FREE BODY DIAGRAM:

\[ \begin{align*}
&H_A \\
&V_A \\
&C \\
&D \\
&E \\
&G \\
&H \\
&\downarrow P \\
&\downarrow H_A \\
&\uparrow V_A \\
&\uparrow V_H
\end{align*} \]

APPLY EQUILIBRIUM TO FIND REACTIONS

\[ \begin{align*}
+ & \quad \sum F_x = 0 \quad \Rightarrow \quad H_A = 0 \\
+ & \quad \sum F_y = 0 \quad \Rightarrow \quad V_A + V_H - P = 0 \\
+ & \quad \sum M_A = 0 \quad \Rightarrow \quad V_H(4L) - P(2L) = 0 \\
\quad \quad \quad \Rightarrow \quad V_H = \frac{P}{2} \\
\quad \quad \quad \Rightarrow \quad V_A = \frac{P}{2}
\end{align*} \]

TO DETERMINE THE DEFLECTION OF D, WE NEED TO EMPLOY COMPATIBILITY & CONSTITUTIVE LAWS.

OUR CONSTITUTIVE LAW FOR BAR DEFORMATION IS:

\[ \sigma_{ij} = \frac{F_{ij} L_{ij}}{AE} \]

SO WE'LL NEED TO SOLVE FOR THE BAR FORCES IN ORDER TO DETERMINE THEIR EXTENSIONS, AND HENCE THE TRUSS DEFLECTION.
Because of symmetry, I only need to find half of the bar forces. All of the pairs mirrored in the D-E axis will have the same bar force:

\[ F_{AC} = F_{CH} \quad F_{BD} = F_{BE} \]
\[ F_{AB} = F_{FH} \quad F_{CD} = F_{DE} \]
\[ F_{BC} = F_{FC} \quad F_{CE} = F_{GE} \]

Solve for independent bar forces:

**MOJ @ A:**

\[ \Sigma F_y = \frac{P}{2} - F_{AB} \cos 45^\circ = 0 \]
\[ F_{AB} = \frac{P}{\sqrt{2}} \]
\[ \Sigma F_x = F_{Ac} + F_{AB} \sin 45^\circ = 0 \]
\[ F_{Ac} = -\frac{P}{2} \]

**MOJ @ B:**

\[ \Sigma F_y = 0 = F_{BC} + (\frac{P}{\sqrt{2}}) \cos 45^\circ \]
\[ F_{BC} = -\frac{P}{2} \]
\[ \Sigma F_x = 0 = F_{BD} - (\frac{P}{\sqrt{2}}) \sin 45^\circ \]
\[ F_{BD} = \frac{P}{2} \]

**MOJ @ C:**

\[ \Sigma F_y = 0 = \frac{P}{2} - F_{CD} \cos 45^\circ \]
\[ F_{CD} = \frac{P}{\sqrt{2}} \]
\[ \Sigma M_B = 0 = -F_{CE} \left( \frac{P}{2} \right) - \frac{P \cdot (\frac{P}{2})}{2} = 0 \]
\[ F_{CE} = -P \]

**MOJ @ E:**

\[ \Sigma F_y = 0 \]
\[ -P - F_{DE} = 0 \]
\[ F_{DE} = -P \]
<table>
<thead>
<tr>
<th>BAR</th>
<th>FORCE ( \frac{F_{ij}}{P} )</th>
<th>LENGTH ( \frac{L_{ij}}{L} )</th>
<th>DEFORMATION ( \frac{\delta_{ij}}{P_L} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>(+\frac{1}{2})</td>
<td>(\sqrt{2})</td>
<td>(+1)</td>
</tr>
<tr>
<td>AC</td>
<td>(-\frac{1}{2})</td>
<td>(1)</td>
<td>(-\frac{1}{2})</td>
</tr>
<tr>
<td>CB</td>
<td>(-\frac{1}{2})</td>
<td>(1)</td>
<td>(-\frac{1}{2})</td>
</tr>
<tr>
<td>CE</td>
<td>(-1)</td>
<td>(1)</td>
<td>(-1)</td>
</tr>
<tr>
<td>CD</td>
<td>(+\frac{1}{\sqrt{2}})</td>
<td>(\sqrt{2})</td>
<td>(+1)</td>
</tr>
<tr>
<td>BD</td>
<td>(+\frac{1}{2})</td>
<td>(1)</td>
<td>(+\frac{1}{2})</td>
</tr>
<tr>
<td>ED</td>
<td>(-1)</td>
<td>(1)</td>
<td>(-1)</td>
</tr>
<tr>
<td>EG</td>
<td>(-1)</td>
<td>(1)</td>
<td>(-1)</td>
</tr>
<tr>
<td>DG</td>
<td>(+\frac{1}{\sqrt{2}})</td>
<td>(\sqrt{2})</td>
<td>(+1)</td>
</tr>
<tr>
<td>DF</td>
<td>(+\frac{1}{2})</td>
<td>(1)</td>
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</tr>
<tr>
<td>GF</td>
<td>(-\frac{1}{2})</td>
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<td>(-\frac{1}{2})</td>
</tr>
<tr>
<td>GH</td>
<td>(-\frac{1}{2})</td>
<td>(1)</td>
<td>(-\frac{1}{2})</td>
</tr>
<tr>
<td>FH</td>
<td>(+\frac{1}{\sqrt{2}})</td>
<td>(\sqrt{2})</td>
<td>(+1)</td>
</tr>
</tbody>
</table>

Now we can go ahead and plot our truss deflection diagram.
\[ \sigma_0 = \frac{PL}{AE} \]

In my hinge point \( A' \) ends up displaced from my origin by \( \sigma_{A_x} \) and \( \sigma_{A_y} \). Then my origin OD is displaced from \( A' \) by \(-\sigma_{A_x} \) and \(-\sigma_{A_y} \).

If I now consider the fixed frame OA, where A and \( A' \) are the same, I can find the deflection of \( D' \) in the fixed frame, which is just its displacement from \( A' \), namely

\[ -\sigma_{A_x} \text{ and } -\sigma_{A_y} \]

The joint \( D \) will translate down by \( \frac{13}{2} \frac{PL}{AE} \) and left by \( \frac{3}{2} \frac{PL}{AE} \).
E. S T I M A T E  O F  T R U S S  D E F L E C T I O N S

B A R S  I N  E X P E R I M E N T A L  T R U S S  M A D E  O F  S T E E L
- H O L L O W  W I T H  2 2 m m  O U T E R  D I A M E T E R  A N D
1.5 m m  W A L L  T H I C K N E S S  ( I G N O R E  E N D  F I T T I N G S )

\[ A \approx 2 \pi r t \approx 10.5 \times 10^{-3} \times 2 \times \pi \times 1.5 \times 10^{-3} \]
\[ \approx 100 \text{ mm}^2 \]

\[ L = 0.5 \text{ m} \]
\[ E = 210 \text{ GPa} \]

C E N T E R  P O I N T  D E F L E C T I O N

\[ \frac{\sigma_o}{P} = \left( \frac{0.5}{100 \times 10^{-6} \times 210 \times 10^9} \right) \left( -\frac{13}{8} - \frac{3t}{2} \right) \]

\[ \frac{\sigma_o}{P} = -3.1 \times 10^{-7} \frac{j}{E} - 7.1 \times 10^{-8} \frac{e}{2} \]