8.902 Fall 2023 - Problem Set #2

Due Tuesday, October 3

For problem 1, it is recommended not to write your own orbit integrator from scratch. Many packages (python, IDL, matlab, mathematica) have routines available for integrating differential equations (see also Numerical Recipes for an excellent discussion). However, you should understand how the solution progresses at each time step. For full credit, you should include in your solutions the equations you used as far as you can on paper, and then print out and submit your software code with the materials you turn in.

1 Orbits in a Logarithmic Potential

Consider a star orbiting in a spherically symmetric logarithmic potential,

$$\Phi(r) = v_c^2 \ln(r/r_{\rm ref})$$

Suppose that at apocenter it has a speed $v_a = v_c/2$. Integrate the orbit numerically, and plot the position in the orbital plane at equal timesteps of roughly 1/50th the radial period. Plot 50 to 100 orbits. Depending on the integration scheme you use, you may need to use time steps that are smaller than the plotting interval. Plot orbits both with continuous lines and with a symbol that gives some idea of the probability density and briefly comment on the results. **(Triple weight; counts 3x as much as Problem 2).**

2 The Mestel Disk

It is possible to define potentials that are rotationally symmetric rather than spherically symmetric. One well known example is the Mestel disk, a flattened potential written as:

$$\Phi(r,\theta) = v_c^2(\ln \frac{r}{r_{\rm ref}} + \ln \frac{1+|cos\theta|}{2})$$

A) Show that the Mestel potential produces a flat rotation curve in the disk plane.

B) Show that for this potential, the density distribution has the unusual property of being zero for all locations outside of the z = 0 plane, and infinite within this plane.

C) Now define the surface density of the disk, as the total density projected in the direction of the z axis. Show that in this configuration, the surface density for the Mestel disk is actually finite, and given as

$$\Sigma(r,\theta) = \int_{-\infty}^{\infty} \rho(\vec{r})dz = \frac{v_c^2}{2\pi G r}$$