3.E Stellar population synthesis

So far, we have only viewed stars as massive particles without other features. But:

- Stars are constantly born at a star formation rate (SFR) $\psi(t)$
- Stars are born with a certain mass spectrum. This is the initial stellar mass function (IMF) $\phi(m)$
- Stars emit light with flux at different wavelengths F_λ.

The galactic spectrum is a superposition of stellar spectra. Adding the stellar spectra taking into account $\psi(t)$ and $\phi(m)$ allows us to constrain the initial mass function and star formation rate of galaxies. We can use this to learn about the stellar population and galaxy evolution.

Star formation rate:
The units of star formation rate are usually $[\psi] = \mathrm{M}_\odot/\mathrm{yr}$. For the Milky Way, $\psi(t) \sim 3\mathrm{M}_\odot/\mathrm{yr}$

$$(t) = \frac{\mathrm{d}m}{\mathrm{d}t} \; . \tag{218}$$

There are a few observational indications for the star formation rate:

- Far infrared (FIR) emission from dust around young stars:

$$\frac{\mathrm{SFR}_{\mathrm{FIR}}}{\mathrm{M}_\odot/\mathrm{yr}} \sim \frac{L_{\mathrm{FIR}}}{5.8\times 10^9 \mathrm{L}_\odot} \tag{219}$$

- H_α emission from HII regions around young stars:

$$\frac{\mathrm{SFR}_{\mathrm{H}_\alpha}}{\mathrm{M}_\odot/\mathrm{yr}} \sim \frac{L_{H_\alpha}}{1.3\times 10^{41}\mathrm{erg/s}} \tag{220}$$

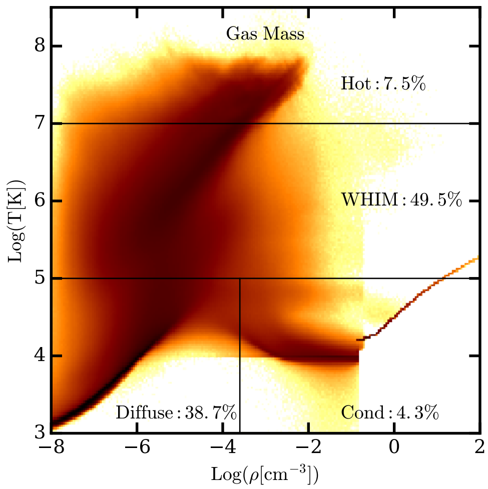
- Ultraviolet (UV) radiation from young stars:

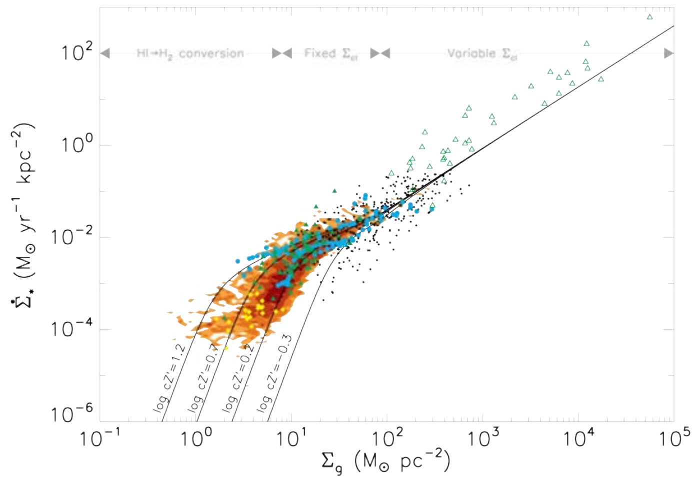
$$\frac{\mathrm{SFR}_{\mathrm{UV}}}{\mathrm{M}_\odot/\mathrm{yr}} \sim \frac{L_{\mathrm{UV}}}{7.2\times 10^{27}\mathrm{erg/s}} \tag{221}$$

We also have theoretical models, for example, the exponential model

$$(t) \propto e^{-t/\tau} \; . \tag{222}$$

For a given galaxy, the star formation rate depends on the density and temperature of the gas. When gas is cold and dense, it is able to collapse into stars. The star formation rate can be roughly approximated by dividing the gas mass by the free-fall time.

