8.902 Fall 2023 - Midterm Problem Set

Due Tuesday, October 24

This problem set is replacing the midterm exam, so its question style is slightly different. The same set of questions were given as a midterm in previous years, so it will be useful when studying for the final exam to refer to this assignment for question style. Please complete this assignment as a regular problem set, e.g. use notes, collaborate with other students, etc. It will be graded and weighted as a normal problem set.

Question 1 (25 Points)

Consider a gas disk of surface density Σ and radius R, rotating with angular speed Ω. We will examine the orbit of a test particle at the periphery of the disk. Assume as usual that the angular momentum is conserved in the system.

A) What is the radial epicyclic frequency κ for nearly circular orbits at the outer radius R of the disk?

B) What is the minimum radius $R_{\min}$ for which $\kappa^2 > 0$?

C) What happens if $R < R_{\min}$?

D) Why doesn't this occur routinely in the Milky Way for disk structures with $R < R_{\min}$?

Question 2 (25 Points)

In class we derived a simple closed-box model describing the chemical evolution of the ISM. The mass-weighted metallicity Z of the gas at any time in this model has a simple relation to the stellar yield y, assuming y is time-independent:

$$Z = -y \ln \mu$$

Here μ represents the mass fraction of the total (stars+gas) system contained in the gas phase at that time. For this problem, we will consider the metallicities of all stars.

A) Derive and explain why in this model the total fraction of stellar mass with metallicity less than a given value Z is given by the following epxression:

$$M_s(< Z) = \frac{1 - e^{-Z/y}}{1 - e^{-Z_0/y}}$$

where Z_0 represents the currecnt value of the ISM's gas-phase metallicity. To obtain full credit, you must write 2-3 sentences explaining the reasoning behind your asnwer – a formula by itself will receive only half credit for this section. You may use the usual instantaenous recycling approximation.

B) The result from (A) specifies the cumulative metallicity distribution of stars. Typically observers measure metallicities in logarithmic units. Derive an expression for the differential metallicity distribution $\mathrm{d}M_S/\mathrm{d}(\log Z)$ and sketch the result. At what value of Z does this distribution peak? Express your answers in terms of y and Z_0.

Question 3 (15 points)

If only 2-body relaxation processes are taken into account, will a galaxy made up of 10^6 solar mass globular clusters relax more or less quickly than a galaxy of the same mass, but made up of individual stars? Give both a quantitative answer, and a short qualitative explanation of why the result turns out that way.

Question 4 (15 points)

Use the Virial Theorem to derive the typical velocity of stars in the Milky Way.

Short Answer Section (5 points each, 20 total)

Provide answers to the following questions in a few sentences to a paragraph. Use diagrams to support your text where appropriate.

A) What is the Navarrao-Frenk-White (NFW) profile?

B) What is violent relaxation? How does it work? Why is it important?

C) Write down the collisionless Boltzmann equation. How can you derive the Jeans equation from this?

D) How likely is a strong encounter between two stars in the Milky Way?

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