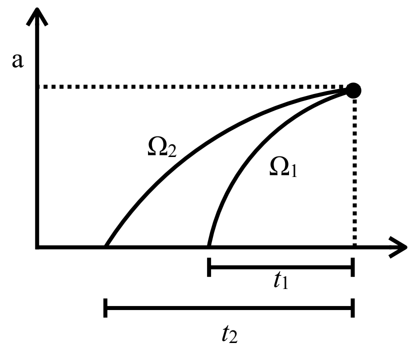
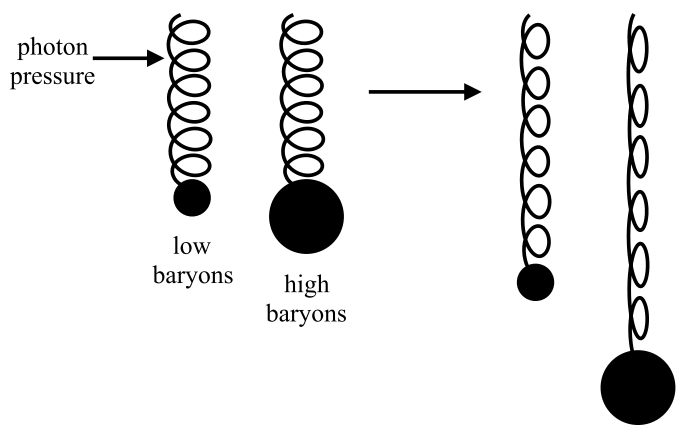



$\Omega_1 > \Omega_2$ leads to $t_1 < t_2$, so there's not as much time to form structures and we get a smaller peak!

Note: based on the first peak, we get $\Omega_m$ and $\Omega_k$, so also $\Omega_\Lambda$ (assuming flat universe)!

- Derive $\Omega_b$ from the second peak:
  $\Omega_b$ is degenerate with $\Omega_m$, so we need the second peak. This can also be derived from Big Bang nucleosynthesis.

  The idea is that a higher baryon mass is like adding mass to a spring ("baryon loading"). More mass causes a deeper fall:




  With more loading, the mass falls deeper but rebounds to the same position. Thus, odd peaks are associated with compression—i.e., how deep the baryons fall into the well. Those peaks get enhanced with more baryons, so the second peak is compressed compared to the first peak. We can therefore constrain $\Omega_b$ with the ratio of the two peaks.

# 2 Thermal history of the Universe

The main idea is that the Universe was very hot in the beginning since $T \propto (1+z)$. For a particle with mass $m_x$ and temperature such that $kT \gtrsim m_x c^2$, we have creation and annihilation reactions. Once $T$ falls low enough, we get *freeze-out* and the reactions stop, freezing the abundance of those particles. (Note: $1\,\mathrm{eV} = 1.1605 \times 10^4\,k_B\mathrm{K} \Rightarrow 1\,\mathrm{eV} \leftrightarrow 10^4\,\mathrm{K}$.)

We first discuss in some more depth the freeze-out of dark matter, which happens around $10-100\,\mathrm{GeV}$. We then briefly discuss the remaining thermal history for temperatures below $\sim 16\,\mathrm{GeV}$ (standard model physics). Big Bang nucleosynthesis will be discussed in the next chapter.

## 2.A Thermal history of dark matter

Where does dark matter come from? At early times ($T > 10^{12}$ K $\cong$ 100 GeV), we have $kT \geq m_x c^2$ for leading dak matter candidates. For non-relativistic particles in equilibrium:

$$n_{\mathrm{eq}} = g \left(\frac{mkT}{2\pi\hbar^2}\right)^{3/2} e^{-\frac{mc^2}{kT}} \,. \tag{466}$$

We have equilibrium between creation and annihilation:

$$x + \bar{x} \leftrightharpoons 2\gamma \,. \tag{467}$$

For creation rate $\psi$ and annihilation rate $n^2\langle\sigma v\rangle$, where $\langle\sigma v\rangle$ is the velocity averaged cross section for annihilation, we have:

$$= n_{\mathrm{eq}}^2 \langle\sigma v\rangle \,. \tag{468}$$

At late times, $kT$ falls below $mc^2$, so $e^{-mc^2/kT} \to 0$. If annihilation continues to happen, no particles will be left since new particles cannot be created at low temperatures, which would leave no *relic abundance*. However, the annihilation rate $n^2\langle\sigma v\rangle$ also goes down since $n \propto a^{-3}$. If there's no creation or annihilation:

$$\begin{aligned}
\text{comoving: } \frac{\mathrm{d}n_c}{\mathrm{d}t} &= 0 \quad \left(n_c = n\left(\frac{a}{a_0}\right)^{-3}\right) \\
\frac{\mathrm{d}n}{\mathrm{d}t} &= -3\frac{\dot{a}}{a}n \\
\Rightarrow \frac{\mathrm{d}n}{\mathrm{d}t} &+ 3Hn = 0
\end{aligned} \tag{469}$$

Thus, annihilation will stop and then there will be a relic abundance. The abundance equations with reactions is:

$$\frac{\mathrm{d}n_c}{\mathrm{d}t} = -\langle\sigma v\rangle \left(n_c^2 - n_{c,\mathrm{eq}}^2\right) \tag{470}$$

so the reactions drive $n_c$ towards the equilibrium value.

