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# Cosmography in a Robertson-Walker universe

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# 1 Introduction

Cosmography is the study of the large-scale structure of spacetime. The goal of cosmography is to measure spacetime geometry or, in other words, to determine the spacetime metric. For obvious practical reasons astronomers cannot use clocks and measuring rods or even lasers to measure spacetime structure, making this a challenging endeavor that has yet to fully succeed.

Observational tests of the Robertson-Walker family of cosmological models (or any others) are made by comparing predictions with observations of astronomically measurable quantities such as the redshift, age, angular size, flux (of electromagnetic radiation) and abundance of objects of various types (galaxies, quasars, etc.). Because of the non-Minkowski geometry of spacetime the dependence of these quantities on distance — and even the meaning of distance itself — are more complicated than in special relativity.

In these notes we derive the predictions relating observables in standard cosmological models. Much of this material is also given in Peacock (1999, sections 3.4 and 5.1). Another concise summary is given by Hogg (1999, astro-ph/9905116).

We assume throughout a Robertson-Walker spacetime metric. For most purposes this metric is thought to be a satisfactory approximation even in a universe with perturbations like our own.

There are five standard cosmological tests plus several variants:

- 1. The age of the universe in units of the Hubble time  $H_0^{-1}$ ;
- 2. Number counts of sources as a function of redshift;
- 3. Angular size as a function of redshift;
- 4. Magnitude (logarithm of flux) as a function of redshift (Hubble diagram); and

5. Number counts of sources versus magnitude.

Some other tests, such as galaxy surface brightness as a function of redshift, are closely related to one or more of our five tests. We will not consider here other tests such as those involving gravitational lensing. An old but excellent review of cosmological tests is given by Sandage (1988; Ann. Rev. Astr. Ap., 26, 561).

A historical perspective is useful. For six decades after Hubble's discovery of cosmological expansion, these five classic tests were the main activity of observational cosmology. Progress was frustrated by the difficulty of separating intrinsic evolution of source properties (e.g. galaxies at high redshift are intrinsically more luminous than nearby ones) from cosmological effects due to the non-Minkowski spacetime geometry. During the 1990s, new tests have been developed based on gravitational lensing, quasar absorption line systems and, especially, the microwave background radiation. Very recently, one of the classic tests (the Hubble diagram for distant supernovae) has been improved sufficiently to yield good (but not conclusive) evidence for accelerated expansion.

The present notes summarize the theoretical elements of cosmology needed for the five classic tests but do not present the detailed results of their application. Chapter 5 of Peacock (1999) presents a recent summary of the observations. The newer tests will be discussed later in the course.

# 2 Proper Distance

Before proceeding to the tests, we begin by writing the Robertson-Walker line element in the preferred form used in these notes (note that we restore the speed of light, c):

$$ds^{2} = a^{2}(\tau) \left[ -c^{2}d\tau^{2} + d\chi^{2} + r^{2}(\chi) \left( d\theta^{2} + \sin^{2}\theta \, d\phi^{2} \right) \right] , \qquad (1)$$

where

$$r(\chi) = \begin{cases} k^{-1/2} \sin(k^{1/2} \chi) , & k > 0 ;\\ (-k)^{-1/2} \sinh[(-k)^{1/2} \chi] , & k < 0 ;\\ \chi , & k = 0 . \end{cases}$$
(2)

Beware that Peacock (1999) uses r for our  $\chi$ ; his notation is nonstandard and one must be careful not to confuse his r with the angular radius appearing in the line element. He and many other authors choose units so that  $k = \pm 1$  or k = 0, in which case the factors of k are absent above. Our  $r(\chi)$  is  $S_k(r)$  in Peacock's notation.

