1) A “passive” scalar quantity $s$ (such as the concentration of a flow tracer dye, say) which does not diffuse significantly obeys

$$\frac{D s}{D t} = 0$$

which simply states that $s$ does not change at a particular point moving with the fluid.

a) Determine how the gradient of $s$ evolves in the flow:

$$\frac{D(\nabla s)}{D t} = ?$$

Which kinematic components of the fluid motion contribute to the changes in magnitude and/or direction of $\nabla s$?

b) A small amount of the passive material is deposited in a flow as a small round blob, so that $\nabla s$ points radially inward to the blob center. This is then subjected to a simple 2-D shear flow.

$$\vec{u} = Ky\hat{i} + 0\hat{j}$$

Sketch the blob boundaries, indicating $\nabla s$, a short time later. Verify that the gradient evolved as indicated by your result from 1a).

2) The boundary layer which exists on the floor of a wind tunnel is taken through a horizontal turn in the tunnel as shown below.

Because the boundary layer’s growth due to viscosity is relatively slow, the inviscid vorticity convection equation

$$\frac{D\vec{\omega}}{D t} = \vec{\omega} \cdot \nabla \vec{u}$$

is approximately valid for rapid flow changes. Determine qualitatively the vorticity orientation in the boundary layer after it is turned a small amount, and the associated velocity pattern.
3) Determine which of the following equations (1) – (5) are invariant under the Galilean coordinate transformation \((\vec{r}, t) \to (\vec{r}', t')\), with \(\vec{c}\) some constant frame-translation velocity.

\[
\begin{align*}
\vec{r}' &= \vec{r} - \vec{c}t \\
t' &= t \\
\text{also: } \vec{u}' &= \vec{u} - \vec{c}
\end{align*}
\]

\[
\begin{align*}
\nabla \cdot \vec{u} &= 0 & \text{Invariant} \\
\frac{\partial \vec{u}}{\partial t} &= 0 & \text{not invariant} \\
\frac{D\vec{u}}{Dt} &= 0 & \text{Invariant} \\
\frac{\partial^2 \rho}{\partial t^2} - c^2 \nabla^2 \rho &= 0 & \text{not invariant}
\end{align*}
\]