There are two competing timescales: the expansion of the Universe and the mean interaction timescale. We can rewrite the above equation:

$$\begin{aligned}
\frac{\mathrm{d}n_c}{\mathrm{d}a}\underbrace{\frac{\mathrm{d}a}{\mathrm{d}t}}_{\dot{a}} &= -\langle\sigma v\rangle n_{c,\mathrm{eq}}^2 \left[\left(\frac{n_c}{n_{c,\mathrm{eq}}}\right)^2 - 1\right] \\
&\left(H = \frac{\dot{a}}{a}\right) \\
\frac{a}{n_{c,\mathrm{eq}}}\frac{\mathrm{d}n_c}{\mathrm{d}a} &= -\frac{\langle\sigma v\rangle n_{c,\mathrm{eq}}}{H}\left[\left(\frac{n_c}{n_{c,\mathrm{eq}}}\right)^2 - 1\right] \,.
\end{aligned} \tag{471}$$

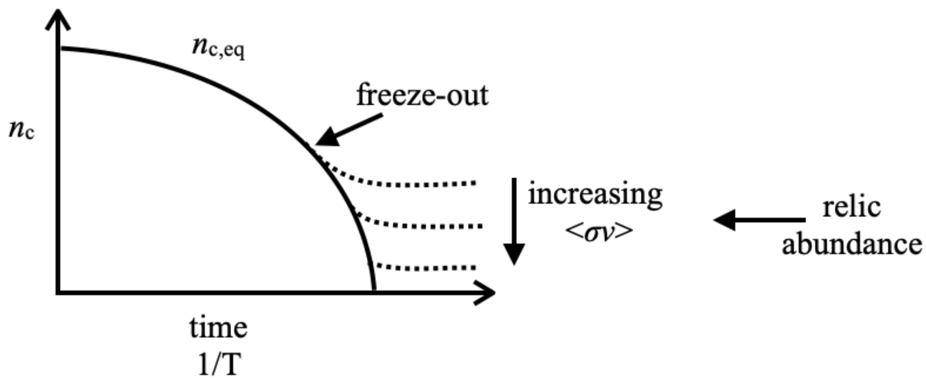
There are two timescales in this equation: $\tau = 1/H$ and $\tau_{\text{coll}} = 1/(n_{\text{eq}}\langle\sigma v\rangle)$. Then

$$\frac{a}{n_{c,\text{eq}}}\frac{\mathrm{d}n_c}{\mathrm{d}a} = -\frac{\tau_H}{\tau_{\text{coll}}}\left[\left(\frac{n_c}{n_{c,\text{eq}}}\right)^2 - 1\right] . \tag{472}$$

We have two regimes that give us two solutions:

- At early times, $\tau_{\text{coll}} \ll \tau_H \Rightarrow n_c \approx n_{c,\text{eq}}$.
- At late times, $\tau_{\text{coll}} \gg \tau_h \Rightarrow n_c \approx \text{constant} \approx n_{c,\text{eq}}(z_{\text{freeze}})$.

At redshift $z_{\text{freeze}}$, we have $\tau_{\text{coll}} \sim \tau_H$, so particles freeze out of equilibrium and the comoving number density stays fixed.




From observed relic abundances, we get $m$ and $\sigma$ at the electroweak scale, which is predicted for WIMPs! This is known as the WIMP miracle. So far, however, nothing has been detected.

**Hot and cold dark matter**:
Are particles moving relativistically (hot dark matter) or non-relativistically (cold dark matter) at freeze-out? Particles become non-relativistic when:

$$3kT(t_{\text{nr}}) \approx mc^2 . \tag{473}$$

There are two cases:

- $t_{\text{nr}} > t_{\text{freeze}} \Rightarrow$ hot relic and hot dark matter
- $t_{\text{nr}} < t_{\text{freeze}} \Rightarrow$ cold relic and cold dark matter

**Hot dark matter**:
Consider an analogy to the Jeans length, the free-streaming length:

$$\lambda = v\sqrt{\frac{\pi}{G\rho}} . \tag{474}$$

This wipes out structures on small scales! Since $v \approx c$, scales below the horizon are suppressed. But particles slow down due to expansion:

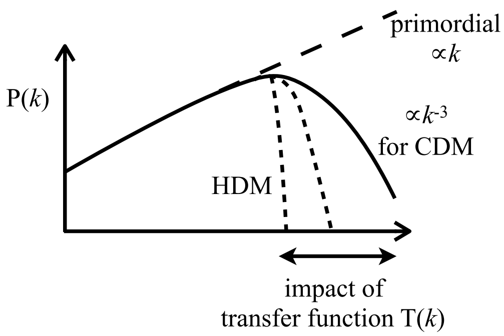
$$v_{\text{pec}} = v - Hd \Rightarrow v \propto a^{-1} \tag{475}$$

and the particle becomes non-relativistic once $mc^2 \sim 3kT$ (relic time and temperature), so:

