Lecture S8 Muddiest Points

General Comments

Today, we did a little review of Laplace transforms, and saw how to use them in the analysis of systems. The most confusion seems to be about the region of convergence.

Responses to Muddiest-Part-of-the-Lecture Cards

(17 cards)

1. **You always want us to out the region of convergence, but what does it mean to converge or not converge? (1 student)** Consider the LT of \( g(t) = e^{at}\sigma(t) \). The LT integral is given by
   \[
   G(s) = \int_0^\infty e^{at}e^{-st} \, dt = \int_0^\infty e^{(a-s)t} \, dt
   \]
   As \( t \to \infty \), the integrand either goes to zero or goes to infinity, depending on whether \( a-s \) is negative or positive. If \( a-s \) is negative, the integrand goes to zero exponentially fast, which means the integral is finite (there is finite are under the graph of \( e^{(a-s)t} \)), so we say the integral converges. if \( a-s \) is positive, the integrand blows up, so the integral is infinite— it doesn’t converge. So the LT is only well-defined for \( s > a \).

2. **Could you explain more clearly what the [pole-zero and region of convergence plot] means? (1)** The LT integral converges for some values of \( s \). The hashed region in the plot represents the complex values of \( s \) for which the integral converges. The x’s on the plot represent the poles of the system. The o’s on the plot represent the zeros of the system.

3. **Confused as to why \( \text{Re}[s] > 0 \) in most cases. It seems to have something to do with the value in the exponential. (1)** For unilateral LTs, the region of convergence is always of the form \( \text{Re}[s] > \sigma_0 \), for some \( \sigma_0 \). The reason is that if the integral converges for some value of \( s \), the integrand of the LT integral will get smaller if the real part of \( s \) is increased. That is, the factor \( e^{-st} \) gets smaller as \( s \) gets more positive.

4. **How do you write the region of convergence for**
   \[
   G(s) = \mathcal{L}[e^{-at}] = \frac{1}{s+a}
   \]
   **Should it be \( s > a \) or \( s > -a \)? (1)** \( s > -a \). The pole of \( G(s) \) is where the denominator is zero, namely, at \( s = -a \). The region of convergence is to the right of the pole.

5. **What’s a pole? (1)** The pole of a transfer function \( G(s) \) are those values of \( s \) for which \( G(s) = \infty \). This happens whenever the denominator of \( G(s) \) is zero. Actually, my definition of a pole above is too sloppy to be correct, but will do for now.

6. **Often, the differential equations don’t come with initial values. Should we just solve with them unknown? (1)** No. Usually, there will be a specific right thing to do, and it won’t involve solving with unknown ICs. For example, when you find the impulse response, the ICs are implicitly zero.
7. For a system with impulse response \( g(t) = \sigma(t) \), how does that lead to

\[
y(t) = \int_{-\infty}^{t} u(\tau) \, d\tau
\]

(1) The response to arbitrary input \( u(t) \) is

\[
y(t) = g(t) * u(t) = \int_{-\infty}^{\infty} \sigma(t - \tau) u(\tau) \, d\tau = \int_{-\infty}^{t} u(\tau) \, d\tau
\]

since \( \sigma(t - \tau) = 1 \) for \( \tau < t \).

8. Why is it that the integral

\[
\int g
\]

sometimes instead of (from last lecture)

\[
\int g'
\]

(1) I have to admit that I don’t understand your question. Please ask again at next lecture, or at the recitation.

9. Please do some real examples (not RLC circuits). (1) Although RLC circuits are real examples, I take your point, and will try.

10. No mud. (8)