The Friedmann equation is

$$H^{2} = \left(\frac{1}{a^{2}}\frac{da}{d\tau}\right)^{2} = \frac{8\pi G\rho}{3} - \frac{kc^{2}}{a^{2}} = \Omega H^{2} - \frac{kc^{2}}{a^{2}} .$$
(3)

Note that  $\tau$ , called conformal time (and denoted  $\eta$  by Peacock), is not the proper time measured by a comoving (fundamental) observer:  $ds^2 = -a^2 d\tau^2$  so that the proper time is  $t = \int a(\tau) d\tau$  and therefore the Hubble parameter is  $H(t) = d \ln a/dt = a^{-2} da/d\tau$ .

All density contributions (including vacuum energy, i.e., a cosmological constant) are included in the density parameter  $\Omega$ . Using the Friedmann equation we may write the curvature constant as

$$k = \left(\frac{a_0 H_0}{c}\right)^2 \left(\Omega_0 - 1\right) \,. \tag{4}$$

Present values will be written with a subscript 0.

Redshift is widely used as a measure of time through the relation  $1 + z = a_0/a(\tau)$ where  $a_0 = a(\tau_0)$  is the present value of the expansion scale factor. (Often one sets  $a_0 = 1$  for convenience.) Likewise, the coordinate distance  $\chi$  is implicitly a function of redshift through the relation for a radial null geodesic,

$$\chi = c(\tau_0 - \tau) = c \int_0^z \left(-\frac{dz}{d\tau}\right)^{-1} dz .$$
(5)

Redshift can therefore be used as a measure of distance and vice versa. These relations are more complicated if one uses cosmic proper time t instead of conformal time.

Using the Friedmann equation to evaluate  $dz/d\tau$ , we get the **proper distance** to redshift z:

$$a_0\chi(z) = \frac{c}{H_0} d(z) , \quad d(z) \equiv \int_0^z \frac{H_0 dz'}{H(z')} , \quad \frac{H_0}{H(z)} = \left[\Omega_0 \frac{\rho(z)}{\rho_0} + (1 - \Omega_0)(1 + z)^2\right]^{-1/2} .$$
(6)

If we normalize the expansion factor as usual,  $a_0 = 1$ , then  $\chi(z)$  is really the **comoving distance**. The comoving distance happens to equal (up to a factor  $a_0$ ) the proper distance at conformal time  $\tau_0$ .

Let's pause to ask what do we mean by proper distance in a nonstatic spacetime. The meaning is the same as in special relativity. The proper distance is measured between two spacetime events with the same  $\tau$ . For equation (6), one of the events is the observer today ( $\tau = \tau_0, \chi = 0$ ) and the other is elsewhere (radial coordinate  $\chi(z)$ ), also today. It is the distance that one would measure if one could travel infinitely fast and string a tape measure across the universe. Since we are not tachyons, such a measurement is impossible. The proper distance is useful principally as an intermediate step in theoretical calculations of the physically measurable distances to be discussed below.

The redshift-dependence of the density  $\rho(z)$  in equation (6) follows from the equation of state  $p = p(\rho)$  and the equation of energy conservation,

$$d\rho/dz = \frac{3}{1+z} \left(\rho + pc^{-2}\right)$$
 (7)

Nonrelativistic matter (with  $p \ll \rho c^2$ ) has  $\rho_{\rm m} \propto (1+z)^3$ , radiation (with  $3p = \rho c^2$ ) has  $\rho_{\rm r} \propto (1+z)^4$  and vacuum energy (with  $p = -\rho c^2$ ) has  $\rho_{\rm v} \propto (1+z)^0$ . Other dependences are possible for more exotic equations of state, e.g. quantum fields.

