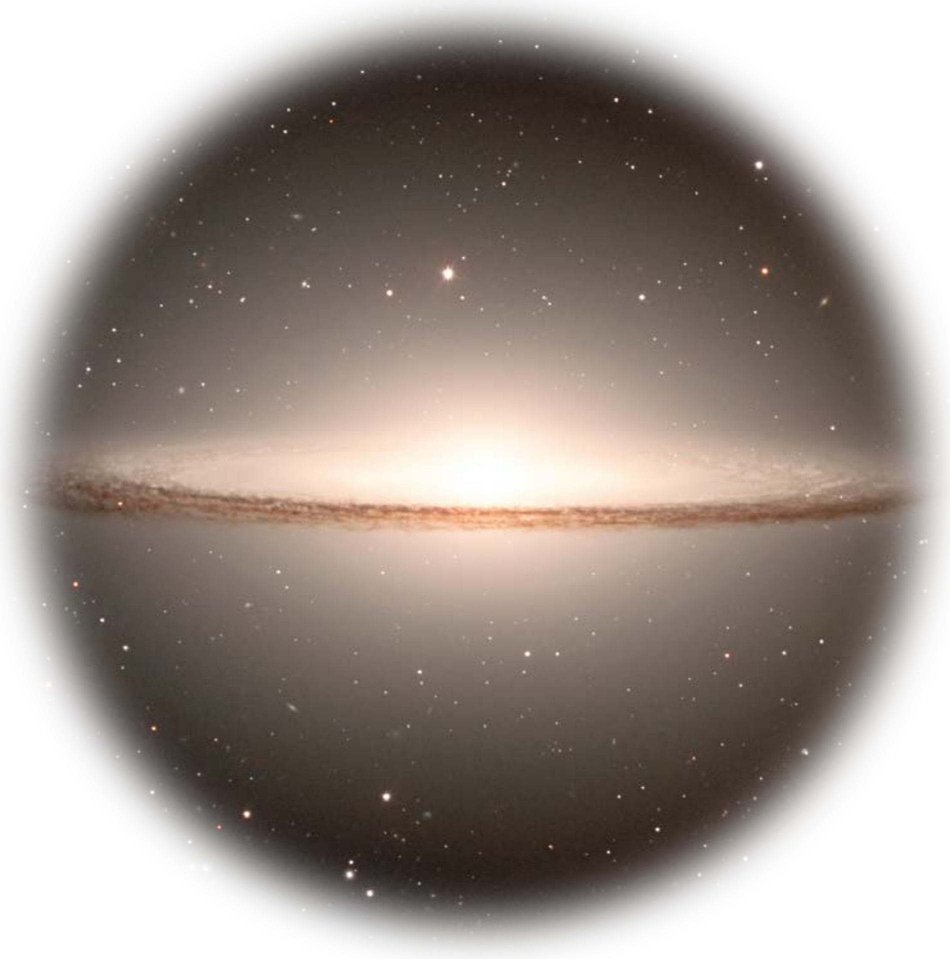
Part I

Galaxies



Messier 104, Sombrero Galaxy. Credit: ESO/P. Barthel (Kapteyn Institute, Groningen). License CC-BY.

1 Key observations of galaxies

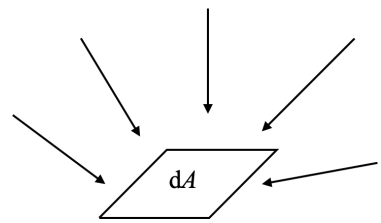
1.A Basic units of radiative transfer

We first define some fundamental quantities.

Flux:

$$\boxed{F_\nu = \frac{\mathrm{d}E_\nu}{\mathrm{d}A\ \mathrm{d}t\ \mathrm{d}\nu}} \tag{1}$$

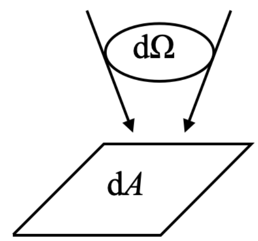
with units $[F_\nu] = \mathrm{erg\ s^{-1}cm^{-2}Hz^{-1}}$.
F_ν is the flux at a specific frequency ν.



