1a) At separation \( \frac{dH^*}{dH} = 0 \). The IBLT matrix becomes:

\[
\begin{bmatrix}
1 & 0 & H+2 \\
0 & 0 & 1-H \\
0 & a & 1
\end{bmatrix}
\]

In the classical case, \( a = 0 \), therefore the matrix is singular. In the IBLT case, the matrix is invertible since \( a = \frac{-8*}{H-H^*} \neq 0 \). This nonzero displacement effect \((H-H^*)\) modifies \( u_c(x) \).

1b) Classical case separates at \( x = 0.382 \). For a given \( H(x) \) or \( u_c(x) \), a laminar BL has the same \( H(x) \) for any \( Re \), so separation, where \( H = H_\infty \), is always at the same \( x \).

In IBLT, \( u_c(x) \) is modified by \( \delta^* \) which depends on \( Re \). \( \delta^*(x) \) also depends on \( Re \), so \( x_{sep} \) moves upstream toward the classical \( x_{sep} = 0.382 \) as \( Re \) is increased.

2) \( x_{sep} \) moves gradually downstream as \( H_{crit} \) is increased.

3) \( o_1 \) is minimum when \( H_{crit} = 4 \). Minimum loss occurs when \( \delta^* \) and \( \delta^*_L \) are dominant laminar flow is maximized as long as separation does not occur.

\[ \frac{d\theta}{dx} = \frac{\theta}{2} - (H+2) \frac{\partial u_c}{u_c} \]

If \( H < 4 \), \( \int_0^1 \frac{\theta}{2} dx \) is dominant linear (penalty).

If \( H > 4 \), \( \int_0^1 -(H+2) \frac{\partial u_c}{u_c} dx \) is dominant linear. This describes mixing losses which occur during reattachment. Form drag for airflow.
To allow design calculations, simplest approach is to introduce a new variable
\[ \beta_n = \frac{\chi}{h} \frac{dh}{dx}, \]
and augment system into a 4x4.

\[
\begin{bmatrix}
1 & 0 & H+2 & 0 \\
-H* \frac{dh}{dx} & H* \frac{dh}{dx} & 1-H & 0 \\
0 & -\delta & 1 & 1-H* \delta \\
-1 & 1 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\beta_n \\
\beta_n \\
\beta_n \\
\beta_n
\end{bmatrix}
= \begin{bmatrix}
\chi \frac{\chi}{h} \delta^2 \\
\chi \left( \frac{\chi}{h} \delta^2 - \delta \right)
\end{bmatrix}
\]

For design condition \( (U_c/U_0) = 0.5 \)

- Minimum length \( x \approx 0.35 \) at \( H_{spec} = 2.35 \)
- Minimum \( \beta(1) \approx 0.0043 \) at \( H_{spec} = 2.0 \)

See attached plots.

Effusion shape:

\[
\text{Effusion shape:}
\]

0 0.1 0.2 0.3 0.4

5a) Integral Momentum Eqn:
\[
\frac{dh}{dx} = \frac{\delta}{2} - (2H) \frac{\delta}{Uc} \frac{dUc}{dx} + (\frac{Vw}{Uc})
\]

Kinetic Energy Shape Parameter Eqn:
\[
\frac{\partial}{\partial x} \frac{dH^2}{H^2} = \frac{2\chi}{H^2} \frac{\chi}{Uc} - \frac{\delta}{2} - (\frac{Vw}{Uc}) (1 - H^*) + (H - 1) \frac{1}{Uc} \frac{dUc}{dx}
\]

The equations are valid for laminar and turbulent flows with suction. However, turbulent closure \( \chi \) have to be modified to account for the effect of suction on the stream lines near the wall (turbulent \( \chi \) and \( \delta \) have to be modified).
Modified interaction law

\[ u(x) = u_0 + \int_0^x \rho \nu \omega dx - \rho u_c (h - \delta x) \]

\[ \Rightarrow \frac{\partial u_c}{\partial x} = \frac{u_c}{h - \delta x} \left[ \frac{d\delta x}{dX} - \frac{dh}{dX} + \frac{\nu}{u_c} \right] \]

56) \hspace{1cm} \eta_i = 0.07 \hspace{1cm} \eta_{air} = 50 \hspace{1cm} Re = 10^6

Ca = 0.013 \hspace{1cm} (1.3\%) required to suppress separation

There is no minimum \( \eta_c = 0 \) with microjet injection

56) \hspace{1cm} Introduce \( \frac{\nu}{u_c} \) as a design variable. Segment system similar to part 4

\[
\begin{bmatrix}
1 & 0 & (2+H) & -\left(\frac{x}{10}\right)
\end{bmatrix}
\begin{bmatrix}
\beta_0 \\
\beta_\delta \\
\beta_u \\
\frac{\nu}{u_c}
\end{bmatrix}
= \begin{bmatrix}
\vdots
\end{bmatrix}
\]

There is no minimum \( C_{pt} \) as in (56) for laminar flow. Attached plot shows \( u_c(x) \) for \( H_{spec} = 4 \) and \( H_{spec} = 2.0 \) (min)

\[ H_{spec} = 4 \hspace{1cm} 2 \]
\[ Ca = -0.009 \hspace{1cm} -0.038 \]
\[ C_{pt} = 0.025 \hspace{1cm} 0.0105 \]
Problem 4 (i)
Problem 4 (ii)