that gives you a little bit of a feeling for what you can do with this effective field theory and how you can kind of exploit symmetry, as well as the calculability of the effective theory to make lots of interesting predictions. OK? So any questions about that? All right. So that's all I have to say about that effective theory.

So we're now going to move on to our final effective field theory, [SIGHS] to soft-collinear effective theory. So what I intend to do with this part of the course, which will take us through the rest of the year, is I'm going to hand out lecture notes for you to read, which I'm going to write. [CHUCKLES]

So I haven't done that yet. I'm going to continue to post-- I mean, I have some rough copy of them, but I'm going to make it pretty and beautiful for you, or more beautiful than it currently is. I'm also going to continue to post the lecture notes of what I present in lecture, so you'll have sort of two versions that you can read, either the LaTeX version or the handwritten version, and depending on your taste you can look at one or the other.

So what we'll do today is we'll just sort of briefly introduce ourselves to this effective theory. And by way of introduction to any effective theory, the kind of things that we should talk about or why-- so we'll talk about why. And then we should talk about what the degrees of freedom are. And in this case, we'll also talk about kind of what are convenient coordinates to be using for this effective theory. So that's what we'll start covering today.

So what is SCET? So it's an effective field theory that you can use for multiple things. One is for energetic hadrons. So the energy of the hadron should be something that's a large scale, which I'll call Q, conventional notation. And by energetic it means much more energetic than the mass of the hadrons, which is of order lambda QCD.

It's also an effective theory for energetic jets. And the situation is in some ways similar to the hadrons. You have a jet energy that's large, of order some Q, and that's much bigger than the jet mass, which is the root squared of some forward momentum.

And by the name, you should also imagine that really what it is something to do with QCD. Well, maybe not by the name, but it is something to do with QCD. And it's kind of what you get if you start with QCD and you focus your attention on a certain set of interactions, which are collinear and soft interactions.

So we've met several different examples of effective field theory so far in the course. Some of them were bottom-up. Some of them were top-down. This is going to be a top-down effective field theory, just like HQET was a top-down effective field theory. So we'll be able to start with the QCD Lagrangian and derive this theory by taking certain limits, and that's what we'll do.

So why study this? We'll come to that in a bit. But what are the kind of features of this from an effective field theory point of view that make it interesting? Well, we'll talk about the answer to both those questions.

So first, why study this? What's this effective theory good for? So one thing that's good for is understanding factorization. And if you think about what we're doing when we probe for short-distance physics at the LHC, we're probing for short-distance physics by colliding things at high energy. So we're carrying out hard collisions, energetic collisions of particles.

So the fact that this effective field theory deals with energetic particles may ring a bell as it being something useful for understanding short-distance physics. And so what this effective theory allows you to do is disentangle the short-distance physics that you're interested in from the longer-distance physics that's associated to binding together into hadrons or producing these energetic jets. So it's really disentangling the long- and short-distance physics in the standard model, or the standard model plus new physics. And basically what that amounts to is disentangling long- and short-distance physics in QCD.

So let me write it this way. Disentangling the physics of QCD with addition of electroweak or BSM physics. It requires a separation of scales. We have to decide what's short distance, what's long distance. If we're interested in a process like the LHC, where we're colliding together hadrons, then that's of course absolutely necessary because the hadrons are long-distance things, and we're interested in short-distance physics, and we can do that with SCET, so that's perhaps the biggest motivation for us today.

When we make hard collisions, these jets and energetic hadrons are very common. They're the most common things that are produced. And from a physics point of view, that means that there's many, many different processes that you can actually use this effective field theory for, and that's one of the reasons it's popular. So I want to give you some examples.

You could do deep inelastic scattering. In this effective theory, deep inelastic scattering involves a hard collision. And even classic textbook deep inelastic scattering is made very much simpler by this effective field theory, and we'll show you that with one of our examples later on. You could do Drell-Yan with either protons and antiprotons or protons and protons. You could do Higgs production, and people use this effective field theory to study Higgs production at the LHC.

You could do, by way of trying to make the examples more varied, e plus e minus to jets. This is another thing that the effective theory is used for. You could do quarkonium physics, e plus or minus, producing a J psi and an X. Here, you need a combination if the J psi is produced at large energy of kind of a nonrelativistic QCD for this cc-bar bound state, as well as an SCT for the fact that it's an energetic state. So this would be actually a combination of two different effective field theories. People use it for thinking about jet substructure. So all sorts of things that you might be interested in and thinking about having to do with hard scattering.