Specific intensity:

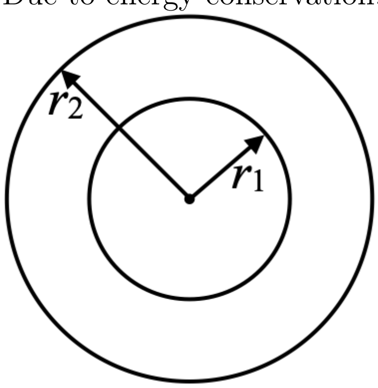
$$\boxed{I_\nu = \frac{\mathrm{d}E_\nu}{\mathrm{d}A\ \mathrm{d}t\ \mathrm{d}\nu\ \mathrm{d}\Omega}} \tag{2}$$

with units $[I_\nu] = \mathrm{erg\ s^{-1}cm^{-2}Hz^{-1}sr^{-1}}$.
This is the flux per solid angle.



Note:

- $F_\nu(r) \propto \frac{1}{r^2}$.
  Due to energy conservation:



$$\begin{aligned} &F_\nu(r_1) \times 4\pi r_1^2 = \\ &(\mathrm{d}E_\nu)_1 = (\mathrm{d}E_\nu)_2 = \\ &F_\nu(r_2)^2 \times 4\pi r_2^2 \end{aligned} \tag{3}$$

$$\begin{aligned} &\Rightarrow \frac{F_\nu(r_1)}{F_\nu(r_2)} = \left(\frac{r_2}{r_1}\right)^2 \\ &\Rightarrow F_\nu(r) \propto \frac{1}{r^2} \end{aligned} \tag{4}$$

- $I_\nu(r) \propto$ constant because:

$$\begin{aligned} &I_\nu = \frac{F_\nu}{\mathrm{d}\Omega} \text{ and } \mathrm{d}\Omega \propto \frac{1}{r^2} \\ &\Rightarrow F_\nu \propto \frac{1}{r^2} \text{ and } \mathrm{d}\Omega \propto \frac{1}{r^2} \\ &\Rightarrow I_\nu \propto \text{constant} \end{aligned} \tag{5}$$

Magnitude scale:
Define the *apparent magnitude* m, i.e. how bright an object appears:

$$m_1 - m_2 = -2.5 \log \left(\frac{(F_\nu)_1}{(F_\nu)_2} \right) \tag{6}$$

With this definition, a brighter object has a lower magnitude. There are two main magnitude systems: Vega and AB.

The *Vega system* is calibrated using the flux of the AO V star Vega $(F_\nu)_{\mathrm{Vega}}$, which has a non-flat distribution (flux changes for different frequencies). The *AB system* is calibrated to

a hypothetical source with flux

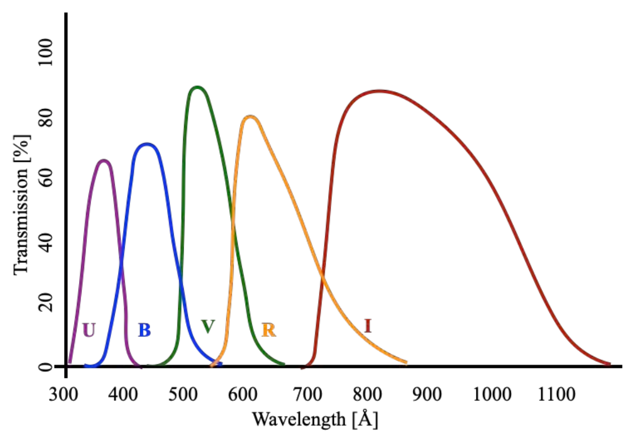
$$(F_\nu)_{\mathrm{AB}} = 3.63 \times 10^{-20} \mathrm{erg\ s^{-1} cm^{-2} Hz^{-1}} \tag{7}$$

which has a flat distribution.

We also have the *monochromatic magnitude*, i.e. the magnitude at a single wavelength, defined for each system:

$$\begin{aligned} \mathrm{Vega} : m_\nu &= -2.5 \log \left(\frac{F_\nu}{(F_\nu)_{\mathrm{Vega}}} \right) \\ \mathrm{AB} : m_\nu &= -2.5 \log \left(\frac{F_\nu}{(F_\nu)_{\mathrm{AB}}} \right) \end{aligned} \tag{8}$$

A more practical quantity is the *band magnitude*. In most observations, the fluxes are integrated over a filter bandpass with a transmission function $T_X(\nu)$ for band X. An example of a set of filters (U,B,V,R,I) and the transmission function (what percent of the flux is let through at a given frequency or wavelength) is shown to the right.