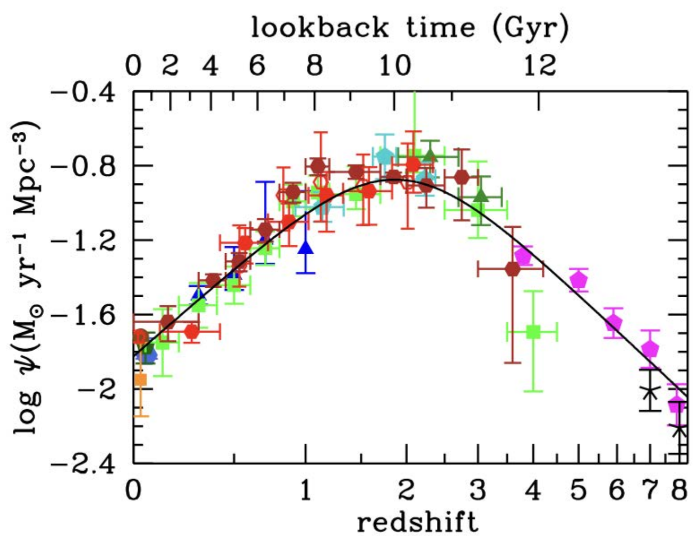




The above plot on the left is from Torrey et al. 2019 shows a phase diagram of gas in the IllustrisTNG simulations at $z = 0$ and is split into various phases of the ISM. Darker regions show a higher gas mass, and the percentages show the total fraction of gas mass in each phase. The condensed material is in the lower right corner, and stars form along the thin line. The plot to the right is from Krumholz et al. 2009 shows the star formation rate surface density as a function of gas surface density. Each point is a different galaxy, compiled from several sources (different colors).

The star formation rate of a galaxy depends primarily on the molecular gas within a galaxy rather than the total gas. However, it can be difficult to predict what fraction of a galaxy's gas is in the molecular phase. This fraction depends on the total gas density, metallicity, and clumping on small scales. To get more precise predictions for star formation, it is also necessary to consider events such as supernovae and shocks.

Over cosmic history, the star formation rate (across all galaxies) started low and increased, peaked at $z \approx 2$, and has been decreasing since.



Initial stellar mass function:
$\phi(m)\mathrm{d}m$ is the relative number of stars born with masses in $(m, m + \mathrm{d}m)$. Note that the units are $[\phi] = \text{mass}^{-2}$, the number of stars formed per mass interval per total mass. This

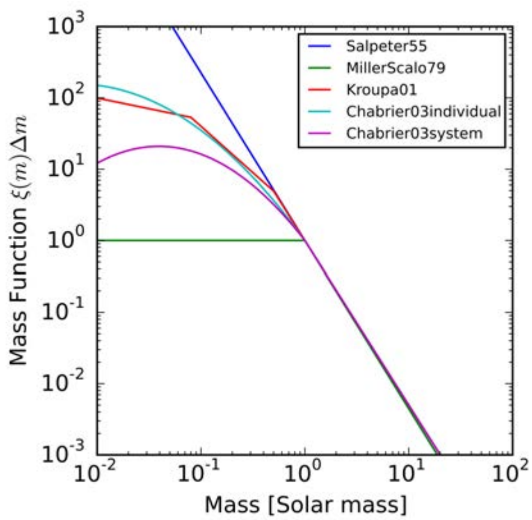
is normalized so

$$\int_{m_l}^{m_h} m\phi(m)\mathrm{d}m = 1\mathrm{M}_\odot \tag{223}$$

$m_l \sim 0.08\mathrm{M}_\odot$ since hydrogen fusion can't occur in stars lower than this and $m_h \sim 100\mathrm{M}_\odot$ since the Eddington limit prevents stars larger than this.

