Brequet Range Equation

\[ \text{Range} = \int_{t_i}^{t_f} V \, dt \]

In level flight at const speed:

\[ L = W, \text{ Lift} = \text{Weight} \]
\[ T = D, \text{ Thrust} = \text{Drag} \]

The aircraft weight changes during flight due to use of fuel. Relate weight change to time change:

\[ dW = \frac{dW}{dt} \, dt \]
\[ = -\frac{\text{fuel weight}}{\text{time}} \, dt \]
\[ = -\frac{\text{fuel weight}}{\text{time}} \frac{T}{t} \, dt \]

The quantity:

\[ \frac{\text{fuel weight}}{\text{time}} \frac{1}{T} \]

is known as the specific fuel consumption or sfc. It has units of:

\[ \text{sfc units} = \frac{\text{lb}^{(\text{fuel})}}{\text{lb} \cdot \text{hr} \text{ or force} \cdot \text{time}} \]

\[ \Rightarrow dW = -\text{sfc} \cdot T \cdot dt \]

\[ \Rightarrow \text{Range} = -\int_{w}^{w} \frac{V}{sfc} \frac{1}{dW} \]

But since \( T = D \) and \( L = W \) we have:

\[ \text{Range} = -\int_{w}^{w} \frac{V \cdot L \cdot dW}{sfc \cdot D \cdot W} \]
This is the general form of the range equation. The Breguet range equation is found by assuming that $\frac{V}{L} \frac{D}{sfc}$ is constant for the entire flight. In that case:

$$\text{Range} = -\int_{w_f}^{w_i} V \frac{L}{sfc} \frac{dW}{D} = -\int_{w_f}^{w_i} \frac{V}{sfc} \frac{L}{D} \frac{dW}{W}$$

Breguet range equation:

$$\text{Range} = \frac{V}{sfc} \frac{L}{D} \log \frac{W_i}{W_f}$$