For small redshifts the Hubble expansion rate may be expanded in a power series, yielding the standard result

$$d(z) = z - \frac{1}{2}(1+q_0)z^2 + O(z^3) .$$
(8)

We have introduced the deceleration parameter

$$q \equiv -\frac{a \left(\frac{d^2 a}{dt^2}\right)}{(da/dt)^2} = \frac{4\pi G}{3H^2} \left(\rho + 3pc^{-2}\right) = \frac{\Omega}{2} \left(1 + \frac{3p}{\rho c^2}\right) . \tag{9}$$

Note that if the universe is matter-dominated,  $q_0 = \Omega_0/2$ . However, there is no compelling observational or theoretical reason to believe that  $|p| \ll \rho c^2$ . Physical equations of state can be constructed theoretically with  $-\rho c^2 \leq p \leq \rho c^2$ .

At small redshifts  $(z \ll 1)$  the large-scale geometry of spacetime is fully specified by three numbers,  $H_0$ ,  $\Omega_0$  and  $q_0$ . For this reason, observational cosmology through the 1960s was sometimes described as a search for three numbers. However, this is no longer true as our cosmological models have become more complete and observations test them to higher redshifts.

At large redshifts (as  $z \to \infty$ ) the proper distance approaches a constant provided  $\rho(z)$  increases as least as rapidly as  $(1 + z)^2$ . This condition is equivalent to q > 0 (decelerated expansion). The limiting proper distance as  $z \to \infty$  is called the **causal** (or particle) **horizon distance**. The actual universe may be far larger (either finite or infinite) than the causal horizon distance but the latter in principle is the limiting distance to which we can see today. If  $q \ge 0$ , the causal horizon distance is approximately  $c H_0^{-1}$  multiplied by a numerical factor of order unity that scales inversely with q. For k = 0 and q being a positive constant, the coefficient is exactly 1/q.

If q < 0 (accelerated expansion) at large z, the causal horizon distance is much larger and may even be infinite. Inflationary universe models have this behavior.

## 3 Age-Hubble Parameter Relation

The simplest cosmological test relies on measurements made at low redshift — the age of the universe (current proper time  $t_0$  measured by a fundamental observer) in units of the Hubble time  $H_0^{-1}$ . Using  $d\tau = dt/a$  together with equations (5) and (6) we obtain

$$H_0[t_0 - t(z)] = \int_0^z \frac{H_0}{H(z')} \frac{dz'}{1 + z'} .$$
(10)

For practically all equations of state in a Robertson-Walker universe,  $t_0$  is finite and equation (10) converges as  $z \to \infty$ . However, if the universe was dominated by vacuum energy as  $z \to \infty$  (with q = -1), H(z) was a constant and the age diverges logarithmically. This behavior occurs in Linde's eternal chaotic inflation model. However, this result has no practical consequences for the astronomer, who cannot measure the total age of the universe. Rather, astronomers at best can measure the ages of old stars and galaxies, giving the proper time elapsed since the redshift of formation of these objects. Therefore cosmological tests based on  $H_0t_0$  should be performed by truncating the integral in equation (10) at some formation redshift, plausibly in the range  $5 \lesssim z \lesssim 100$  for the oldest objects.

For a Friedmann universe, i.e., a universe dominated by nonrelativistic matter so that  $p \ll \rho c^2$ , equation (10) may be evaluated analytically. For  $\Omega_0 = 1$  (the Einstein-de Sitter universe),  $H_0 t_0 = 2/3$ . For  $\Omega_0 < 1$ ,

$$H_0 t_0 = \frac{1}{(1 - \Omega_0)^{1/2}} - \frac{\Omega_0}{2(1 - \Omega_0)^{3/2}} \ln\left[\frac{2}{\Omega_0} - 1 + \frac{2}{\Omega_0}(1 - \Omega_0)^{1/2}\right] \text{ for } p = 0 \text{ and } \Omega_0 < 1.$$
(11)

One may verify that for  $0 \le \Omega_0 \le 1$ ,  $1 \ge H_0 t_0 \ge 2/3$ .

A formula similar to equation (11) holds for  $\Omega_0 > 1$ , with  $H_0 t_0 < 2/3$ . Thus, matterdominated universes have  $H_0 t_0 < 1$ . Adding the radiation that we know (the microwave background radiation) decreases  $H_0 t_0$  very slightly.

