Lecture F4 Mud: Dimensional Analysis
(37 respondents)

1. I don’t understand what \( \Pi \) products are. How do you get them? (3 students)
A \( \Pi \) product is a dimensionless combination of parameters which influence a physical situation (e.g. flow over an airfoil). They can be obtained by inspection, by trial & error, by insight, or by linear algebra as a last resort (see Anderson 1.7). The choices are not unique. Instead of \( V_\infty/a_\infty \), we could have chosen \( a_\infty/V_\infty \). But \( V/a \) is more traditional, so that’s what we use.

2. Can I get additional \( \Pi \) products just by combining the ones I already have? (1 student)
No. You can get alternative \( \Pi \) products that way, but not additional ones. Here are two possible sets you can use:

\[
\begin{align*}
\Pi_1 & = \alpha \\
\Pi_2 & = \rho_\infty V_\infty c/\mu_\infty \\
\Pi_3 & = V_\infty/a_\infty \\
\end{align*}
\]

\[
\begin{align*}
\Pi_1 & = \alpha^2 \\
\Pi_2 & = \rho_\infty a_\infty c/\mu_\infty \\
\Pi_3 & = \alpha a_\infty/V_\infty \\
\end{align*}
\]

Set 2 is usable, but Set 1 is traditional and easier to use (that’s why it’s traditional!).

3. What is the function \( \bar{g}(c_\ell, \alpha, M_\infty, Re) = 0 \)? (student)
This is a conceptual way to say that the four parameters interrelate in some way for a physical situation. The amazing thing is that with Dimensional Analysis (and Buckingham Pi Theorem) we are able to identify these parameters without knowing what the dependencies actually are quantitatively.

4. It seems we’re extrapolating a lot of info from small observations? (3 students)
Yep. Dimensional analysis is amazingly powerful.

5. How does Buckinghams’ Theorem relate to Dimensional Analysis? (1 student)
B’s Theorem is one step of Dimensional Analysis. Other steps include identifying parameters, deriving other dimensionless quantities (\( C_p \), etc.)

6. When two airfoils have the same \( \Pi \) products, what do you mean that they are the “same”? (1 student)
They may look different, but they are the same only when expressed, examined, or plotted in the dimensionless variables. For example, \( L_1' \neq L_2' \), but \( c_\ell_1 = c_\ell_2 \).

7. What is this stuff used for? (3 students)
Lots of uses. One use is applying wind tunnel data to full size. To get full-size lift \( L_1 \) from the wind-tunnel lift \( L_2 \), we would compute

\[
\begin{align*}
C_{L_2} & = L_2 / 0.5\rho_2 V_2^2 S_2 \\
C_{L_1} & = C_{L_2} \\
L_1 & = 0.5\rho_1 V_1^2 S_1 \\
\end{align*}
\]
8. **How does the air viscosity vary with temperature?** (1 student)
   It’s a complicated relationship. Look up “Sutherland’s Law” for gasses. We can frequently approximate this as \( \mu \sim \sqrt{T} \), so gas viscosity always increases with increasing temperature. Liquid viscosity usually decreases with increasing temperature.

9. **What does Reynolds Number mean?** (1 student)
   It’s a measure of how “viscous” or “sticky” the flow is, or how big inertial forces are relative to viscous or friction forces. This comparison depends not only on the viscosity of the fluid, but also on density, velocity, and body size. The Reynolds Number lumps all these effects into one significant parameter.

10. **Can you bring more demos to class?** (1 student)
    I will if it’s feasible. Unfortunately, in-class wind tunnel demos are not very practical. The small tunnel we had in class is too small for most demos of interest.

11. **No mud** (18 students)