- $T \sim 10^{12}$ GeV, $t \sim 10^{-30}$ s:
  Peccei-Quinn phase transition, if PQ mechanism is the correct explanation for the strong CP problem
- $T \sim 10\text{s} - 100\text{s}$ GeV, $t \sim 10^{-8}$ s:
  WIMPs freeze out
- $T \sim 100 - 300\text{s}$ MeV, $t \sim 10^{-5}$ s:
  quark-hadron phase transition: quarks and gluons first become bound into neurons and protons
- $T \sim 0.1$ MeV $- 10$ MeV, $t \sim$ seconds – minutes:
  Big Bang nucleosynthesis (BBN): neutrons and protons first combine to form D, ^{4}He, ^{3}He, and ^{7}Li nuclei
- $T \sim$ keV, $t \sim 1$ day:
  photons fall out of equilibrium, and the number density of photons is conserved
- $T \sim 3$ eV, $t \sim 10^{4-5}$ yrs:
  matter-radiation equality: energy density is dominated by photons at earlier times
- $T \sim$ eV, $t \sim 400,000$ yrs: electrons and protons combine to form hydrogen
- $T \sim 10^{-3}$ eV, $t \sim 10^9$ yrs:
  first stars and galaxies form
- $T \sim 10^{-4}$ eV, $t \sim 10^{10}$ yrs:
  today

3 Big Bang nucleosynthesis

Once protons and neutrons become available, they can fuse into elements. This allows detailed predictions about the abundance of the first stars.

Proton/neutron reactions:
After n, p production from the gluon-gluon plasma:

$$\begin{aligned} n + \nu_e &\leftrightharpoons p + e^- \\ n + e^+ &\leftrightharpoons p + \bar{\nu}_e \end{aligned} \tag{477}$$

with weak interactions mediated by neutrinos and

$$n_{\text{eq}} = g \left(\frac{mkT}{2\pi\hbar} \right)^{3/2} e^{-\frac{mc^2}{kT}} \; . \tag{478}$$

Protons and neutrons have $g_n = g_p = 2$, so

$$\frac{n_n}{n_p} = \left(\frac{m_n}{m_p} \right)^{3/2} e^{-\frac{(m_m - m_p)c^2}{kT}} \; . \tag{479}$$

So before freeze-out of the above reactions, we have:

$$\frac{n_n}{n_p} \approx e^{-\frac{1.29\,\mathrm{MeV}}{kT}} \,. \tag{480}$$

Weak interactions stop once neutrinos freeze out, $T \sim 0.8\,\mathrm{MeV}, t \sim 1\,\mathrm{s}$:

$$\frac{n_n}{n_p} \approx e^{-1.29/0.8} \approx 0.2 \,, \tag{481}$$

a 5-to-1 ratio. Now nucleosynthesis starts.

Deuteron fusion:
We have the strong reaction:

$$p + n \leftrightharpoons D + \gamma \tag{482}$$

with the binding energy of deuteron approximately 2.22 MeV. At the time of neutrino freeze-out, the temperature is already smaller than 2.22 MeV, but there are so many more photons than baryons. The high energy tail of photons is still sufficient to destroy deuteron, so we need $k_B T \ll 2.22\,\mathrm{MeV}$ to efficiently form deuteron! (The deuteron fusion bottleneck means that 4_{He} fusion afterwards is quick.)

The time delay needed for the temperature to drop below 2.22 MeV causes neutrons to decay through β-decay before they can fuse to deuteron. Without fusion to deuteron, all neutrons would be gone!

Note: If we assume the deuteron fusion is instantaneous, what helium/baryon mass fraction (Y) would we get?
All neutrons would fuse into 4_{He}:

- think of a group of 12 nucleons: $10p + 2n$ (5:1 ratio, see above)
- all neutrons fuse into 4_{He}, so we get one 4_{He} atom and eight free protons
- $$\Rightarrow Y = \frac{4}{4+8} = \frac{4}{12} \approx 0.33 \tag{483}$$

  but we observe 0.24, which is lower due to β-decay.