Historically, equation (11) has been a crucial test of cosmological theories and the source of many newspaper articles declaring crisis in cosmology. Before the late 1940s, the Hubble constant was estimated to be several times larger than it is now, and for a time this was in clear conflict with the age of the Earth. This discrepancy gave impetus to the steady-state theory, in which  $H_0t_0$  is unbounded. More recently, during the 1990s estimates of both  $H_0$  and  $t_0$  improved and there again seemed to be a significant discrepancy with standard models.

This discrepancy has lessened, if not entirely disappeared, during the last two years. To translate the test into more common units, we write

$$H_0 t_0 = 1.023 \, \left(\frac{h \, t_0}{10 \, \text{Gyr}}\right) \tag{12}$$

where  $H_0 = 100 \,h\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$  and 1 Gyr=10<sup>9</sup> yr. The latest results of the Hubble Space Telescope Cepheid Key Project indicate  $h = 0.71 \pm 0.07$  (Sakai et al. 1999, astro-ph/9909269). The error bar is dominated by systematic errors, the largest of which remains the distance to the Large Magellanic Cloud, a nearby dwarf galaxy containing many Cepheid variables.

For a long time the ages of the oldest stars in our galaxy was thought to exceed 16 Gyr, which was a real embarrassment for matter-dominated models and especially the simplest cold dark matter model (with  $\Omega_{\rm m} = 1$  which predicted  $H_0 t_0 = 2/3$ . However, recently the distances to globular clusters in our galaxy were revised upwards based indirectly on astrometric observations by the *Hipparcos* satellite. For a given measured flux, this meant that the stars must be more luminous, hence younger. Chaboyer et al (1998, ApJ 449, 96) now estimate the age of the oldest globular clusters as  $11.5 \pm 1.3$ 

Gyr. Even if it took 1 Gyr before the globular clusters could form (and I suspect that is an overestimate),  $H_0 t_0 \approx 0.9$  is now estimated to be less than unity. While this value is still too high for the Einstein-de Sitter model, it is consistent with the currently favored model, which has a cosmological constant.

Cosmological models with a cosmological constant plus nonrelativistic matter are called Friedmann-Lemaitre models (for a review see Felten and Isaacman 1986; *Rev. Mod. Phys.*, **58**, 689). A cosmological constant is equivalent to a vacuum energy density  $\rho_v = \Lambda/(8\pi G)$ . The general solution for  $H_0 t_0$  in such a model requires numerical quadrature of equation (10). However, in the special case of a flat (k = 0) universe, the solution may be obtained in closed form:

$$H_0 t_0 = \frac{2}{3} \Omega_{\rm v}^{-1/2} \ln \left[ \frac{1 + \Omega_{\rm v}^{1/2}}{\Omega_{\rm m}^{1/2}} \right] \quad \text{for} \quad \Omega_{\rm m} + \Omega_{\rm v} = 1 \ (k = 0) \ . \tag{13}$$

Here we have defined  $\Omega_{\rm m} = 8\pi G \rho_{\rm m}/(3H^2)$  and  $\Omega_{\rm v} = 8\pi G \rho_{\rm v}/(3H^2) = \Lambda/(3H^2)$ . Present values are implied although the subscript 0 is deleted for notational convenience. From equation (13) we see that a model with a cosmological constant can have arbitrarily large  $H_0 t_0$  provided that  $\Omega_{\rm m} \ll 1$ . This restriction can be relaxed in a universe with curvature. For example, for  $\Omega_{\rm m} = 0.2$  (consistent with several determinations),  $H_0 t_0 \rightarrow \infty$  for  $\Omega_{\rm v} \approx 1.5476$ .