Example:
M_* is the total mass of newly formed stars. Then the total number $\mathrm{d}N(m)$ and total mass $\mathrm{d}M(m)$ of stars born in $(m, m + \mathrm{d}m)$ are

$$\begin{aligned} \mathrm{d}N(m) &= \frac{M_*}{\mathrm{M}_\odot}\phi(m)\mathrm{d}m \\ \mathrm{d}M(m) &= \frac{M_*}{\mathrm{M}_\odot}m\phi(m)\mathrm{d}m\ . \end{aligned} \tag{224}$$



The initial mass function is often assumed to follow the Salpeter mass function:

$$\phi(m) \propto m^{-(1+x)},\ x = 1.35\ . \tag{225}$$

There are other forms, like the Chabrier function, although the form of the initial mass function is uncertain since it depends on relating luminosity and mass and we observe the present day mass function, not the initial mass function.

Stellar spectra:
The stellar spectrum of a star is given by its luminosity L, effective temperature T_{eff}, and chemical composition z. The evolution of a star in the (L, T_{eff}) plane (stellar evolutionary or HR diagram track) only depends on the initial mass and initial metallicity. Once the initial mass and metallicity are known, one can calculate a stellar spectrum.

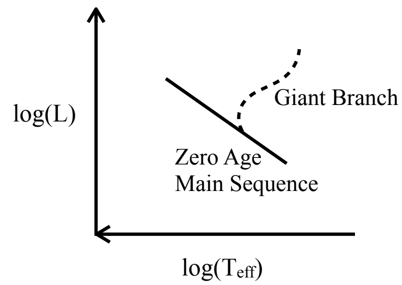
From the Stefan-Boltzmann law $L = 4\pi R^2\sigma T_{\mathrm{eff}}^4$, we can see that L and T are linearly related on the (logarithmic) HR diagram through R:

$$\log(L) = 2\log(R) + 4\log(T)\ . \tag{226}$$

Stars along the $R = 1$ line form the main sequence.

Stars move in the HR diagram as they age and go through the stellar stages (main sequence, red giant, white dwarf, etc.).
Time spend on the main sequence is

$$\boxed{\tau_{\rm MS} \propto M^{-3}} \ . \tag{227}$$



This comes from assuming that the lifetime of a star depends on how much fuel it has (its mass) and how fast it burns that fuel (rate of energy burning, luminosity)

$$\tau_{\rm MS} \propto \frac{M}{L} \ . \tag{228}$$

From the observed mass luminosity relationship, $L \propto M^4$ so

$$\tau_{\rm MS} \propto \frac{M}{L} \propto \frac{M}{M^4} \propto M^{-3} \ . \tag{229}$$

For low mass stars ($M < 0.7\,{\rm M}_\odot$), $L \propto M^3$ so $\tau_{\rm MS} \propto M^{-2}$.

Population synthesis:
A galaxy spectrum is a superposition of stellar spectrum:

$$L_\lambda = \int_0^t \mathcal{L}_\lambda^{\rm cp}(t - t', Z(t'))\psi(t'){\rm d}t' \ . \tag{230}$$

We can measure time from t' that the stars formed so $\tau = t - t'$ and $\tau_0 = t'$. $\psi(t')$ is the star formation rate at t'. $\mathcal{L}_\lambda$ is the luminosity at λ per unit stellar mass of all stars of a coeval population of age τ with initial metallicity $Z(\tau_0)$:

$$\mathcal{L}_\lambda^{\rm cp}(\tau, Z(\tau_0)) = \int \mathcal{L}_\lambda(m, Z(\tau_0), \tau)\frac{\phi(m)}{M_\odot}{\rm d}m \tag{231}$$

where $\mathcal{L}_\lambda$ is the luminosity at wavelength λ of a star with initial mass m and initial metallicity $Z(\tau_0)$ at time τ.

A few notes:

- $L_\lambda(t)$ is a convolution of ϕ, ψ, and $\mathcal{L}_\lambda$.
- ϕ and ψ are not known precisely.
- There are sophisticated codes available to numerically iterate to figure out ϕ and ψ:
  - assume an initial mass function ϕ
  - impose a star formation rate
  - run convolution
  - compare with data (can break some degeneracies with spectral features)
  - adjust SFR and IMF and repeat.

3.F Chemical evolution of galaxies

We have used stellar population synthesis to constrain the initial mass function and the star formation rate. Chemical evolution can also be used to learn about the baryonic history of a galaxy. We use heavy elements as a chronometer. The general chemical evolution follows:

- $t = 0$: no heavy elements
- stellar nucleosynthesis generates heavy elements
- supernovae eject heavy elements into the interstellar medium
- heavy elements are incorporated into new stars.