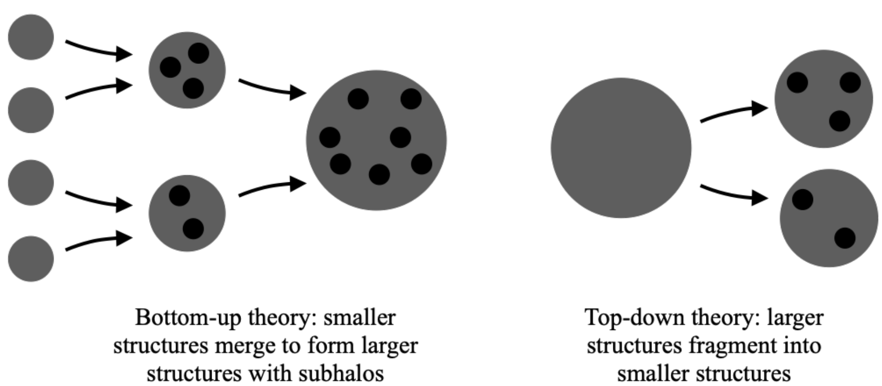
$$\begin{aligned} t &\sim 2 \times 10^{12} s \left( \frac{mc^2}{2\,\text{keV}} \right)^{-2} \\ l_h &\sim ct_h \sim 60\,\text{Mpc} \left( \frac{mc^2}{3\,\text{keV}} \right)^{-1} \end{aligned} \tag{476}$$

so hot dark matter erases all structures below $l_h$ due to free-streaming.

**Cold dark matter**:
Cold dark matter is already non-relativistic at freeze-out, so structures can grow. It still has some free-streaming scale, but it is much smaller.




Hot dark matter would not be captured by small potential wells, so it needs large potential wells to form structures. This leads to top-down structure formation, where large large structures form first and fragment into smaller structures. Cold dark matter can form small halos that merge into larger ones in bottom-up formation.




## 2.B Thermal history of the Universe and other particles

- $T \sim 10^{19}$ GeV, $t \sim 10^{-43}$ s:
  quantum gravity regime
- $T \sim 10^{16}$ GeV, $t \sim 10^{-38}$ s:
  GUT phase transition: strong and electroweak interactions are indistinguishable at earlier times

- $T \sim 10^{12}\,\text{GeV}, t \sim 10^{-30}\,\text{s}$:
  Peccei-Quinn phase transition, if PQ mechanism is the correct explanation for the strong CP problem

- $T \sim 10\text{s} - 100\text{s}\,\text{GeV}, t \sim 10^{-8}\,\text{s}$:
  WIMPs freeze out

- $T \sim 100 - 300\text{s}\,\text{MeV}, t \sim 10^{-5}\,\text{s}$:
  quark-hadron phase transition: quarks and gluons first become bound into neurons and protons

- $T \sim 0.1\,\text{MeV} - 10\,\text{MeV}, t \sim \text{seconds} - \text{minutes}$:
  Big Bang nucleosynthesis (BBN): neutrons and protons first combine to form D, $^4$He, $^3$He, and $^7$Li nuclei

- $T \sim \text{ keV}, t \sim 1\,\text{day}$:
  photons fall out of equilibrium, and the number density of photons is conserved

- $T \sim 3\,\text{eV}, t \sim 10^{4-5}\,\text{yrs}$:
  matter-radiation equality: energy density is dominated by photons at earlier times

- $T \sim \text{eV}, t \sim 400,000\,\text{yrs}$: electrons and protons combine to form hydrogen

- $T \sim 10^{-3}\,\text{eV}, t \sim 10^{9}\,\text{yrs}$:
  first stars and galaxies form

- $T \sim 10^{-4}\,\text{eV}, t \sim 10^{10}\,\text{yrs}$:
  today

# 3 Big Bang nucleosynthesis

Once protons and neutrons become available, they can fuse into elements. This allows detailed predictions about the abundance of the first stars.

**Proton/neutron reactions**:
After $n, p$ production from the gluon-gluon plasma:

$$\begin{aligned} n + \nu_e &\leftrightharpoons p + e^- \\ n + e^+ &\leftrightharpoons p + \bar{\nu}_e \end{aligned} \tag{477}$$

with weak interactions mediated by neutrinos and

$$n_{\text{eq}} = g \left( \frac{mkT}{2\pi\hbar} \right)^{3/2} e^{-\frac{mc^2}{kT}} \; . \tag{478}$$

Protons and neutrons have $g_n = g_p = 2$, so

$$\frac{n_n}{n_p} = \left( \frac{m_n}{m_p} \right)^{3/2} e^{-\frac{(m_m - m_p)c^2}{kT}} \; . \tag{479}$$