## 4 Proper Motion Distance and Mattig Formula

Most cosmological tests rely, directly or indirectly, on the angular part of the Robertson-Walker metric and not only on the radial part used above. Before introducing the other standard cosmological tests, we introduce the **proper motion distance** to redshift z, which makes use of the angular radius  $r(\chi)$ :

$$a_0 r(z) = a_0 r(\chi(z)) = \frac{c}{H_0} d(z) F(d(z)) ,$$
  

$$F(d) = \frac{r(\chi)}{\chi} = \begin{cases} \frac{\sin[d(\Omega_0 - 1)^{1/2}]}{d(\Omega_0 - 1)^{1/2}} , & \Omega_0 > 1 \ (k > 0) ; \\ \frac{\sinh[d(1 - \Omega_0)^{1/2}]}{d(1 - \Omega_0)^{1/2}} , & \Omega_0 < 1 \ (k < 0) ; \\ 1 \ , & \Omega_0 = 1 \ (k = 0) . \end{cases}$$
(14)

The proper motion distance relates the transverse speed  $v_{\perp}$  (perpendicular to the radial direction) and the angular speed  $d\theta/dt_0$  (assuming  $d\phi = 0$ ) by  $v_{\perp} = a_0 r d\theta/dt_0$ . If the angular and physical velocities of distant objects could be measured at  $z \gtrsim 1$  (currently there is no way to do this), then a powerful cosmological test could be performed by observationally determining r(z).

In the general case the proper motion distance  $a_0r(z)$  is rather complicated and can be evaluated only by using equation (14) with d(z) obtained by numerical quadrature of equation (6). (For small z, eq. 6 gives the correct leading behavior for  $a_0H_0r/c$ .) The proper motion distance simplifies for a Friedmann (matter-dominated) universe. In this case one obtains a simple result known as the Mattig formula (Mattig 1958; *Astron. Nachr.*, **284**, 109):

$$\left(\frac{a_0 H_0}{c}\right) r(z) = d(z) F(d(z)) = \frac{2}{\Omega_0^2 (1+z)} \left[ (2 - \Omega_0)(1 - \sqrt{1 + \Omega_0 z}) + \Omega_0 z \right] \quad \text{for } p = 0.$$
(15)

Many of the discussions of distances in the cosmology literature implicitly assume a Friedmann universe or a Friedmann-Lemaitre universe (one dominated by matter plus a cosmological constant). (Unfortunately, analytical results for the proper motion distance exist only for the Friedmann models and not those with a cosmological constant.) However, the modern view of physical cosmology is that almost arbitrary equations of state may be possible, so that the cosmological proper motion distance-redshift relation should not in general be restricted to equation (15).

Stated another way, measurement of r(z) is really a test of the equation of state (implicit in  $\rho(z)$  in eq. 6) at least as much as it is a test of the geometry of spacetime.

## 5 Number Counts

Now consider the volume corresponding to a given range of coordinates. The **proper** volume element at conformal time  $\tau$  is dVol =  $a^3r^2(\chi)d\chi d\Omega$ , giving the differential proper volume element

$$\left(\frac{H_0}{c}\right)^3 \frac{d\mathbf{Vol}}{dz \, d\Omega} = (1+z)^{-3} \frac{H_0}{H(z)} \left(\frac{a_0 H_0 r}{c}\right)^2 = (1+z)^{-3} \frac{H_0}{H(z)} \, d^2(z) \, F^2(d(z)) \;. \tag{16}$$

The comoving volume element is  $dV_c \equiv (1+z)^3 d$ Vol.

