**Falkner-Skan Flows**

For the family of flows, we assume that the edge velocity, \( u_e(x) \) is of the following form:

\[
u_e(x) = Kx^m \
K = \text{arbitrary constant}
\]

The pressure can be calculated from the Bernoulli in the outer, inviscid flow:

\[
p_e + \frac{1}{2} \rho u_e^2 = \text{const.}
\]

\[
\Rightarrow \frac{dp_e}{dx} = -\rho u_e \frac{du_e}{dx}
\]

\[
\Rightarrow \frac{dp_e}{dx} = -\rho K^2 x^{2m-1}
\]

if \( m > 0 \) then \( \frac{dp_e}{dx} < 0 \Rightarrow \) favorable pressure gradient

if \( m < 0 \) then \( \frac{dp_e}{dx} > 0 \Rightarrow \) adverse pressure gradient

These edge velocities result from the following inviscid flows:

\[
\beta \equiv \frac{2m}{1 + m}
\]

- Flow around a corner (diffusion) \( -2 \leq \beta \leq 0 \)
- Wedge flow \( 0 \leq \beta \leq 2 \)
Some important cases:

\[ \beta = 0, m = 0: \text{ flat plat (Blasius) } \]
\[ \beta = 1, m = 1: \text{ plane stagnant point } \]

The boundary layer independent variable \( \eta \) from the Blasius solution generalizes to:

\[ \eta \equiv y \sqrt{\frac{m+1}{2} \frac{u_e(x)}{v_x}} \quad \text{and} \quad u(x, y) = u_e(x) f' (m) \]

An interesting case in \( \beta = 1, m = 1 \), i.e. stagnation point flow:

\[ u_e = K_x \]

inviscid flow velocity increases away from stag. pt. at \( x = 0 \)

\[ \eta = y \sqrt{\frac{1+1}{2} \frac{K_v}{v_x}} \]

\[ \Rightarrow \quad \eta = y \sqrt{\frac{K}{v}} \quad \Rightarrow \eta \text{ is independent of } x \]

\[ \Rightarrow \text{Boundary layer at a stagnation point does not grow with } x! \]

The skin friction can be found from:

\[ \tau_w = \mu \left. \frac{\partial u}{\partial y} \right|_{y=0} = \mu u_e(x) \left. \frac{d^2 f}{d\eta^2} \right|_{\eta=0} \left. \frac{\partial \eta}{\partial y} \right|_{y=0} \]

\[ f''(o) \]
Since \( \eta = y \sqrt{\frac{m + 1}{\frac{v_x}{u_x}}} \Rightarrow \frac{\partial \eta}{\partial y} = \sqrt{\frac{m + 1}{\frac{v_x}{u_x}}} \)

\[ \Rightarrow \tau_w = \mu u_e(x) \left( \frac{m + 1}{\frac{v_x}{u_x}} \right) \frac{v}{2} f^{11}(o) \]

The skin friction coefficient is normalized by \( \frac{1}{2} \rho u_e^2(x) \):

\[ C_f(x) \equiv \frac{\tau_w}{\frac{1}{2} \rho u_e^2(x)} = 2 \sqrt{\frac{m + 1}{\frac{v_x}{u_x}}} \frac{v}{2} u_e(x)x f^{11}(o) \]

\[ \Rightarrow C_f = \frac{2 \sqrt{\frac{m + 1}{\frac{v_x}{u_x}}} f^{11}(o)}{\sqrt{Re_x}} \]

\[ Re_x \equiv \frac{u_e(x)x}{v} \]

Note: separation occurs when \( C_f = 0 \) which means \( f^{11}(o) = 0 \). From the table, this occurs for \( \beta = -0.19884 \)

\[ \Rightarrow \text{This is only an angle of } 18^\circ \]