8.902 Fall 2023 - Problem Set #3

Due Thursday, October 12

1 Oort Constants

Assume the Milky Way is infinitely thin, and that its stellar velocities are perfectly described by a differentially rotating disk with velocity profile $v_c(R)$.

A) Under these assumptions, show that the radial and transverse components of the observed velocities of stars relative to the Sun are given by:

$$v_r = Ad\sin(2l) \qquad\qquad v_T = d(A\cos(2l) + B)$$

where l represents galactic longitude, d is the distance between the Sun and a nearby star and

$$A = \frac{1}{2}\left(\frac{v_c}{R} - \frac{dv_c}{dR}\right)_{R_0} \qquad\qquad B = -\frac{1}{2}\left(\frac{v_c}{R} + \frac{dv_c}{dR}\right)_{R_0}$$

where R_0 is the orbital radius of the Sun. Assume that $d << R_0$.

Measurements of stellar proper motions and radial velocities in the solar neighborhood give the following values for the Oort constant (found from the Hipparcos satellite):

$$A = 14.82 \pm 0.84 \text{ km/s/kpc} \qquad\qquad B = -12.37 \pm 0.64 \text{ km/s/kpc}$$

Assuming R_0=8.5 kpc, derive expressions for the following quantities in terms of Oort's A and B and R_0, and calculate their numerical values at the solar orbit (i.e., the local standard of rest). (For these calculations use best-fit values of A and B given above–you need not propagate errors):

B) The circular velocity v_0 and the orbital period of the Sun about the Galactic center.

C) The local velocity shear, $\frac{dv_0}{dR}$

D) The local epicyclic frequency, k.

E) The number of epicyclic oscillations experienced during each orbit of the guiding center about the galactic center

2 Free-Fall Time (B&T 3.4)

Prove that if a homogenous sphere of a pressureless fluid with density ρ is released from rest, it will collapse to a point in time $t_{\rm ff} = \sqrt{\frac{3\pi}{32G\rho}}$. The time $t_{\rm ff}$ is called the **free-fall time** of a system with density ρ.

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