The proper and comoving volume elements are useful for computing number densities from redshift distributions. Suppose that there exists a homogeneous distribution of objects with comoving number density  $n_{\rm c}(z)$ , corresponding to a proper number density  $(1+z)^3n_{\rm c}(z)$  ( $n_{\rm c}$  is constant if the objects are conserved). Then the differential number of objects per unit redshift and solid angle is

$$\frac{dN}{dz\,d\Omega} = n_{\rm c}(z)\,\frac{H_0}{H(z)}\,d^2(z)\,F^2(d(z))\;.$$
(17)

If a population of objects such as galaxies were known to have constant comoving number density and to be visible at all redshifts, measurement of dN/dz would provide a test of the cosmological model (the number-redshift test) through the dependence of dN/dz on  $H_0/H(z)$ . This test is complicated by the fact that the number density of galaxies shows significant inhomogeneity on scales of  $\Delta z \lesssim .05$ . In addition, acquiring much leverage on cosmological parameters requires observations extending to  $z \gtrsim 1$ . Many different kinds of objects have been proposed for this test, including galaxies, quasars, radio galaxies, gravitational lenses, X-ray clusters, and Lyman alpha clouds. Several difficulties have rendered this test difficult and inconclusive.

The biggest difficulty is that all known astronomical objects evolve with redshift. The comoving number density is not constant. Indeed, number counts are now used more to constrain the evolution of the source population more than to test the geometry of the universe. A second problem is the practical difficulty of measuring redshifts for a complete and well-defined sample of objects at high redshift. This difficulty has been ameliorated in recent years with the advent of large telescopes such as Keck and the development of powerful spectrophotometric redshift estimation methods for galaxies, as well as large observational surveys conducted with Hubble Space Telescope and ground-based instruments. As a result, number counts have become a useful tool for studying galaxy (and other source) evolution.

To complete the discussion of number count tests, we derive the **total comoving volume** to redshift z:

$$V_{\rm c}(z) = 4\pi a_0^3 \int_0^{\chi(z)} r^2(\chi') d\chi' = \frac{4\pi}{3} \left(\frac{c}{H_0}\right)^3 d^3(z) \left(\frac{3\left[1 - F(2d)\right]}{2\left(\Omega_0 - 1\right)d^2}\right) .$$
(18)

For  $\Omega_0 = 1$  the factor in parentheses in equation (18) is 1. We noted earlier that d(z) increases with decreasing q. Therefore, the total comoving volume is larger in models with small q, e.g., models with vacuum energy.

# 6 Angular-Diameter Test

Next we define the **angular-diameter distance**:

$$d_{\rm A}(z) = a r(z) = \frac{a_0 r(z)}{1+z} = \frac{c}{H_0} \frac{d(z)F(d(z))}{1+z} .$$
(19)

The angular-diameter distance is defined so that an object of proper size l at redshift z subtends an angle  $\theta = l/d_A(z)$  (for  $\theta^2 \ll 1$ ; for large angles trigonometric functions appear).

If a population of objects of known proper size ("standard meter-sticks") existed throughout the universe, cosmological models could be tested using measurements of  $d_{\rm A}(z)$ . This is known as the angular diameter-redshift test.

Unfortunately, good meter-sticks are hard to find in cosmology. Gurvits et al (1999, A&A, 342, 378) have used "compact radio sources" (typically, relativistic jets of plasma emitting synchrotron radiation, powered by massive black holes in quasars and radio galaxies) but they have not demonstrated rigorously that the objects should have the same physical size. A better test will come from measurements of hot gas in clusters of galaxies by combining X-ray emission with the Sunyaev-Zel'dovich effect (shadowing

of the microwave background). Birkinshaw (1999, Phys. Rep., 310, 97) gives a detailed review. Finally, other possibilities have been proposed based on the clustering of quasar absorption lines (Hui et al 1999, ApJ, 511, L5) or high-redshift galaxies (Nair 1999, ApJ, 522, 569). These methods are statistical and are based on comparing clustering in the radial and transverse directions, following a suggestion by Alcock & Paczynski (1979, Nature, 281, 358).

The cosmic microwave background anisotropy spectrum offers the cleanest version of the angular diameter test, based on the acoustic horizon size (the distance sound waves travel since the big bang) at the time of photon decoupling. This test will be discussed later in the course.

