4.5 Integral Methods

A. BL behavior example
B. Sep. behavior
C. Separation in TFL Context

Ready: Handout paper

B. BL Behavior estimate Example

Goal: Gain insight into how various terms in the 2-ozn method drive BL behavior

Problem:

\[
\frac{u_{e_2}}{u_{e_1}} = 0.9
\]

- Sudden decrease (10%) will UE on a flat plate BL
  a. Will BL separate?
  b. What is the k increase?

Use logarithmic form of Reyn & K-E eqn:

\[
\frac{d}{dx} \ln \theta = \frac{1}{\theta} \left( \frac{G}{2} - (2+H) \frac{d}{dx} \ln \theta \right)
\]

\[
\frac{d}{dx} (\ln \theta^*) = \frac{1}{\theta} \left[ \frac{2G}{H^*} - \frac{G}{2} \right] + (H-1) \frac{d}{dx} \ln (u_e)
\]

To check for separation we use k-e shape parameter eqn:

\[
\int_{x_1}^{x_2} \{ \}
\]

\[
\frac{H_{2}^{*}}{H_{1}^{*}} = \exp \left[ \frac{2G}{H^*} - \frac{G}{2} \right] \frac{1}{\theta} (x_2-x_1) \frac{(u_{e_2})^{H-1}}{u_{e_1}}
\]
Examine limits in K.E expr $\rightarrow \text{decr, } K < 0 \Rightarrow H^* \text{ gets smaller for sufficiently fast deceleration } x_2 - x_1 \rightarrow 0$

$\Rightarrow \exp \frac{g}{f} \rightarrow 1$

$$(\frac{H_2^*}{H_1^*}) = (0.9)^{(H-1)\text{avg}}$$

For laminar flow

$H_1 = 2.6, \quad H_1^* = 1.55$

Turbulent $H_1 = 1.4, \quad H_1^* = 1.85$

$H_2^* = (0.9)^{1.6}, \quad H_1^* = 0.84, \quad H_1^* = 1.75 \Rightarrow \text{flow will separate below } H^* = 1.5 \text{ sep. limit}$

$H_2^* = (0.9)^{0.9}, \quad H_1^* = 1.78 \Rightarrow \text{far from separation}$

Estimate $\theta_2$ for turbulent flow

$$\ln \frac{\theta_2}{\theta_1} = -(H+2) \ln 0.9$$

$$\Rightarrow \frac{\theta_2}{\theta_1} = 1.43 \text{ sudden increase in } \theta$$

If $\Delta x$ is large, there will be additional contribution from $\theta$ limit for slower deceleration. One limiting factor is multiply $\Delta x$ and allow the effects of the pressure gradient, but add to $\theta$ increase

$$\frac{2C_0}{H^*} - \frac{g/2}{f}$$

As $H$ increases $\frac{2C_0}{H^*}$ becomes more prominent and drive $H^*$ bigger to alleviate the tendency to separate.
Going back to laminar case

\[ H^* = 1.3 \]

which is below the new "permitted" by \( H^*(H) \) correlation function. In reality, the flow will separate and moderate the \( \nu \) decrease via the blockage or displacement effect (next slide on IBLT) so that \( H^* \to \) approach 1.5

\[ \frac{dH}{dx} = \frac{1}{dH*/dH} \left\{ \frac{1}{\nu} \left( \frac{d\nu}{dH^*} - \frac{1}{2} \right) + (H-1) \frac{1}{\nu} \frac{d\nu}{dx} \right\} \]

Towards separation point with \( \nu(x) \) prescribed

\[ H^* = 1.5 + k(H-4)^2 \]

\[ \frac{dH^*}{dH} = k(H-4) \]

\[ \frac{dH}{dx} = \frac{1}{k(H-4)} \]

\[ \frac{dH}{dx} = \frac{\alpha}{H-4} \]

\[ \epsilon(H-4) = \sqrt{\frac{\alpha}{x}} \]

As \( H \to 4 \), \( \frac{dH^*}{dH} \to 0 \), \( \frac{dH}{dx} \to \infty \), \( \frac{d\nu}{dx} \), \( \frac{dC}{dx} \), \( \frac{dC_p}{dx} \) \to \infty

Called "Goldstein singularity". Purely numerical artifact occurs when \( \nu(x) \) is imposed at separation.
We can see that \( \frac{dH}{dx} \) is finite only if

\[
\left\{ \frac{2c_0}{H^*} - \frac{q}{2} \right\} - \left( H-1 \right) \frac{1}{H} \frac{dH}{dx} = 0 \text{ at separation}
\]

or \( \frac{dH}{dx} \) is such that,

\[
\frac{H^*}{H} = \frac{2c_0}{H^*} - \frac{q}{2} \quad \text{at separation}
\]

\[
\frac{\frac{dH}{dx}}{H^*} = \frac{2c_0}{H^*} - \frac{q}{2} H^* \quad \text{at separation}
\]

\[
= \text{boundary layer determining } \frac{dH}{dx} \text{ (wall channel example in 'blockage' or } \delta^* \text{) mechanism}.
\]

\[
\frac{dH}{dx} \text{ is determined in } y \text{ of}
\]

In other words,

\[
\text{This requires IBLT displacement effect, so that } BL \text{ can modify } \frac{dH}{dx} \text{ so that } \frac{dH}{dx} \text{ reaches the "admissible" value.}
\]

\[
\frac{dH}{dx} \text{ is quite small in separated flow regions.}
\]

\[
\text{We imposed by } \delta^* \quad \text{We imposed by } BL
\]

\[
\rho_{in \text{ count}} \Rightarrow \text{ We in count}
\]

\[
\text{nearly stagnant, re-incoming flow} \Rightarrow \text{nearly count pressure}
\]
we can deal with limited separation

* TSL assumption (approx. reasonably valid)
  - $\frac{d\delta}{dx} \ll 1$
  - $\frac{\partial p}{\partial y}$ small

1. i.e. stall

2. total or
   - i.e. stall $\ll 1$ x
     large scale unreliability

3. diffuser sep.
   - $0.1$ ok.

4. $\approx 1.0$ x
   large scale unreliability

$\Rightarrow$ leads to IBLT feedback.