Massachusetts Institute of Technology
Department of Physics

Physics 8.942 Fall 2001

Problem Set #4
Due in class Tuesday, October 16, 2001.

1. **Temperature vs. time**
Assume that we live in a Robertson-Walker universe with matter, radiation and curvature. The present mass density is $\rho_m = 3\Omega_0 H_0^2/(8\pi G)$, where $H_0 = 100\,h\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ and $\Omega_0(1+\rho_{\mathrm{r},0}/\rho_{\mathrm{m},0}) \le 1$ (i.e., $k \le 0$). The present radiation temperature is $T_0 = 2.725\,\mathrm{K}$. Assume that only photons, with present temperature $T_0 = 2.725\,\mathrm{K}$ contribute to the radiation; ignore neutrinos in this problem.

   a) At what redshift did the radiation energy density equal the matter density? How old was the universe then and what was the radiation temperature? Your answers should scale appropriately with Ω_m and h.

   b) How old was the universe when the radiation temperature was 1 MeV? 1 GeV? 10^{14} GeV? (Hint: you need g_*, the effective number of relativistic spin states contributing to the energy density. At 1 GeV $g_* = 61.75$ and at 10^{14} GeV, $g_* = 106.75$ without supersymmetry or double this with SUSY.)

2. **Massless and massive neutrinos**
Besides photons, there are 3 flavors of neutrinos, each with 2 spin states (neglecting right-handed neutrinos) which may be relativistic today. Assume that the numbers of neutrinos and antineutrinos of each flavor are equal.

   a) Assuming that all neutrinos are massless and that the photon temperature is increased by a factor $(11/4)^{1/3}$ as a result of e^+e^- annihilation, derive z_{eq}, the redshift at which (nonrelativistic) matter and radiation (relativistic particles, including photons and neutrinos), have equal energy densities (Peacock eq. 9.3.) What is the ratio of energy densities in neutrinos and photons?

   b) Now suppose that one neutrino flavor (e.g., ν_τ and $\bar{\nu}_\tau$) has nonzero rest mass. Derive the neutrino mass that is needed to close the universe, assuming that the contribution made by other massive particles (nucleons and cold dark matter) is small (Peacock eq. 9.31). Note that the shape of the phase space distribution is invariant after neutrino decoupling. (This calculation was first made by Cowsik and McClelland 1972, *Phys. Rev. Lett.*, **29**, 669.)

3. **Equilibrium recombination**
In terms of the conserved baryon/photon ratio η (eq. 9.83 of Peacock), find the CMB temperature and redshift at which recombination ended, as defined by the condition that the photon mean-free scattering rate equals the expansion rate, $n_e \sigma_T c = H$. Use the Saha equation, eq. 9.45 of Peacock, assuming the parameters of the ΛCDM model of Problem Sets 2 and 3 plus the present CMB temperature $T_0 = 2.725$ K and the baryon abundance $\Omega_B h^2 = 0.02$. (See Burles et al. 2001, ApJ 552, L1 for conversion of $\Omega_B h^2$ to η.) Compare your resulting electron fraction and redshift with the graphs presented in Seager et al., ApJS 128, 407. Is your recombination redshift too high or too low compared with an exact calculation? Why?

4. **Nonstandard nucleosynthesis**
Under which of the following suppositions would primordial nucleosynthesis have produced less ^{4}He than the standard model? Less ^{2}H (deuterium)?

a) Suppose that the baryond density in the universe today is larger than we think.

b) Suppose that there is a fourth neutrino flavor with mass much less than 1 MeV. (It must be "sterile," e.g. right-handed, not to have been detected in Z^0 decay at LEP.)

c) Suppose that there are many more neutrinos than antineutrinos or photons in the cosmic background today.

d) Suppose that there are many more antineutrinos than neutrinos or photons in the cosmic background today.

e) Suppose that there is a significant contribution of gravitational radiation to the total energy density of the universe, comparable in magnitude with the energy density of the microwave background radiation.