

Problem Set #3
Due in class Tuesday, October 2, 2001.

1. Horizons

The particle (or causal) horizon distance $l_H(t)$ is defined to be the coordinate distance χ travelled by a photon on a radial geodesic since $t = 0$, using the Robertson-Walker metric with line element in the form

$$ds^2 = -dt^2 + a^2(t) \left[d\chi^2 + r^2(\chi)(d\theta^2 + \sin^2\theta d\phi^2) \right]. \quad (1)$$

For simplicity consider the $k = 0$ case so that $r(\chi) = \chi$. Find $l_H(t)$ for the following cases: (1) $a(t) \propto t^{2/3}$ (Einstein-de Sitter, matter dominated universe); (2) $a(t) \propto t^{1/2}$ (radiation dominated universe); and (3) $a(t) \propto \exp(Ht)$ with $H = \text{constant}$ (vacuum energy dominated universe, known also as the de Sitter universe). Show that in cases (1) and (2), a photon emitted at $t = t_0$ can reach any coordinate distance given sufficient time but not so in the de Sitter universe. This result shows that the de Sitter universe has an event horizon. Draw a spacetime diagram to explain what is going on.

2. Angle subtended by causal horizon

Consider two comoving observers A and B , each at the same comoving distance χ from you, the observer. At conformal time τ_e , A and B come into causal contact for the first time: a photon emitted by A at $\tau = 0$ reaches B (and vice versa) at $\tau = \tau_e$. At $\tau = \tau_e$ both A and B emit photons along radial paths toward you which you receive today at $\tau = \tau_0$. For simplicity, ignore the gravitational effect of radiation in the universe; this is a fair approximation after $z = 10^4$.

- a) Assuming that you live in an Einstein-de Sitter universe, what angular separation do you measure between A and B ? (You may use small-angle formulae.)
- b) Evaluate the angle in degrees if $1 + z_e = 1100$ so that A and B are on the cosmic photosphere at recombination (when the cosmic plasma became transparent). Without an earlier inflationary era, places on the photosphere separated by more than this amount are causally disconnected.
- c) Redo part b) for the Λ CDM and Ω CDM models of Problem Set 2. For the same Ω_m in both models, does the angle subtended by the causal horizon at recombination depend more strongly on Ω_k or Ω_Λ ?

3. Power-Law Inflation: Peacock 11.1

Verify that the potential

$$V(\phi) \propto \exp\left(\sqrt{\frac{16\pi}{p m_{\text{Pl}}^2}} \phi\right)$$

leads to inflation with a scale-factor dependence $a(t) \propto t^p$. (Here $m_{\text{Pl}} = G^{-1/2}$ is the Planck mass in units where $c = \hbar = 1$.) Since this is not de Sitter space, why does the resulting behavior still lead to zero comoving spatial curvature?

4. The Future of Inflation

- a) Show that inflation can keep the observed universe homogeneous only for a time $t < t_{\text{crit}}$ where

$$t_{\text{crit}} \sim \frac{t_i^2}{t_f} \exp 2H(t_f - t_i)$$

where t_i and t_f are the times at which inflation started and ended and H is the Hubble parameter during inflation.

- b) Consider a $k > 0$ universe which went through an inflationary phase and never inflates thereafter. Since such a model is expected to recollapse eventually, observations will eventually reveal that $\Omega > 1$ at, say, $t > T$. Estimate T .