Part III

CMB, BBN, and Thermal History of the Universe

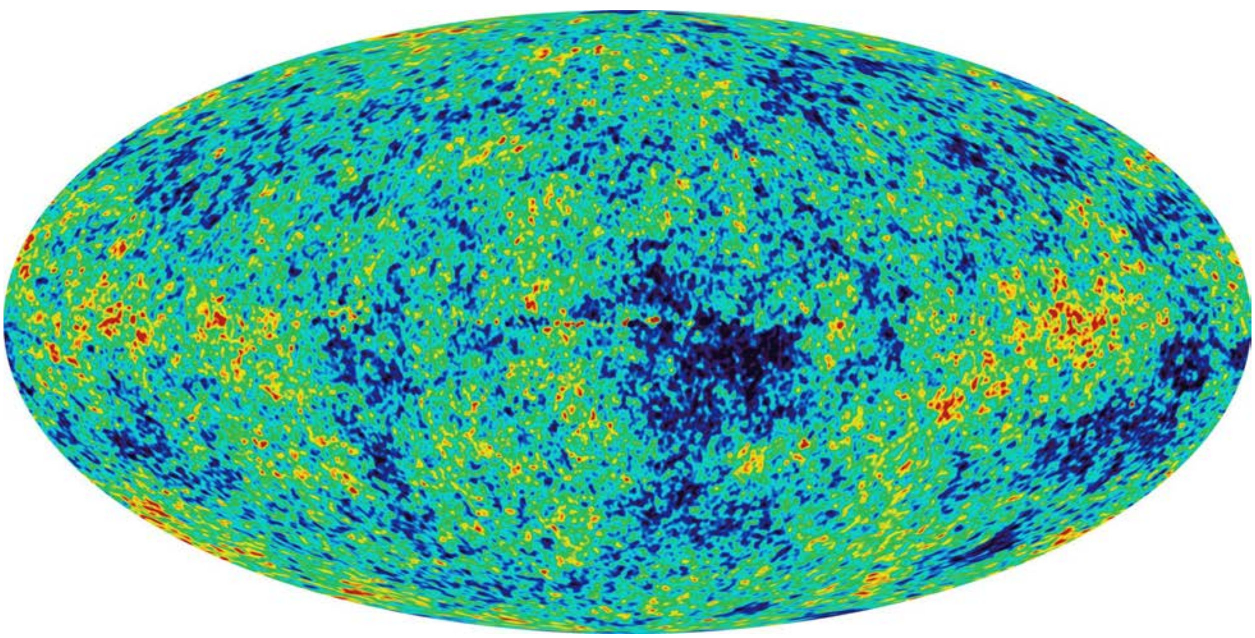


Image: NASA. Image is in the public domain.

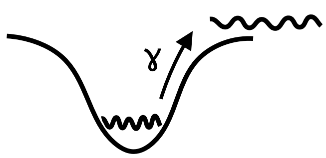
So far, we have mostly discussed the late-time evolution of the Universe (except inflation). We now study the early phases.

1 The Cosmic Microwave Background

1.A Basic picture of the CMB

At $z \sim 1000$, photons decouple from matter (previously coupled due to Thomson scattering). At that time, the dark matter has already formed dark matter potential wells, which leads to perturbations in baryons. This leads to temperature fluctuations in the CMB $\frac{\delta T}{T} \sim 10^{-5}$ (note: without dark matter, we would expect $\frac{\delta T}{T} \sim 10^{-3}$). This leads to anisotropies in the CMB.

Primary anisotropies:
These are anisotropies caused by properties of the CMB.

- Large scales:
  - 

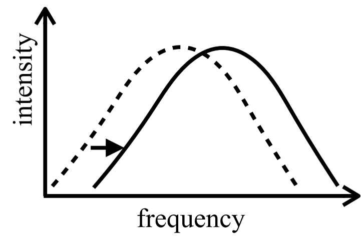
    Dark matter potential wells lead to gravitational redshift and gravitational time delay. Photons were scattered earlier (and then delayed), so they were higher temperature.

    Both effects (gravitational redshift and time delay) always happen together. This is the *Sachs-Wolfe effect*.
  - Doppler effect due to the peculiar motion of electrons.
- On scales larger than the horizon, baryons follow dark matter, leading to higher temperatures in dark matter wells.
- On scales smaller than the horizon, baryons feel radiation pressure. This leads to *baryonic acoustic oscillations* (BAO).
- On very small scales, the imperfect coupling between photons and electrons leads to diffusion. Fluctuations are smeared out and damped on scales $\leq 5'$. This is called *Silk damping*.

Secondary anisotropies:
These impact the measurements of the CMB due to effects on photons as the travel from the CMB to us.

- Thomson scattering of CMB photons: the Universe was reionized by the first stars, galaxies, and quasars between $z \sim 1000$ and $z \sim 6$. These photons then experience Thomson scattering with free electrons as they travel through space. The scattering is isotropic, so it results in an overall reduction of CMB anisotropies.