The *metallicity* Z of a star is

$$Z = \frac{\text{mass of heavy elements}}{\text{total mass}} \tag{232}$$

and is often quoted as a fraction of the solar metallicity $Z/\mathrm{Z}_\odot$ with $\mathrm{Z}_\odot \approx 0.02$.

The abundance $\left[\frac{X}{Y}\right]$ of elements is a comparison between two elements X and Y, e.g. $\left[\frac{\mathrm{Fe}}{\mathrm{H}}\right]$. We report it as the fraction of the log of the solar abundance:

$$\left[\frac{X}{Y}\right] = \log\left(\frac{n_x/n_y}{(n_x/n_y)_\odot}\right) \tag{233}$$

so $\left[\frac{X}{Y}\right] = 0$ means the star has the same abundance as the sun, $\left[\frac{X}{Y}\right] = -1$ means the star has $\frac{1}{10}$ of the solar abundance, $\left[\frac{X}{Y}\right] = -2$ means the star has $\frac{1}{100}$ of the solar abundance, etc.

Note that the metallicity measures by mass and abundance measures by number.

Modelling chemical evolution:

M is the total mass

M_s is the mass in stars

M_g is the mass in gas

f is the gas inflow rate

e is the gas outflow rate

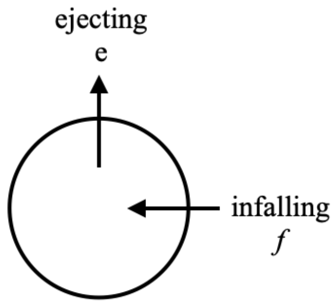
is the star formation rate

E is the gas ejection rate

E_Z is the ejection rate of metals from stars, supernovae, etc.

$Z_f f$ is the infalling metals per time

ZM_g is the mass of metals in gas



$$\begin{aligned} M &= M_s + M_g \\ \frac{\mathrm{d}M}{\mathrm{d}t} &= f - e \\ \frac{\mathrm{d}M_g}{\mathrm{d}t} &= \psi - E \\ \frac{\mathrm{d}M_g}{\mathrm{d}t} &= -\psi + E + f - e \\ \frac{\mathrm{d}(ZM_g)}{\mathrm{d}t} &= -Z\psi + E_Z + Z_f f - Ze \end{aligned} \tag{234}$$

This forms a complete chemical model. We can figure out individual terms and then solve. We will use some approximations for an analytical solution.

$$E(t) = \int_{m_t}^{\infty} (m - w_m)\psi\left(t - \tau_{\rm MS}(m)\right)\phi(m){\rm d}m \tag{235}$$

m_t: Main sequence turnoff mass. This is the lowest mass of stars dying at time t.

$m - w_m$: The ejected mass; w_m is the remnant mass.

$\tau_m(m)$: Main sequence lifetime at mass m.

${}_{t-\tau_m(m)}\phi(m)$: Birth rate of stars of mass m at time $t - \tau_m(m)$, which is the death rate at time t.

$$E_z(t) = \int_{m_t}^{\infty} \left[(m - w_m)Z(t - \tau_{\rm MS}(m)) + m\rho_{Zm}\right]\psi(t - \tau_{\rm MS}(m))\phi(m){\rm d}m \tag{236}$$

$(m - w_m)z(t - \tau_{\rm MS}(m))$: mass of metals that at time $t - \tau_{\rm MS}(m)$ were locked in a star of mass m and are now ejected with the envelop at time t.

$m\rho_{Zm}$: new metals produced by a star of mass m. (Note: some elements get destroyed, for example lithium has a $\rho_{zm} < 0$.)