There's also another way that you could get energetic particles, even if you don't force them to be energetic, and that is if you had a very heavy particle decaying. So if you had a B meson decaying and it decayed to light stuff, this would also be the right effective theory for that. So for example, in the process B to s gamma, this effective theory plays a role, in the process B to D pi, where the pion comes out very energetic, or B to pi pi, which is something very interesting for studying CP violation. You can use this effective theory to understand the short- and long-distance physics of those processes, as well.

OK? And this is coming about because this B meson is heavy. So that's the sort of range of examples, and it's not really complete in any sense. There's just lots of different things you can study with this theory.

So this I already said, but let me write it. I sort of alluded to separating perturbative physics at short distances from the nonperturbative physics at long distances. And the nonperturbative physics, if you're lucky, may be only encoded in parton distribution functions or something if you're doing a p-p collision, and you'd like to separate that from shorter-distance physics that you could calculate perturbatively, and this is a way of doing that.

And then there's a renormalization group in this effective theory, and that renormalization group actually allows you to do something cool, too, and that is that you can sum up something called Sudakov logarithms, which are algorithms that appear in our canonical way as alpha s times a logarithm. But instead of one logarithm, you actually get two. And so the renormalization group in this effective theory will allow you to handle those Sudakov logarithms.

OK? So that's some set of things. What are we going to have to do that we haven't done before? What are the kind of new ingredients that we're going to run into? So sort of a prelude to things to come, or another way of saying what makes SCET different from the effective theories we've seen so far.

Let's have a little list for that, as well. So perhaps the most interesting one is that we are going to have more than one field for each degree of freedom. So we're going to have more than one field-- that's not quite the right way of saying it. We're going to have more than one field for the same particle. Just a slightly different statement.

So we're going to have a field, which we'll talk about, that I'll call c sub n, which is a collinear quark field for some quark, say an up quark. And there's also going to be another field, q sub s, which is a soft quark field for the same up quark. So let's say for the up quark [INAUDIBLE] just to make it definite.

And the reason that we're going to have these two different fields is because they're going to describe the same particle but in different regions of momentum space. So if you like, they're describing different degrees of freedom of the same particle because they're in different regions of momentum space. But it still is the same particle, and that's not something we've had to deal with in any of our examples so far. We always had one field for each particle. Now we're going to have more than one, so that's one complication.

The second one we have actually encountered before, and that is that we're going to integrate out off-shell modes of a field, but not the entire particle, not the entire degree of freedom. And we encountered that when we talked about HQET. So the situation here will be a little more complicated than HQET, but this second point is like HQET, and we'll use some of our experience that we gained from studying HQET.

AUDIENCE: [INAUDIBLE] soft and collinear? Which one--

IAIN STEWART: It never can be both. [CHUCKLES]

AUDIENCE: Why?

IAIN STEWART: We'll have to decide whether it's one or the other. So an important thing-- exactly this issue of which it is an important thing because you're going to be describing with these fields certain fluctuations, and you don't want overlap. So you're going to have to-- if you point your finger somewhere in momentum space, we'll have to decide is it in this category or this category. It can't be in both. But that will be important in a bit. That will come back.

AUDIENCE: So you're partioning--

IAIN STEWART: Yeah, two regions, and no overlap. But we'll have to work a little bit to make sure that's true. Yeah, we want to make sure to not have double counting in the effective theory, where you could write down something that looks like it could be described by either one. And if that was true, then we would have a problem.

So another thing that makes SCET interesting from an EFT point of view is that we have convolutions. So usually in an effective theory you think about having multiple operators that have the same quantum numbers. We sum over those operators, say at some order in the power counting, sum over i of Wilson coefficients times operators. And this sum becomes a continuous integral in SCET. You'll see how that works.

And that actually leads to convolutions. In fact, exactly this little manipulation here is what leads to the convolution of the parton distribution function with hard scattering. If you think about this operator is describing-- the matrix elements of this operator is describing a parton distribution function. The Wilson's coefficients is describing the hard scattering. Exactly this integral is what leads to the integral the convolutes together parton distribution functions and hard scattering matrix elements. Come back to that.

So there's going to be a power counting for this effective theory. And we're going to call the power counting parameter lambda, and it's not going to be related to the mass dimension of fields. So we'll have to figure out how to do the power counting. There's going to be something called the Wilson line, which will show up all over the place in this effective theory.

So these are lines in space that are coupled to the gauge field, A. And we'll see why they're showing up, but it's related actually to the fact that this effective theory has a very interesting and perhaps subtle gauge symmetry structure. One way of actually already seeing why that is true is because I told you over here that we have two types of quarks.