- Integrated Sachs-Wolfe effect: photons experience gravitational potential and time delays as they travel through structures in the Universe.
- Gravitational lensing from structures in the Universe.
- Sunyaev-Zel'dovich (SZ) effect: CMB photons passing through the hot intergalactic medium of galaxies Thomson scatter with electrons. This reduces the intensity for lower frequencies and increases the intensity for large frequencies, resulting in a shift in the spectrum.



1.B Describing anisotropies and the fluctuation spectrum:

We focus now on understanding the primary anisotropies. We have three main effects:

- Large scales: Sachs-Wolfe and Doppler effects roughly compensate each other. The photons then provide an imprint of the dark matter distribution.
- Smaller scales: Baryonic acoustic oscillations of the photon-baryon plasma.
- Smallest scales: Silk damping due to photon diffusion.

We need to quantify the temperature fluctuations on the sky. We decompose the fluctuations into spherical harmonics:

$$T(\vec{\theta}) = \sum_{l,m} a_{lm} Y_l^m(\vec{\theta}) \tag{459}$$

where $\vec{\theta} = (\theta, \varphi)$ and a_{lm} are the complex coefficients

$$a_{lm} = \int_0^{2\pi} \mathrm{d}\varphi \int_0^{\pi} \mathrm{d}\theta \sin\theta T(\theta,\varphi) Y_{l_1}^{m*}(\theta,\phi) \tag{460}$$

since

$$\int_0^{2\pi} \mathrm{d}\varphi \int_0^{\pi} \mathrm{d}\theta \sin\theta Y_{l_1}^{m_1 *}(\theta,\varphi) Y_{l_2}^{m_2}(\theta,\varphi) = \delta_{l_1 l_2}\delta_{m_1 m_2} \ . \tag{461}$$

We define the *power spectrum*:

$$C_l = \left\langle |a_{lm}|^2 \right\rangle \tag{462}$$

averaging over m. We often plot $l(l+1)C_l$ and define this as the amplitude of fluctuations on the angular scale $\theta \sim \frac{\pi}{l} = \frac{180^\circ}{l}$. $l = 1$ is the dipole anisotropy due to the motion of Earth and $l = 2$ is the quadrupole anisotropy.

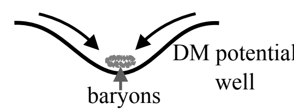
Fluctuations on large scales:
At $z = z_{\mathrm{rec}} \sim 1000$, there is a characteristic scale the horizon with angle:

$$\varphi_{\mathrm{horizon,rec}} \approx 1.7^\circ \sqrt{\Omega_{m,0}} \tag{463}$$

so for a flat universe: $\varphi_{\text{horizon,rec}} \approx 1.7°$. Then for $\theta \gg 1.7°$, large-scale effects dominate (Sachs-Wolfe and Doppler) and there are no baryonic acoustic oscillations. Then C_l reflect the matter power spectrum $P(k)$ on large scales. For $P(k) \propto k$ (Harrison-Zel'dovich spectrum), $l(l+1)C_l$ is approximately constant for $l \ll \frac{180°}{1.7°} \approx 100$.

Fluctuations on small scales:
For $\theta \ll 1.7°$, physical effects can act.

The baryon-photon fluid has a sound speed of $c_s \approx c/\sqrt{3}$. Then the largest wavelengths such that the wave can have half an oscillation (compression) until $z_{\text{rec}}, t_{\text{rec}}$ is:



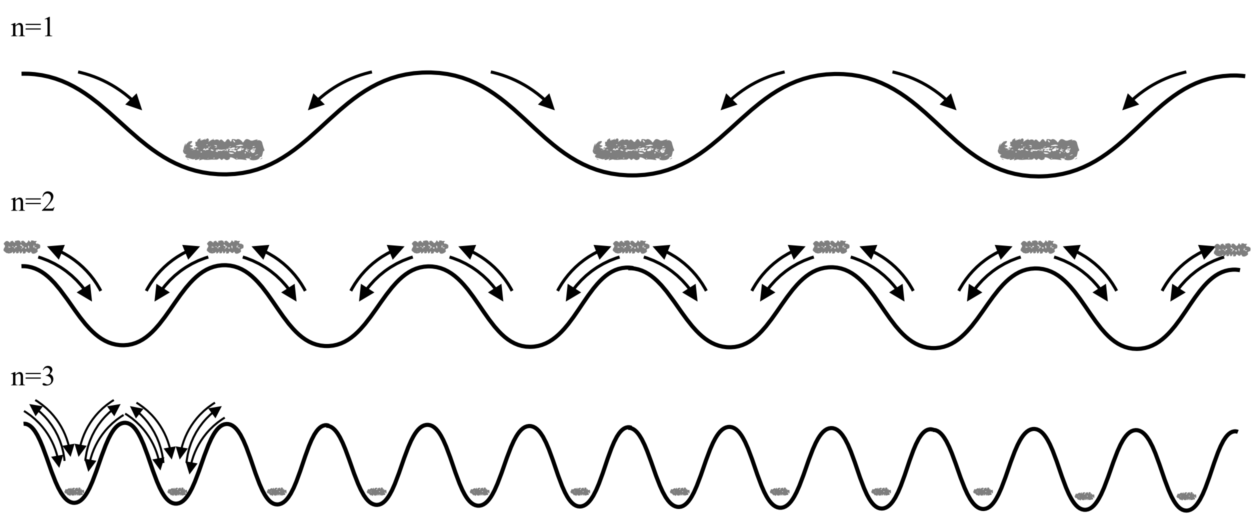
$$\lambda_{\max} = t_{\text{rec}} c_s = t_{\text{rec}} \frac{c}{\sqrt{3}} \,. \tag{464}$$