Instantaneous recycling approximation: (IRA)
We assume that the mass and elements of stars are returned to the interstellar medium without delay and the ejecta are fully mixed immediately. This only works for massive enough stars, $m > m_{\rm lim}$.

$$\psi(t - \tau_m(m)) \approx \psi(t) \tag{237}$$

then

$$E(t) \approx \psi(t)\int_{m_{\rm lim}}^{\infty} \left[m - w_m\right]\phi(m){\rm d}m = \psi(t)R\,. \tag{238}$$

Stars below $m_{\rm lim}$ never lose mass while stars greater than $m_{\rm lim}$ immediately lose mass. This is because massive stars get off the main sequence so quickly ($\tau_{\rm MS} \propto M^{-3}$) so this process is essentially instantaneous, $\tau_{\rm MS} \approx 0$. R is the returned mass per star formed:

$$R = \int_{m_{\rm lim}}^{\infty} (m - w_m)\phi(m){\rm d}m\,. \tag{239}$$

Then

$$\begin{aligned} E_z(t) &\approx \int_{m_{\rm lim}}^{\infty} \left[(m - w_m)Z(t) + m\rho_{Zm}\right]\phi(m){\rm d}m \\ &= \psi(t)Z(t)R + \psi(t)\int_{m_{\rm lim}}^{\infty} m\rho_{Zm}\phi(m){\rm d}m \\ &= \psi(t)Z(t)R + (1 - R)y\psi(t) \end{aligned} \tag{240}$$

where y is the mass of produced metals per remnant mass (white dwarfs, neutron stars, etc.)

$$y = \frac{1}{1-R} \int_{m_{\mathrm{lim}}}^{\infty} m \rho_{Zm} \phi(m) \mathrm{d}m \; . \tag{241}$$

This gives us the equations of chemical evolution in the instantaneous recycling approximation:

$$\begin{aligned}
M &= M_s + M_g \\
\frac{\mathrm{d}M}{\mathrm{d}t} &= f - e \\
\frac{\mathrm{d}M_s}{\mathrm{d}t} &= (1-R)\psi(t) \\
\frac{\mathrm{d}M_g}{\mathrm{d}t} &= -(1-R)\psi(t) + f - e \\
\frac{\mathrm{d}(ZM_g)}{\mathrm{d}t} &= -z\psi + RZ(t)\psi(t) + (1-R)y\psi(t) + Z_f f - Ze \\
&= (1-R)(-Z+y)\psi + Z_f f - Ze
\end{aligned} \tag{242}$$

We can combine the last two equations for $\frac{\mathrm{d}M_g}{\mathrm{d}t}$ and $\frac{\mathrm{d}(ZM_g)}{\mathrm{d}t}$ to get

$$M_g \frac{\mathrm{d}Z}{\mathrm{d}t} = (1-R)y\psi(t) + (Z_f - Z)f + Ze \; . \tag{243}$$

Closed-box model:
The simplest evolution model is to assume a closed box ($f = e = 0$) containing only gas ($M_g(0) = M, M_s(0) = 0$ with zero metallicity ($Z(0) = 0$)). The equations then simplify:

$$\begin{aligned}
M &= M_s + M_g \\
\frac{\mathrm{d}M}{\mathrm{d}t} &= 0 \\
\frac{\mathrm{d}M_s}{\mathrm{d}t} &= (1-R)\psi(t) \\
\frac{\mathrm{d}M_g}{\mathrm{d}t} &= -(1-R)\psi(t) \\
\frac{\mathrm{d}(ZM_g)}{\mathrm{d}t} &= (1-R)(-Z+y)\psi \\
M_g \frac{\mathrm{d}Z}{\mathrm{d}t} &= (1-R)y\psi(t) \; .
\end{aligned} \tag{244}$$

We can divide $\frac{\mathrm{d}M_g}{\mathrm{d}t}$ by $M_g \frac{\mathrm{d}Z}{\mathrm{d}t}$ to get

$$\frac{1}{M_g} \frac{\mathrm{d}M_g}{\mathrm{d}Z} = -\frac{1}{y} \tag{245}$$

and integrate

$$\begin{aligned}
\ln(M_g)\big|_{M}^{M_g(t)} = \ln\left(\frac{M_g(t)}{M}\right) &= \int_0^{Z(t)} -\frac{\mathrm{d}Z}{y} = -\frac{Z(t)}{y} \\
\Rightarrow Z(t) = y \ln\left(\frac{M}{M_g(t)}\right)
\end{aligned} \tag{246}$$

so we have:

$$\boxed{Z(t) = y \ln \left(\frac{M_g(t=0)}{M_g(t)} \right)} \tag{247}$$

which is the metallicity of gas as a function of only $M_g(t)$.