Everything I say about quarks is also true of gluons, so there's going to be two types of gluons, an A n gluon and an A s gluon. And once there's two types of gluons, you should say, well, what's the gauge group of this gluon and what's the gauge group for that gluon? Well, they're both the same gauge group. It's QCD. So we partitioned QCD in some kind of weird way, where we have two fields for the gauge group, rather than just one. Part of that story is going to be related to these Wilson lines.

And finally, one final kind of interesting thing that's going to happen is we're going to see when we do one-loop calculations in this theory that we find 1 over epsilon squared divergences. So usually, when you do an effective field theory calculation, at one loop you find a 1 over epsilon pole, not a 1 over epsilon squared. And I denote it by epsilon u v because both poles here require u v renormalization.

We need to add a counterterm to cancel this double pole. And exactly the presence of that double pole is what leads to these Sudakov logs that I was mentioning, the fact that we have these double logs going along with each alpha s. So we'll see that taking care of this with renormalization, which is little different than the renormalization you're used to, will allow us to sum up those Sudakov logarithms, OK?

So that's the kind of hint of the things that we will be studying. You're not supposed to understand any of them yet unless you've study SCET, but we will understand all of these by the end of the course. OK? Any question? Anything I missed on my list? [CHUCKLES] All right.

So let's spend some time talking about degrees of freedom. The way I like to think about degrees of freedom for any effective field theory is this is where you have to think about the physics of what you're doing, OK? So let's think about some physical process. I'm going to consider two different physical processes. Actually, today, I'll only start talking about one of them, and then we'll talk about the other one next time.

Think about some physical process and try to identify what the right degrees of freedom are. So you could look at the process of a B meson changing into a D meson and a pion. The advantage of starting with this one is that we've already talked about HQET, so we understand how to do B mesons and D mesons, and we just have to understand how to do the pion.

So we have a B meson. Let's think about it in the rest frame. Decays to a very heavy D meson and a very light pion. So the D meson we can treat with HQET. The B meson we can treat with HQET. The problem is this pion because it's very energetic. If you look at the momentum of that pion and the way I've drawn it, and you put in some numbers just to get a sense of what we're talking about, then it's got an energy, which is slightly bigger than the momentum, and that's because it has a mass, but it's a pretty small mass.

e squared minus p squared is 135 m e V squared. And so if you look at that, you see that this momentum of the pion is very close to the light cone. The light cone would be if these two were equal. That's what a massless particle would do. A pion is pretty light, and it's moving relativistically, and there's no sense in which we can think about 2.3 G e V as a small scale. We're not going to expand in 2.3 G e V over m B because that's a pretty lousy expansion.

So p pi mu here is basically some scale times a light-like vector, where n mu is 1, 0, 0, minus 1, and that's to a good approximation. We'll see how good over there. So we're going to use light-like vectors to talk about particles like a pion. So n mu is a light-like vector. n squared is 0.

And what we're going to be expanding in is, instead of expanding in Q over m B, Q over pion, we're going to think about the dynamics with this scale Q scale, 2.3 G e V as being much bigger than lambda QCD. So we're going to use, for particles like this pion that are very energetic, light cone coordinates.

So when you have a light cone vector like n and you want to decompose any vector in terms of it, you need another auxiliary vector, which I'm going to call n bar. So these are two light-like vectors. They both square to 0. And the reason I need two of them is because I need some notion of orthogonality, so you should think of n bar as like the complementary vector to n. It's the dual vector in the orthogonal sense.

So with those two vectors, you can decompose any vector in terms of-- use them as a basis. This is how it works, my conventions. So you have two components that you can decompose along n and n bar, and the value of the momentum along that direction is the complementary vector dotted into p.

And so we usually-- with this notation, we call n bar dot p p minus. So that's just a shorthand. And n dot p is p plus, OK? So it's going to be useful to decompose things along these directions for the same reason that you kind of saw here, that the pion was naturally decomposing itself along a particular light-like direction. So we're going to want to do that more generally, and that's how we'll do it for momenta.

If we talk about p squared, and I just square the components here, n dotted into "perp" is 0, and bar dotted into "perp" is 0. There's a nontrivial-- that's square root of 0. That's square root of 0. There's a nontrivial cross-term between those two, which is this. This guy can be squared. If we use our shorthand, then it's this. And if we want to use Euclidean notation, then I can-- I'll switch and I'll say that p "perp" squared, which is a Minkowski negative quantity, is minus p vector "perp" squared, where p vector "perp" squared is Euclidean. So sometimes I'll-- use for both of those notations.

All right, so I think we're out of time for today. We'll continue with this discussion of degrees of freedom next time and talk a little bit more about coordinates, as well.