The sound horizon is $\frac{1}{\sqrt{3}}$ times smaller than the horizon. The angular scale is then

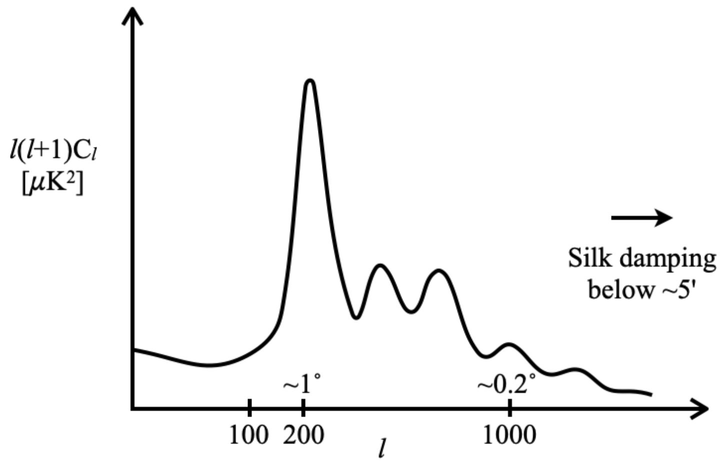
$$\begin{aligned} \theta_1 &\approx \frac{1.7°}{\sqrt{3}} \sim 1° \\ l_1 &\approx 200 \end{aligned} \tag{465}$$

so we can expect the first peak in $l(l+1)C_l$ there since baryons are compressed. Adiabatic compression and the Doppler effect lead to temperature fluctuations on that scale.

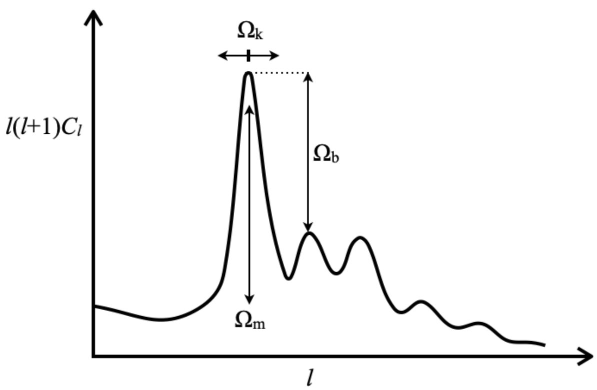
The second peak occurs for scales for which one full oscillation is possible and so forth:



Peaks happen at stationary points of oscillations. We can draw the power spectrum:

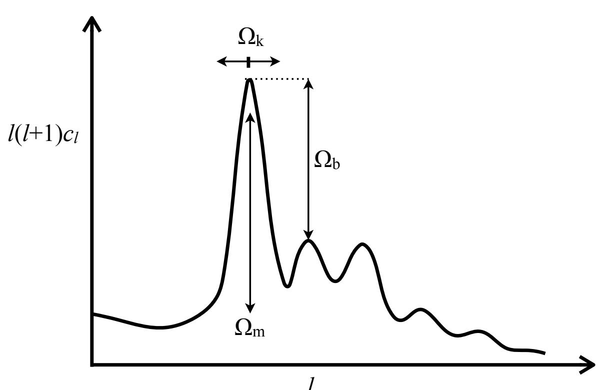


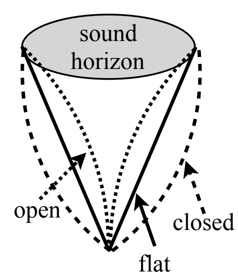
1.C Cosmology with the CMB



We can derive the cosmological parameters Ω_k, Ω_m, and Ω_b from the first and second peaks of the power spectrum.

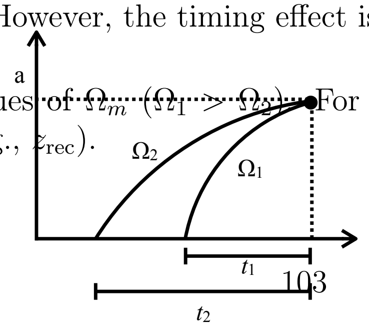
- Derive Ω_k f...
  The actual ...
  scale/angul...

  In an open ...
  will appear ...
  In a closed ...
  for the first ...

- Derive Ω_m from the first peak:
  A naive assumption might lead us to believe that more matter means more gravity and so bigger peaks. However, the timing effect is more important!

  Consider two values of Ω_m ($\Omega_1 > \Omega_2$). For larger Ω_m, the universe is younger for a given redshift (e.g., z_{rec}).