Metallicity of stars:
In the closed box model, stars and gas must contain all the metals ever produced.

$$Z_s M_s + Z M_g = \int_0^t \int_0^\infty m \rho_{Zm} \psi(t') \phi(m) \mathrm{d}m \mathrm{d}t' \tag{248}$$

where Z_s is the average metallicity of stars and the right side of the equation represents the mass of all metals injected into the interstellar medium until time t. Then

$$Z_s M_s + Z M_g = \int_0^t (1-R) y \psi(t') \mathrm{d}t' \approx (1-R) y \bar{\psi}(t) t \; . \tag{249}$$

We can integrate our equation

$$\frac{\mathrm{d}M_s}{\mathrm{d}t} = (1-R)\psi(t) \tag{250}$$

to get the mass

$$M_s = (1-R)\bar{\psi}(t) t \tag{251}$$

which matches the second equality in equation above. This makes sense since $\bar{\psi}(t)$ is the total stellar mass and $(1-R)$ subtracts the remnants. We can substitute $u = M_g/M$ to get

$$Z_s = y - Z\frac{M_g}{M - M_g} = y - Z\frac{u}{1-u} \; . \tag{252}$$

So we finally get

$$\boxed{\begin{aligned} \text{gas}: Z(s) &= y \ln \left(\frac{1}{u(t)} \right) \\ \text{stars}: Z_s(t) &= y - Z(t)\frac{u(t)}{1-u(t)} \end{aligned}} \tag{253}$$

where u is the gas fraction M_g/M and M is constant for the closed box model. As $u \to 0, Z_s \to y$, which gives the typical metallicity of stars. This must be less than or equal to the typical yield.

G-dwarf problem:
We want to measure the metallicity distribution of G stars. These stars have not evolved much and are still on the main sequence. Their age is so high that they formed from a very low metallicity gas, since $Z(0) = 0$. We can use the closed box result to predict their metallicity distribution. However, we cannot use an average Z_s since we are looking for a distribution.

We apply the closed-box model:

$$M_s(\geq u) = (1-u)M \Rightarrow \frac{M_s(\geq u)}{M_{s,0}} = \frac{1-u}{1-u_0} \tag{254}$$

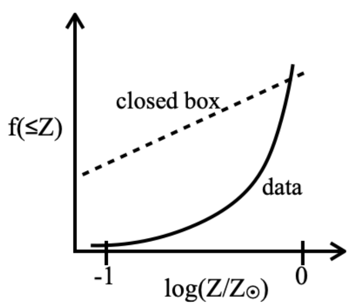
where $M_s(\geq u)$ is the mass of stars formed while the gas fraction was $\geq u$ and $..._0$ refer to present-day values.

Then the stellar mass fraction $M_s(\geq u)/M$ was made from gas with $Z \leq y\ln(1/u)$ and we can rewrite the stellar mass fraction using $u = e^{-Z/y}$ and $u_0 = e^{-Z_0/y}$:

$$\frac{M_s(\leq Z)}{M_{s,0}} = \frac{1-e^{-Z/y}}{1-u_0} = \frac{1-u_0^{Z/Z_0}}{1-u_0}\ . \tag{255}$$

Then we can get the fraction of stellar mass with metallicity $\leq Z$:

$$f(\leq Z) = \frac{M_s(\leq Z)}{M_{s,0}} = \frac{1-u_0^{Z/Z_0}}{1-u_0}\ . \tag{256}$$



However, the model does not agree well with the data because the model is incomplete (infalls, variations in the IMF, etc.).

3.G Active galaxies (AGN)

AGN is Active Galactic Nucleus.
Definition:

- Galaxies whose total luminosity is dominated by radiation not produced in stars. Stars produce near-UV, optical, and near-IR light in blackbodies. Other sources may emit radio or X-ray light.
- The energy generation is associated with a point-like source at the nucleus of the galaxy ($\sim$ black hole with mass $10^6 - 10^9 M_\odot$).

AGN types:

- Radio galaxies: