Can you describe what the physical meaning of a moment is, and how do you choose \( r \) in \( \mathbf{r} \times \mathbf{F} \). A moment (or more correctly a moment of a force) is the effect of a force acting about an axis to cause an angular acceleration (or rotation). \( r \) is to some extent arbitrary. It is the position vector of some point on the line of application of the force \( \mathbf{F} \).

What are \( i \) and \( j \) in the force moment equations, why do you use this notation: \( \mathbf{R}^{ij} \) is the force acting on particle \( i \) due to particle \( j \). I use the superscripts because I reserve a subscript to define a tensor quantity (which we will see later in the term). \( \mathbf{r} \) is the position vector of point \( i \).

When we write \( \sum r^i \times \mathbf{F}^i + \mathbf{M}^i = \mathbf{0} \), what is \( \mathbf{M}^i \)? It is a pure moment. In reality a pure moment usually results from a force couple (i.e. two equal and opposite forces with parallel lines of action). Given this it can be argued that the \( \mathbf{M}^i \) is superfluous. Note in the particular example I did on the board there were no pure moments applied to the system, so there were no \( \mathbf{M}^i \) terms.

I missed the derivation that resulted in the sum of moments in a system being equal to the sum of the external moments? Confused as to why moments cancel? Please explain why. \( \sum \mathbf{r}^1 \times \mathbf{R}^{13} + \mathbf{r}^3 \times \mathbf{R}^{31} = \mathbf{0} \)?

Given that \( \mathbf{R}^{13} = \mathbf{R}^{31} \) and that the two forces have the same line of action this implies that \( \mathbf{r}^1 \times \mathbf{R}^{13} = \mathbf{r}^3 \times \mathbf{R}^{31} \). Note that this is only true when the particles in the system are connected by the internal reactions. It is not true of an arbitrary system of particles where there are no connections.

If the system is not in an equilibrium state will the internal force appear in the equation? Usually it will not appear in the equation, although there are cases where the overall system is in equilibrium but the individual particles are accelerating - for instance if one has a body with particles connected by springs the springs can be vibrating.

I don't get the moments w/o forces part. Please explain with examples. Please come and talk to me after tomorrow's lecture.

If you find the equilibrium forces for each \( (x, y, z) \) component, do you still need to find the equilibrium moment separately? Absolutely. Equilibrium of forces and equilibrium of moments are separate conditions. As we saw in U3, a pair of forces can constitute a couple, which means they generate no net force, but they do result in a moment. Equally a single force acting through a point mass generates no moment about that point, but will cause the point mass to accelerate unless there is an equal and opposite force applied to it to keep it in equilibrium.

For the last part \( \sum \mathbf{r}' \times \mathbf{P}' + \mathbf{M}' = \mathbf{0} \) only the external forces affect (?) so \( \mathbf{P}'^{(n)} \) is not an external force? No \( \mathbf{P}'^{(n)} \) is an external force, and it appears in the equation. In the notes I used \( \mathbf{R}'^{(i)} \) as the internal forces and \( \mathbf{P} \) as the external forces.
Do we take into account some forces have positive and negative components (with reference to a diagram of force vectors acting on a point mass). Absolutely, when I write $\sum \mathbf{P} = 0$ or $\sum \mathbf{r} \times \mathbf{P} + \mathbf{M} = 0$ or equivalent equilibrium equations, for these equations to make sense some of the components must have positive, and some have negative components.

**How are couple vectors related to the directions of the force vectors?** Given that a pair of forces $\mathbf{F}$ and $-\mathbf{F}$ passing through points with a relative position vector, $\mathbf{r}$ lead to a couple $\mathbf{C} = \mathbf{r} \times \mathbf{F}$, then it follows that $\mathbf{C}$ is defined according to the right hand rule that governs the vector product (x) operation. Thus $\mathbf{C}$ is perpendicular to $\mathbf{r}$, $\mathbf{F}$ and $-\mathbf{F}$ and acts in a direction that is right handed as one goes from $\mathbf{r}$ to $\mathbf{F}$. If you draw a couple of forces in 2-D, you should be able to convince yourself that the moment vector will lie out of the plane, and that the rotation will be consistent with the moment twisting in a "right-handed" (clockwise) fashion about that axis.

**What are $L_w$ and $M_w$?** I used these to represent the equipollent force and moment due to the pressure distribution on a wing (hence the subscript W). $L_w$ is the lift force, and $M_w$ is the accompanying moment.

**What is a moment, briefly?** A moment is the effect of a force acting at some distance about an axis. The effect of a moment is to cause a rotation of the body it acts on about the axis of the moment. In vector terms it is defined by: $\mathbf{M} = \mathbf{r} \times \mathbf{F}$, where $\mathbf{r}$ is a position vector of a point on the line of action of a force $\mathbf{F}$ relative to the point about which moments are being evaluated. In general the moment due to a force will vary with the point of application. The exception to this is in the case of a couple, or pure moment. Review of 8.01 would be very helpful if this concept is still confusing.

I thought Materials and Structures would be more chemistry related (more like materials engineering) is this not the case? We will touch on the edge of this later in the course, but for the most part we will be interested in the mechanical properties of material, and how this translates into performance at the structural level.

**Are we going to build some structures?** Definitely, and better still, break them!

**How are the couple vectors related to the directions of the force vectors?** They will be perpendicular to the force vectors and the vector connecting any two points on the lines of application of the forces. Their direction is then governed by the right hand rule of the vector product.

**In order to $\sum \mathbf{P}$ do we need to change the $\mathbf{P}$ into components and sum the components?** You could do it this way, I would prefer just leaving the quantities as vectors and letting the vector algebra do the work for you - which comes to the same thing.

**There were a couple of questions regarding the PRS questions?** Please review the PRS question and see if further review will clarify matters. I will cover at the beginning of class tomorrow.
Explain \( R \)? Is \( R \) the force acting on the particle or the particles \( F \times F \) force. When I see \( R \), I want to think the particles reaction force to another force, but it is always the force acting on a particle (shouldn’t we label it \( F \))? \( R \) is the internal reaction force due to the particles remaining connected together. I used \( F \) to represent external forces acting on the system of particles.