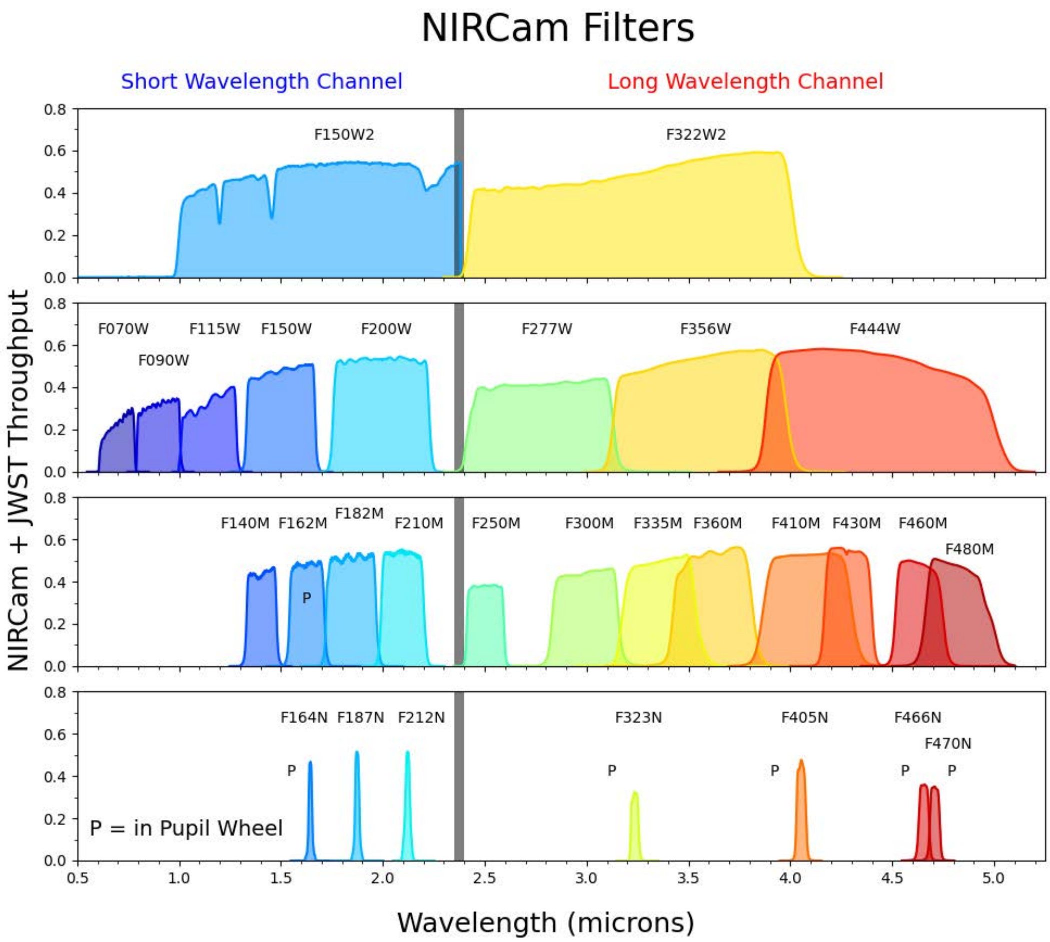


$$\begin{aligned} \mathrm{Vega} : m_X &= -2.5 \log \left(\frac{\int F_\nu T_X(\nu) \mathrm{d}\nu}{\int (F_\nu)_{\mathrm{Vega}} T_X(\nu) \mathrm{d}\nu} \right) \\ \mathrm{AB} : m_X &= -2.5 \log \left(\frac{\int F_\nu T_X(\nu) \mathrm{d}\nu}{\int (F_\nu)_{\mathrm{AB}} T_X(\nu) \mathrm{d}\nu} \right) \end{aligned} \tag{9}$$

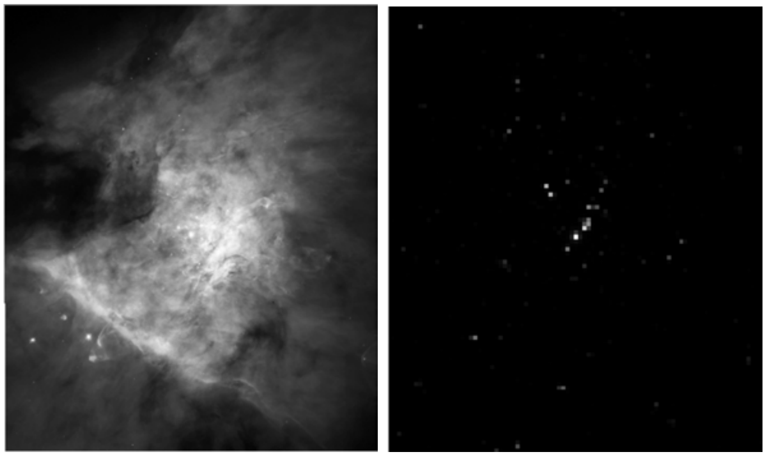
$(F_\nu)_{\mathrm{AB}} = \text{constant}$ and $\int T_X(\nu) \mathrm{d}\nu = 1$, so

$$\int (F_\nu)_{\mathrm{AB}} T_X(\nu) \mathrm{d}\nu = (F_\nu)_{\mathrm{AB}} \ . \tag{10}$$

Telescopes like Hubble and SDSS observe primarily in the visible light spectrum. JWST measures slightly longer wavelengths and is sensitive to the infrared range. The NIRCam instrument filters are shown in below. We show the total throughput (photon-to-election conversion efficiency) for extra-wide, wide, medium, and narrow filters for NIRCam (image from https://jwst-docs.stsci.edu).



Each filter measures a different energy range of electromagnetic waves and therefore probes different physics. As an example of this, we show the Orion Nebula as viewed in visible light from Hubble below on the left and in X-ray from Chandra on the right. In the visible range, we can see the diffuse gas while in the X-ray, we can see point-like sources from stars.



For all filters, $-2.5\log(3.63 \times 10^{-20}) = 48.6$, so (for the AB system)

$$
\begin{aligned}
m_X &= -2.5\log\left(\int F_\nu T_X(\nu)\mathrm{d}\nu\right) - 48.6 \\
m_\nu &= -2.5\log(F_\nu) - 48.6 \; .
\end{aligned} \tag{11}
$$

The value for $(F_\nu)_{\mathrm{AB}}$ was chosen such that $m_V(\mathrm{AB}) = m_V(\mathrm{Vega})$ and they have the same magnitude in the V-band. For other bands, one must apply the conversion

$$
m_X(\mathrm{AB}) - m_X(\mathrm{Vega}) = -2.5\log\left(\frac{\int (F_\nu)_{\mathrm{AB}} T_X(\nu)\mathrm{d}\nu}{(F_\nu)_{\mathrm{Vega}} T_X(\nu)\mathrm{d}\nu}\right) \; . \tag{12}
$$

This gives us, for example:

$$
\begin{aligned}
U_{\mathrm{AB}} &= U_{\mathrm{Vega}} - 0.8 \\
B_{\mathrm{AB}} &= B_{\mathrm{Vega}} - 0.11 \\
V_{\mathrm{AB}} &= V_{\mathrm{Vega}}
\end{aligned} \tag{13}
$$

Be careful which magnitude is quoted! SDSS uses u, g, r, i, z filters.

Define the *absolute magnitude* as the apparent magnitude if the object were at a distance of 10 pc. Apparent magnitude depends on both the brightness of the object and its distance. Absolute magnitude is related to the intrinsic brightness of the object.

$$
m_X - M_X = 5\log\left(\frac{D}{10\,\mathrm{pc}}\right) \equiv \mu \tag{14}
$$

Image of Orion Nebula created by Mark Vogelsberger using SAOImage DS9 image display and visualization tool for astronomical data. https://sites.google.com/cfa.harvard.edu/saoimageds9/home

Since

$$F_{\rm app} = \frac{L}{4\pi D^2} \qquad \text{and} \qquad F_{\rm abs} = \frac{L}{4\pi(10\,{\rm pc})^2}$$

$$\begin{aligned}
\Rightarrow\ m_X &= -2.5\log\left(\frac{L}{4\pi D^2}\right) + \text{constant} \\
M_X &= -2.5\log\left(\frac{L}{4\pi(10\,{\rm pc})^2}\right) + \text{constant} \\
\Rightarrow\ m_X - M_X &= +2.5\log(D^2) - 2.5\log((10\,{\rm pc})^2) \\
&= 5\log(D) - 5\log(10\,{\rm pc}) \\
&= 5\log(D/{\rm pc}) - 5
\end{aligned} \tag{15}$$

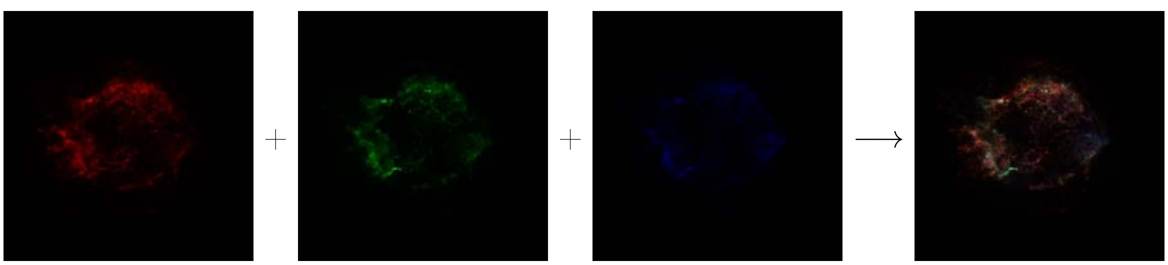
$\mu = m_X - M_X$ is the *distance modulus* (a measure of distance).

Colors:
If observations are made in more than one filter (X, Y), then one can define a color as the difference in magnitudes between the two bands:

$$(X - Y) = m_X - m_Y = M_X - M_Y \tag{16}$$

Stars and galaxies can be “red" or “blue", for example. It is common to use the difference between g and r filters to get $g - r$ color. A higher $g - r$ value is red and a lower value is blue. Note that higher $g - r$ has a higher g relative to r, but a higher magnitude is less bright.

We can take images of the same object through different filters and combine them for a more complete view of the object. Here we show images of the supernova remnant Cassiopeia A taken in three wavelength ranges (0.6-1.65 keV, 1.67-2.25 keV, and 2.25-7.5 keV) shown in red, green, and blue and then combined into a single image.



Surface brightness:
We measure the luminosity ([erg s^{-1}]) per area. This is often called Σ or I. It effectively measures the magnitude per square arcsecond:

$$M \propto -2.5\log(I)\ . \tag{17}$$

Spiral and elliptical galaxies show different surface density profiles:

$$\begin{aligned}
\text{exponential} : I(r) &= I_0 e^{-r/r_s} && \text{(spirals)} \\
\text{de Vaucouleurs} : I(r) &= I_0 e^{-7.67(r/r_e)^{\frac{1}{4}}} && \text{(ellipticals)}
\end{aligned} \tag{18}$$

Images of supernova remnant created by Mark Vogelsberger using SAOImage DS9 image display and visualization tool for astronomical data. https://sites.google.com/cfa.harvard.edu/saoimageds9/home

More generally, we have the Sérsic profile with Sérsic index n:

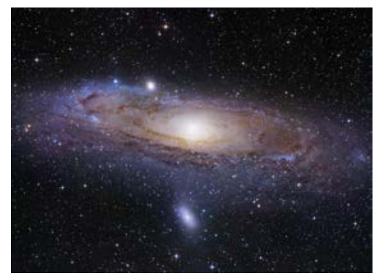
$$I(r) = I_0 e^{-(r/r_0)^{\frac{1}{n}}} \tag{19}$$

Observationally, $n \approx 4$ for ellipticals and $n \approx 1$ for spirals. Theory needs to explain this!

1.B Basic properties of the galaxy population

Types of galaxies:
Images of galaxies show mainly three types:

- Spirals (Sp):
  - dominate in the field (outside clusters)
  - disks with gas and stars
  - young stellar population
  - rotationally supported
  - blue color
  - exponential surface brightness profile
- Ellipticals (E):
  - cluster environment
  - spheroidal
  - old stellar population
  - pressure supported
  - red color
  - de Vaucouleurs surface brightness profile
- Lenticular (SO):
  - stellar disk
  - no gas disk
  - link between spiral and elliptical galaxies



Andromeda

M87

NGC 2787

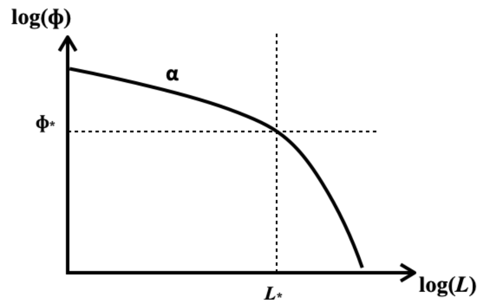
Galaxy luminosity distribution:
The luminosity L of an object is

$$L = \frac{\mathrm{d}E}{dt} = \int I_\nu \mathrm{d}A d\Omega \mathrm{d}\nu \ . \tag{20}$$

What is the distribution function of L for galaxies? We commonly use the *Schechter function* to describe the number density of galaxies at a given luminosity:

$$\phi(L)\mathrm{d}L = \phi_* \left(\frac{L}{L_*}\right)^{\alpha} e^{-L/L_*} \frac{\mathrm{d}L}{L_*} \tag{21}$$

ϕ_*: normalization	$\phi_* \approx 0.02\, h^3 \mathrm{Mpc}^{-3}$
α: faint-end slope	$\alpha \approx -1.09$
L_*: characteristic L at the normalization point	$L_* \approx 10^{10}\, L_\odot h^{-2}$



Velocity structure of galaxies:
Spectral data of galaxies allows us to measure velocities. Spiral galaxies have ordered, circular motion with $V_c \sim 200 \pm 50\,\mathrm{km/s}$. We can measure the circular velocity through the motions of stars and, further out, from the spectral lines of gas. Outside the galaxy, one finds that v_c remains constant, but one would expect:

$$\frac{mv_c^2}{r} = \frac{GMm}{r^2} \tag{22}$$

for a circular orbit. This implies $v_c \propto r^{-1/2}$, for centralized mass, which is not constant. To have v_c constant, we need

$$\begin{aligned} v_c &\propto \frac{1}{r}\int_0^r 4\pi r^2 \rho(r) dr \\ \Rightarrow &\rho(r) \propto r^{-2} \end{aligned} \tag{23}$$

to large radii. This was one of the first hints for dark matter.
What is dark matter? A few things we know:

- It can't be non-luminous gas since we would have seen it through absorption lines
- Dim stars or other dense objects at larger distances (MACHOS: Massive Compact Halo Object) have been ruled out since microlensing (the temporary brightening of a distant object due to a closer massive object bending the light rays closer together) does not occur frequently enough
- Neutrinos have been ruled out since they lead to the wrong structure formation because they move so fast (hot dark matter). Since neutrinos move close to the speed of light, they have too much kinetic energy to be bound in low-mass potential wells.

- It could possibly be WIMPs (Weakly Interacting Massive Particles). However, there are no detections of WIMPs so far.
- General theories:
  - Cold Dark Matter (CDM): dark matter is a particle that moves slowly ($v \ll c$) and is collisionless, interacting solely through gravity.
  - Self-interacting dark matter (SIDM): dark matter interacts through gravity as well as through self-interactions that allow particles to scatter and transfer energy and momentum.
  - Warm dark matter (WDM): dark matter is still collisionless but moves with a faster velocity than CDM, which makes it harder to form less massive halos.
  - Bose-Einstein condensate (very low mass) dark matter: dark matter particles are very low mass such that their de Broglie wavelength is on the length scale of galaxies and leads to interference patterns in halos.
  - Modified Newtonian dynamics (MOND): Dark matter is not a type of matter but is accounted for through modifications to our theory of gravity.

Elliptical galaxies have a random motion velocity structure with velocity dispersion $\sigma_v \sim 200 - 300\,\mathrm{km/s}$. There is negligible circular motion, typically $v_c \sim 50 - 100\,\mathrm{km/s}$.

Spectra can also be used to measure redshift/recession velocity z of galaxies:

$$z = \frac{\lambda_{\mathrm{obs}} - \lambda_0}{\lambda_0} \qquad \text{or} \qquad 1 + z = \frac{\lambda_{\mathrm{obs}}}{\lambda_0} \tag{24}$$

For low z ($z \ll 1$), one finds that the distance d is related to the redshift through the present-day Hubble constant H_0:

$$d \approx \frac{cz}{H_0}\,, \tag{25}$$

which yields

$$\boxed{v = H_0 d \approx cz\,, \quad z \ll 1} \tag{26}$$

The redshift directly yields the recession velocity. A more formal proof of $d \approx \frac{cz}{H_0}$ will be discussed later (low z limit for all distances).

> Note: h is defined so the Hubble constant today is $H_0 = 100\,h\ \mathrm{km/s/Mpc}$

1.C Stellar population synthesis

So far, we have used spectral information only to derive velocities. However, we can also use this information to derive the *spectral energy distribution* (SED) of a galaxy. Stellar SEDs are blackbodies with different temperatures. The types of stars are referred to as O, B, A, F, G, K, M, L, and T, each with different temperatures that contribute differently to the spectrum of the galaxy. Galaxies are a combination of these, so the total flux at a given frequency is a combination of the flux from each star:

$$F_\nu = N_O F_{\nu,O} + N_B F_{\nu,B} + ... \tag{27}$$

Determining $N_O, N_B, N_A, N_F...$ is the basic idea of stellar population synthesis.