

8.902 Fall 2023 - Problem Set #6

Due Tuesday, November 28

Unless otherwise stated, assume a flat Universe with $\Omega_M = 0.3$, $\Omega_\Lambda = 0.7$, and $H_0 = 71$ km/s/Mpc.

1 Spherical Top Hat

Using a spherical top hat model for structure formation in an Einstein-de Sitter universe, work out the following:

A) What is the dynamical (i.e. free-fall) timescale for a forming galaxy which breaks away from the Hubble flow and reaches its maximum radius at $z = 8$? You do not need to rederive the overdensity value (i.e. $\rho_{\text{halo}}/\rho_{\text{universe}}$).

B) Suppose that all of the systems gravitational potential energy is used to heat the baryons upon collapse. What is the final gas temperature after virialization, in the absence of any cooling mechanism? Express your answer in terms of the total mass M of the galaxy, and normalize it to units of $10^{12}M_\odot$. For simplicity, you may assume the density within the sphere is spatially uniform and the baryons behave like an ideal gas.

C) If the baryons start with this temperature, estimate the cooling timescale for the system. Assume that 5% of the mass is in baryons, and that the cooling curve takes on a value of $\Lambda(T) = 10^{-23} \text{ erg cm}^3 \text{ s}^{-1}$ for the temperatures in question. (If needed, there is a nice description of cooling curves in Sparke & Gallagher pp. 105–106).

D) One criterion for galaxy formation is that the cooling timescale should be less than the dynamical timescale. This leads to an order-of-magnitude estimate for the maximum mass of a galaxy. What is this characteristic mass, determined from comparison of your answers to A) and C).

2 Press-Schechter

A) We have discussed the Press-Schechter formalism in class. Show that the normalization of the mass function is wrong; i.e., integrate the volume fraction and show that it only accounts for 1/2 of the mass.

B) There is an alternative approach to deriving the mass function called ‘Excursion Set Formalism’ or ‘Extended Press-Schechter’ (Bond et al. 1991). Read about this and explain the basic idea of this formalism. How does this formalism fix the ‘fudge factor’ problem we discussed in class?

3 Bonus

It's the last problem set! You've learned a lot about galaxies and the universe. Draw a congratulatory galaxy for five bonus points.

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8.902 Astrophysics II

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