Problem 1.

Sketch the root locus for \( L(s) = \frac{s}{(s+1)(s+4)}. \)
\( \phi_R = \frac{180^\circ + 360^\circ}{2} = 180^\circ, \) open loop pole at \( s = -1, s = -4. \) Zero at \( s = 0. \)

Problem 2.

Sketch the root locus for \( L(s) = \frac{s}{(s-1)(s-4)}. \)
\( \phi_R = \frac{180^\circ + 360^\circ}{2} = 180^\circ, \) open loop pole at \( s = 1, s = 4. \) Zero at \( s = 0. \)

To find departure/arrival point from real axis, use characteristic equation:

\[
1 + kL(s) = 0 \rightarrow 1 + \frac{ks}{s^2 - 5s + 4} = 0
\]
\[ s^2 + (k - 5)s + 4 = 0 \]

Use quadratic formula

\[
\frac{-(k - 5)}{2} \pm \frac{\sqrt{(k - 5)^2 - 16}}{2}
\]

The \( \frac{\sqrt{(k - 5)^2 - 16}}{2} \) term may be real or imaginary. If we sent it equal to zero and solve for \( k \), that is the gain at which the transition from real to imaginary occurs.

\[
\frac{\sqrt{(k - 5)^2 - 16}}{2} = 0
\]

\[
(k - 5)^2 = 15
\]

\[
|k - 5| = 4
\]

\[
\rightarrow k = 1, 9
\]

Now need to put \( k \) values back into characteristic equation, and solve for \( s \). This will tell us the location of the roots.

\( k = 1 \rightarrow s^2 - 4s + 4 = 0 \), two roots at \( s = 2 \), \( k = 9 \rightarrow s^2 + 4s + 4 = 0 \).

Two roots at \( s = -2 \).

When \( k = 5 \), the real part of the quadratic equation is zero, so this is the value of \( k \) for when the locus intersects the imaginary axis. Plugging \( k = 5 \) into characteristic equation:

\( s^2 + 4 = 0 \rightarrow \text{Intersects imaginary axis at } s = \pm 2j. \)
Problem 3.

Sketch the locus of \( L(s) = \frac{s+3}{(s+1)(s+2)(s+20)} \)

\[ \alpha = \frac{-1-2+3-20}{3-1} = -10 \]

If the pole at \( s = -20 \) were closer to the zero, the locus would look more like