Lecture 52

9/11/00

From last time:

\[
\frac{i}{R} \quad \text{The constitutive relation is} \quad v = iR \quad "\text{Ohm's law}" \\
\]

Why do we choose the sign convention this way?

If \( v > 0 \ldots \)

+ terminal is at higher potential than
  - terminal, so...
+ charges move toward lower potential,
  - charges move toward higher potential, so...
  current is + if drawn as shown.

Why is Ohm's law linear? Cannot easily derive - it's an experimentally observed fact about many materials.

The battery symbol is

\[
\begin{array}{c}
\text{V} \\
\text{+} \\
\text{−}
\end{array}
\]

What is the constitutive relation for a battery?

(Will say more later...)
The Voltage Source

Constitutive relation:

\[ V = V \text{ for all } i \]

Is this a good model for a battery?

Sometimes, a voltage source is a good model for a battery, sometimes it is not, depending on the current draw and the length of time it is used.

Example Models of a battery

Case 1. Current is low enough that battery voltage is nearly constant. Then the model is

\[ 12V \sim 12V \]

\[ \Rightarrow \]

\( V = 12 \text{ volt} \)
Case 2. Current is significant enough that voltage varies due to current flow.

\[ 12\,V \quad \frac{+}{-} \quad \approx 12\,V \quad \frac{+}{-} \]

\[ R_i = \text{"internal resistance"} \]

\[ \Rightarrow \quad V = 12\,V + i\,R_i \]

Case 3. As in case 3, but battery runs long enough to be discharged, so voltage drops over time.

\[ 12\,V \quad \frac{+}{-} \quad \approx f(Q_b) \quad \frac{+}{-} \]

\[ V = f(Q_b) + i\,R_i \]

\( Q_b = \text{battery charge} \)

\[ = Q_i + \int_0^t i\,dt \quad R_i \text{ might also be a function of } Q_b. \]
The Current Source

\[ I + u = i \]

Constitutive relation:

\[ i = I \text{ for all } u \]

Current sources are useful idealizations of circuits that produce a nearly constant current for a range of loads. They are often seen in transistor amplifier circuits.

Solving networks

By "solving," we mean finding all the branch currents and branch voltages in the circuit.

Simple example:

\[ V_3 + R_1 + R_2 \]

"Parallel" resistors

Find \( V_1, V_2, V_3, i_1, i_2, i_3 \);
\[ V_1 = V_2 = V_3 = V_3 \quad \text{(Why?)} \] (1)

\[ i_1 = \frac{V_1}{R_1} = \frac{V_3}{R_1} \quad \text{(constitutive relation for } R_1) \]

\[ i_2 = \frac{V_2}{R_2} = \frac{V_3}{R_2} \quad \text{(constitutive relation for } R_2) \]

\[ i_3 + i_1 + i_2 = 0 \quad \text{(Why?)} \] (2)

\[ \Rightarrow i_3 = -\left(\frac{1}{R_1} + \frac{1}{R_2}\right) V_3 \]

\[ = -\frac{V_3}{R} \]

\[ R = R_1 || R_2 = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2} \]

To solve, we used the constitutive relations for the elements, plus Kirchhoff's laws:

(1) comes from Kirchhoff's Voltage Law (KVL)

(2) comes from Kirchhoff's Current Law (KCL)