#### 7 Hubble Diagram

Astronomers measure fluxes (or magnitudes, i.e., logarithms of fluxes, if they are optical astronomers) in addition to angular sizes. A radiation source of proper luminosity (i.e., power) L emitting isotropically at redshift z produces a flux at the observer

$$S = \frac{L(1+z)^{-2}}{4\pi a_0^2 r^2} \equiv \frac{L}{4\pi d_{\rm L}^2(z)}$$
(20)

where we have defined the luminosity distance  $d_{\rm L}(z)$ .

Flux is power per unit area. The proper area of the sphere surrounding the source is  $4\pi a_0^2 r^2$  but the power is diminished by two powers of (1 + z): one for the redshifting of energy and another for the diminished rate of arrival of the energy. Note that we are implicitly assuming that the total flux is measured, i.e. the spectral flux integrated over all energy (the bolometric flux). In practice this is usually not possible and a correction must be applied for the redshifting of the spectral bandpass. (In optical astronomy this is called the *K*-correction). From equation (20) we can express the luminosity distance in terms of the angular-size and proper motion distances:

$$d_{\rm L}(z) = a_0 r(z) (1+z) = (1+z)^2 d_{\rm A}(z) = \frac{c}{H_0} (1+z) d(z) F(d(z)) .$$
 (21)

Sources of known proper luminosity ("standard candles") would offer a test of cosmological models (the magnitude-redshift test) through measurement of  $d_{\rm L}(z)$ . The magnitude-redshift test was pioneered by Hubble; the plot of magnitude vs. redshift for a sample of objects such as galaxies or quasars is known as the Hubble diagram.

The magnitude-redshift test has been a mainstay of observational cosmology for more than four decades (e.g. Humason, Mayall, & Sandage 1956, AJ, 61, 97). It is basically an extension to high redshift ('high" means z > 0.5) of the methods used to estimate the Hubble constant. The difficulty lies in finding standard candles. Most objects bright enough to be visible at high redshift (e.g. galaxies, quasars, or, more recently, gamma ray bursts) are not standard but evolve strongly in their properties as a function of cosmic time.

Recently, two groups have refined the use of Type Ia supernovae as standard candles and have used them to measure the angular diameter-distance relation out to a redshift about 1 (Garnavich et al 1998, ApJ, 509, 74; Perlmutter et al 1999, ApJ 517, 565). These results are among the most exciting in cosmology during the last two years. They have garnered widespread attention because of the implication for cosmic acceleration. We will explore the evidence in detail in the lecture and in a problem set.

# 8 Number-Flux Test

As a last cosmological test, suppose that number counts of objects are measured as a function of flux but not redshift. There are many cosmological objects for which redshifts are difficulty to determine (e.g. very faint galaxies and gamma ray bursts). If an object were a standard candle, then its flux would give the luminosity through equations (20) and (21), modulo the uncertain cosmological model. If, however, the evolution of comoving number density were known, then one could combine the angular-diameter and number counts techniques to obtain the number-flux test, commonly known as N(S).

The number-flux test has been used for radio sources and gamma ray bursts, neither of which are standard candles. An assumption must therefore be made about the luminosity distribution of the sources. Unfortunately, it is difficult to disentangle evolution of the luminosity distribution from cosmological effects. Nonetheless, the test is sufficiently clean for gamma-ray bursts so that N(S) gave strong evidence for a cosmological origin (as did the isotropy on the sky) before the first redshifts were measured in 1997 (Metzger et al 1997, Nature, 387, 878). At that point, the debate was over: classical gamma ray bursts occur in high-redshift galaxies. However, their physics is still too poorly understood to allow their use in precision cosmological tests. Hogg & Fruchter (1999, ApJ, 520, 54) have investigated the number-flux-redshift distribution of gamma-ray bursts. They found that while the data do not significantly constrain the spacetime geometry, they do constrain models for the evolution of the source population.