

Problem Set #2
Due in class Tuesday, September 25, 2001.

1. Cosmological Redshift as a Doppler Shift

For nearby objects with recession speed $Hr \ll c$, the first-order Doppler shift gives $\lambda_o/\lambda_e = 1 + Hr/c$. To get the redshift factor $R \equiv \lambda_o/\lambda_e$ for a distant object, we can break the light path into many small segments such that $H\Delta r \ll c$ for each, where Δr is the proper length of a segment. Multiplying together the redshift factors for all segments gives the net redshift factor $R = R_1 R_2 \cdots R_n$.

- a) Using the first-order Doppler shift, show that for one segment, lasting in time from t to $t + dt$, the redshift factor is

$$R_t = \exp\left(\frac{1}{a} \frac{da}{dt} dt\right). \quad (1)$$

- b) Multiply the results from all segments to obtain the usual cosmological redshift formula $\lambda_o/\lambda_e = a_o/a_e$ (where usually one sets $a_o = 1$).

2. Cosmic Time-Redshift-Distance Relation

Newspaper articles presenting cosmological discoveries of distant objects usually do not quote redshifts, but instead give the time since the big bang. One such account reported that a galaxy has been found to be fully formed a mere 1 billion years after the big bang. Suppose that you are asked by an astronomer friend what redshift was measured for the galaxy. Additionally, a non-astronomer friend asks you the distance to the galaxy in light-years. After explaining that distance is ambiguous, your friend says alright, just tell me the comoving distance in light-years.

Estimate the redshift and the comoving distance using the following three cosmological models. In each case, assume that the Hubble constant is $72 \text{ km s}^{-1} \text{ Mpc}^{-1}$.

- a) Λ CDM: flat $\Omega_{\text{cdm}} = 1, \Omega_{\Lambda} = 0$;
b) OCDM: open $\Omega_{\text{cdm}} = 0.35, \Omega_{\Lambda} = 0$;
c) Λ CDM: flat $\Omega_{\text{cdm}} = 0.35, \Omega_{\Lambda} = 0.65$.

Note: you may have to perform a numerical integral. The 8.942 webpage has links to information on using Mathematica, Matlab, Maple, as well as a function (C and F77) to perform numerical integration.

3. Angular Size vs. Redshift

An object of linear size L (perpendicular to the line of sight) subtends a small angle α . Suppose that the object and the observer are both comoving and that the redshift of the object is z . For a fixed linear size the angle is a function of redshift: $\alpha = \alpha(z)$. For $\alpha^2 \ll 1$, $\alpha \propto L$. Find the constant of proportionality in terms of τ_e and τ_o (the conformal times when light is emitted by the object and received by the observer). For an Einstein-de Sitter universe ($k = 0$ and $a \propto t^{2/3}$), $\alpha(z)/L$ has a minimum at a certain redshift. What is that redshift? If galaxies formed long ago and all have about the same typical size, say 5 kpc, what is the minimum angular size of a galaxy in an Einstein-de Sitter universe, in arcseconds? Assume a Hubble constant of $72 \text{ km s}^{-1} \text{ Mpc}^{-1}$.

4. Number Counts

Suppose that there exists a population of luminous sources distributed homogeneously throughout space. For simplicity, assume that all sources have identical proper bolometric luminosity L and that they emit isotropically. (If the sources have a distribution of luminosities we could integrate over that later.) The comoving number density of these sources is $n(t)$, i.e., the proper number density at cosmic time t is $(a_0/a)^3 n(t)$ where $a(t)$ is the expansion scale factor and $a = a_0$ today. A terrestrial astronomer surveys the entire sky and finds N sources with measured bolometric flux greater than S . Assuming that we live in a Robertson-Walker universe, find $N(S)$. Evaluate it fully in the simple case $n(t) = \text{constant}$ and an Einstein-de Sitter universe ($k = 0$ matter-dominated). Show that as $S \rightarrow \infty$, $N(S)$ approximates the Euclidean result $N = (4\pi n/3)(L/4\pi S)^{3/2}$. Why? In the opposite limit, $S \rightarrow 0$, what is the behavior of $N(S)$, and why?