TRUSS DEFLECTION EXAMPLE

Calculate deflection of loading point E in pin-jointed truss shown below. Bars are at 90 or 45° to each other. All bars have cross sectional area A, Young's modulus E.

No temperature change occurs.

![Diagram of truss]

Draw FBD

\[ \sum F_y = 0 \quad V_A - P = 0 \]
\[ \Rightarrow V_A = P \quad (1) \]
\[ \sum F_x = 0 : \quad H_A + H_B = 0 \]
\[ H_A = -H_B \quad (2) \]
\[ \sum M_A = 0 : \quad H_B L - 2LP = 0 \]
\[ H_B = 2P \Rightarrow \]
\[ \Rightarrow H_A = -2P \]
Analyze bar forces. Mo J.

@B

\[ \sum F_y \uparrow = 0 \quad F_{BA} = 0 \leq \]
\[ \sum F_x = 0: \quad F_{BD} + 2P = 0 \Rightarrow F_{BD} = -2P \leq \]

@E

\[ \sum F_y \uparrow = 0: \quad F_{EC} \sin 45^\circ - P = 0 \Rightarrow F_{EC} + P\sqrt{2} \leq \]
\[ \sum F_x = 0: \quad -F_{EC} \cos 45^\circ - F_{ED} = 0 \]
\[ \Rightarrow F_{ED} = -P \leq \]

\[ \sum M_D = 0: \quad +2PL - PL - F_{AC}L = 0 \]
\[ \Rightarrow F_{AC} = +P \leq \]

\[ \sum F_y \uparrow = 0: \quad F_{DC} + \dot{P} = 0 \]
\[ \Rightarrow F_{DC} = -P \leq \]

\[ \sum F_y \uparrow = 0: \quad F_{AD} \cos 45^\circ = 0 \]
\[ \Rightarrow F_{AD} = \sqrt{2}P \leq \]
Bar Deflections given by \( \frac{FL}{AE} \)

<table>
<thead>
<tr>
<th>Bar</th>
<th>Force/P</th>
<th>Length/L</th>
<th>( \frac{\delta}{\left(\frac{FL}{AE}\right)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>BD</td>
<td>-2</td>
<td>1</td>
<td>-2</td>
</tr>
<tr>
<td>AD</td>
<td>+(\sqrt{2})</td>
<td>(\sqrt{2})</td>
<td>(\sqrt{2})</td>
</tr>
<tr>
<td>AC</td>
<td>+1</td>
<td>1</td>
<td>+1</td>
</tr>
<tr>
<td>CD</td>
<td>-1</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>DE</td>
<td>-1</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>CE</td>
<td>+(\sqrt{2})</td>
<td>(\sqrt{2})</td>
<td>+(\sqrt{2})</td>
</tr>
</tbody>
</table>

Deflection Diagram:

1. Fixed points - 0, A, B
2. Locate D’ via extension/rotations of BD & AD
3. Locate C’ via extensions/rotations of AC & CD
4. Locate E’ via extensions/rotations of CE & DE
Displacement diagram (to Scale)

\[ \delta = \frac{PL}{AE} \]

Horizontal Displacement

\[ \Delta x = \frac{PL}{AE} \] to the left

Vertical Displacement

\[ \Delta y = 12.9 \frac{PL}{AE} \]
Statically Indeterminate Trusses

Can set up problem to yield a set of simultaneous equations with unknown reactions and bar forces but known displacements (at certain points - compatibility) and known constitutive behaviors.

Can also use superposition and symmetry (two pretty good principles) to simplify seemingly complicated problems. Since trusses are linear (i.e. if you double the applied load the internal forces and deflections will also double) we can superimpose the effects of multiple force systems in order to solve a problem.

Can extend the idea of deflection diagrams to more complicated trusses - basic principles remain the same:

**Example:** Symmetric 3 bar truss, bars cross sectional area A, Young’s modulus, E
Note: RA = F_{DA}, RB = F_{DB}, RC = F_{DC}

\[ \sum F_y \uparrow = 0 \quad F_{AD} \cos \theta + F_{BD} + F_{CD} \cos \theta - P = 0 \quad (1.) \]

\[ \sum F_x = 0 \quad - F_{AD} \sin \theta + F_{DC} \sin \theta = 0 \]

\[ F_{DC} = F_{AD} \quad \text{(symmetry)} \quad (2.) \]

2 equations; 3 unknowns
cannot take moments - all forces pass through D

Constitutive behavior. No $\Delta T : \delta = \frac{FL}{AE}$

<table>
<thead>
<tr>
<th>Bar</th>
<th>Force</th>
<th>Length</th>
<th>Extension</th>
</tr>
</thead>
<tbody>
<tr>
<td>$AD$</td>
<td>$F_{AD}$</td>
<td>$L$</td>
<td>$\delta_{AD} = \frac{F_{AD}L}{AE}$ (3)</td>
</tr>
<tr>
<td>$BD$</td>
<td>$F_{BD}$</td>
<td>$L\cos \theta$</td>
<td>$\delta_{BD} = \frac{F_{BD}L\cos \theta}{AE}$ (4)</td>
</tr>
<tr>
<td>$CD$</td>
<td>$F_{CD}$</td>
<td>$L$</td>
<td>$\delta_{CD} = \frac{F_{CD}L}{AE}$ (5)</td>
</tr>
</tbody>
</table>

5 equations; 6 unknowns. Two equilibrium equations, 3 constitutive relations

So must invoke compatibility:

bars extend and rotate, but remain attached at $D$: Displacement diagram

Enlarged view of displacement diagram only:
\[ \delta_{AD} = \delta_{CD} = \delta_{BD} \cos \theta \quad (6) \]

Now have 6 equations, 6 unknowns and can solve.

Substitute 3, 4, 5 into 6:

\[ \frac{F_{AD}L}{AE} = \frac{F_{BD}L \cos^2 \theta}{AE} \Rightarrow F_{AD} = F_{BD} \cos^2 \theta \]

Substitute into (1)

\[ 2F_{BD} \cos^3 \theta + F_{BD} - P = 0 \]

\[ F_{BD} = \frac{P}{(1 + 2 \cos^3 \theta)} \]

\[ F_{AD} = F_{CD} = \frac{P \cos^2 \theta}{(1 + 2 \cos^3 \theta)} \]