To begin, label the signals as shown below:

From the problem statement,
\[ y(t) = [x(t) + A] \cos(2\pi f_c t + \theta_c) \]

Define
\[ z(t) = x(t) + A \]
\[ w(t) = \cos(2\pi f_c t + \theta_c) \]

The factor \( w(t) \) can be expanded as
\[ w(t) = \cos(2\pi f_c t + \theta_c) = \cos \theta_c \cos 2\pi f_c t - \sin \theta_c \sin 2\pi f_c t \]

The Fourier transform of \( w(t) \) is then given by
\[
W(f) = \mathcal{F}[\cos(2\pi f_c t + \theta_c)]
\]
\[
= \frac{1}{2} \cos \theta_c \left[ \delta(f - f_c) + \delta(f + f_c) \right] - \frac{1}{2} \sin \theta_c \left[ -j \delta(f - f_c) + j \delta(f + f_c) \right]
\]
\[
= \frac{1}{2} (\cos \theta_c + j \sin \theta_c) \delta(f - f_c) + \frac{1}{2} (\cos \theta_c - j \sin \theta_c) \delta(f + f_c)
\]

The Fourier transform of \( z(t) = x(t) + A \) is given by
\[
Z(f) = \mathcal{F}[z(t)] = X(f) + A \delta(f)
\]

\( Z(f) \) is bandlimited, because \( X(f) \) is, and of course the impulse function is bandlimited. So the FT of \( y(t) \) is given by the convolution
\[
Y(w) = Z(f) * W(f)
\]
\[
= \frac{1}{2} \left[ (\cos \theta_c + j \sin \theta_c) Z(f - f_c) + (\cos \theta_c - j \sin \theta_c) Z(f + f_c) \right]
\]
Next, compute the spectra of \( y_1(t) \) and \( y_2(t) \). To do so, we need the spectra of \( w_1(t) \) and \( w_2(t) \):

\[
W_1(f) = \mathcal{F}[w_1(t)] = \mathcal{F}[\cos 2\pi f_c t] = \frac{1}{2} [\delta(f - f_c) + \delta(f + f_c)]
\]

\[
W_2(f) = \mathcal{F}[w_2(t)] = \mathcal{F}[\sin 2\pi f_c t] = \frac{1}{2} [-j\delta(f - f_c) + j\delta(f + f_c)]
\]

Then

\[
Y_1(f) = W_1(f) * Y(f) = \frac{1}{2} [Y(f - f_c) + Y(f + f_c)]
\]

\[
= \frac{1}{4} \left[ (\cos \theta_c + j \sin \theta_c) Z(f - 2f_c) + (\cos \theta_c - j \sin \theta_c) Z(f) \right] + \frac{1}{4} \left[ (\cos \theta_c + j \sin \theta_c) Z(f) + (\cos \theta_c - j \sin \theta_c) Z(f + 2f_c) \right]
\]

\[
= \frac{1}{2} \cos \theta_c Z(f)
\]

\[
+ \frac{1}{4} \left[ (\cos \theta_c + j \sin \theta_c) Z(f - 2f_c) + (\cos \theta_c - j \sin \theta_c) Z(f + 2f_c) \right]
\]

Similarly,

\[
Y_2(f) = W_2(f) * Y(f) = \frac{1}{2} [-jY(f - f_c) + jY(f - f_c)]
\]

\[
= -\frac{j}{4} \left[ (\cos \theta_c + j \sin \theta_c) Z(f - 2f_c) + (\cos \theta_c - j \sin \theta_c) Z(f) \right] + \frac{j}{4} \left[ (\cos \theta_c + j \sin \theta_c) Z(f) + (\cos \theta_c - j \sin \theta_c) Z(f + 2f_c) \right]
\]

\[
= -\frac{1}{2} \sin \theta_c Z(f)
\]

\[
+ \frac{1}{4} \left[ (-j \cos \theta_c + \sin \theta_c) Z(f - 2f_c) + (j \cos \theta_c + \sin \theta_c) Z(f + 2f_c) \right]
\]

Now, when \( y_1(t) \) and \( y_2(t) \) are passed through the lowpass filters, the \( Z(f - 2f_c) \) and \( Z(f + 2f_c) \) terms are eliminated, and the \( Z(f) \) terms are passed. Therefore,

\[
Y_2(f) = \frac{1}{2} \cos \theta_c Z(f)
\]

\[
Y_5(f) = -\frac{1}{2} \sin \theta_c Z(f)
\]
and

\begin{align*}
y_2(t) &= \frac{1}{2} \cos \theta_c z(t) \\
y_5(t) &= -\frac{1}{2} \sin \theta_c z(t)
\end{align*}

After passing these signals through the squarers, we have

\begin{align*}
y_3(t) &= \frac{1}{4} \cos^2 \theta_c z^2(t) \\
y_6(t) &= \frac{1}{4} \sin^2 \theta_c z^2(t)
\end{align*}

\(y_7(t)\) is the sum of these, so that

\[y_7(t) = y_3(t) + y_7(t) = \frac{1}{4} \left[ \cos^2 \theta_c z^2(t) + \sin^2 \theta_c z^2(t) \right] = \frac{1}{4} z^2(t)\]

Finally, \(r(t)\) is obtained by passing taking the square root of \(y_7(t)\), so that

\[r(t) = \sqrt{\frac{z^2(t)}{4}} = \frac{|z(t)|}{2}\]

if the positive root is always taken. But \(z(t) = x(t) + A\) is always positive, according to the problem statement. Therefore,

\[x(t) = 2r(t) - A\]