We now do the precise calculation:

- $p + n \leftrightharpoons D + \gamma$ never freezes out. It stops once all neutrons are used up. We can use the Saha equation to find the abundance:

  $$\frac{n_D}{n_p n_n} = \frac{g_D}{g_p g_n}\left(\frac{m_D}{m_p m_n}\right)^{3/2}\left(\frac{kT}{2\pi\hbar^2}\right)^{-3/2} e^{\frac{2.22\,\mathrm{MeV}}{kT}} \tag{484}$$

  where $g_p = g_n = 2$ (2 spin configurations) and $g_D = 3$ (3 spin configurations: $\uparrow\uparrow, \downarrow\downarrow, \uparrow\downarrow$). We also know $m_p = m_n = m_D/2$, so:

  $$\frac{n_D}{n_p n_n} = 6\left(\frac{m_n kT}{\pi\hbar^2}\right)^{-3/2} e^{\frac{2.22\,\mathrm{MeV}}{kT}} \tag{485}$$

- Protons always outnumber neutrons, so define the time of deuteron fusion is the time when half the neutrons have fused into deuteron: $n_D = n_n$. Then from the Saha equation:

$$\frac{n_D}{n_n} = 1 = 6n_p \left(\frac{m_n kT}{\pi\hbar^2}\right)^{-3/2} e^{\frac{2.22\,\mathrm{MeV}}{kT}} \tag{486}$$

We now want to know when this happens. We can relate n_p to the temperature to calculate the corresponding temperature and time. We can relate n_p to n_b, which we can relate to n_γ through the fixed baryon-to-photon ratio η.

$$n_b = n_p + n_n = \frac{6}{5} n_p \tag{487}$$

since $n_n = 0.2n_p = \frac{1}{5}n_p$. Then, for a black-body,

$$\frac{n_p}{n_b} = \frac{5}{6} \Rightarrow n_p = \frac{5}{6}\eta n_\gamma = \frac{5}{6}\eta\left(0.24\left(\frac{kT}{\hbar c}\right)^3\right) \tag{488}$$

so we get:

$$\begin{aligned} 1 &= 6\frac{5}{6}\eta\left(\frac{kT}{\hbar c}\right)^3 \cdot 0.24\left(\frac{m_n kT}{\pi\hbar^2}\right)^{-3/2} e^{\frac{2.22\,\mathrm{MeV}}{kT}} \\ &= \frac{0.24\cdot 5}{1.25}\eta\left(\frac{(kT)^2}{)\hbar c)^2}\frac{\pi\hbar^2}{m_n kT}\right)^{3/2} e^{\frac{2.22\,\mathrm{MeV}}{kT}} \\ &= \eta\underbrace{\pi^{3/2}\cdot 1.25}_{5.5\cdot 1.25 = 6.9}\left(\frac{kT}{m_n c^2}\right)^{3/2} e^{\frac{2.22\,\mathrm{MeV}}{kT}} \\ &\approx 6.9\eta\left(\frac{kT}{m_n c^2}\right)^{3/2} e^{\frac{2.22\,\mathrm{MeV}}{kT}} \\ &\text{(fiducial } \eta \sim 5\times 10^{-10}) \\ &\approx 3.4\times 10^{-9}\left(\frac{kT}{m_n c^2}\right)^{3/2} e^{\frac{2.22\,\mathrm{MeV}}{kT}} \end{aligned} \tag{489}$$

So we get

$$T \approx 8\times 10^8\,\mathrm{K}, t \approx 200\,\mathrm{s} \tag{490}$$

for the time of deuteron fusion!

- We lose neutrons through β-decay with a half-life $t_{1/2} = 890\,\mathrm{s}$. After $200\,\mathrm{s}$,

$$\frac{n_n}{n_p} \approx 0.15 < 0.2 \tag{491}$$

so we get a helium-to-baryon ratio:

$$Y = \frac{4n_{\mathrm{He}}}{4n_{\mathrm{He}} + n_{\mathrm{H}}} = \frac{2n_n}{2n_n + (n_p - n_n)} = \frac{2n_n}{n_p + n_n} \tag{492}$$

since $n_{\mathrm{He}} = n_n/2$ (every 4_{He} nucleus has $2n$) and $n_{\mathrm{H}} = n_p - n_n$ (since 4_{He} nucleus has equal number of protons and neutrons), leaving us with

$$Y = \frac{2(n_n/n_p)}{1+(n_n/n_p)} \approx 0.25 \tag{493}$$

Notes:

- Large Ω_b leads to larger η, so deuteron can form earlier and there is less neutron decay. This leads to a larger n_n/n_p, so Y increases with Ω_b.
- Measurements of 4_{He} and D allow us to determine η and Ω_b. Deuteron abundance is a sensitive measure for Ω_b, and can be found with the Lyman-α forest relating line strength of H and D.