HOME ASSIGNMENT #2

Warm-Up Exercises

1. The three-dimensional (3-D) stress-strain equations are written as:

\[ \sigma_{mn} = E_{mnpq} \varepsilon_{pq} \]

These can be reduced for the plane stress case (i.e. \( \sigma_{33} = \sigma_{13} = \sigma_{23} = 0 \)) to a two-dimensional (2-D) representation:

\[ \sigma_{\alpha\beta} = E^*_{\alpha\beta\gamma\delta} \varepsilon_{\gamma\delta} \]

where the asterisk indicates that this is a 2-D elasticity tensor for plane stress whose components do not have a one-to-one correspondence with the 3-D elasticity tensor, \( E_{mnpq} \). For an orthotropic material find the relations between \( E^*_{\alpha\beta\gamma\delta} \) and \( E_{mnpq} \).

2. Repeat the previous problem for an isotropic material.

Practice Problems

3. A unidirectional graphite/epoxy composite material is loaded in the plane of its fibers. This material is transversely isotropic (\( \nu_{12} = \nu_{13} ; E_{22} = E_{33} \)) and has the following four elastic constants:

\[
\begin{align*}
E_{11} &= 130 \text{ GPa} \\
E_{22} &= 10.5 \text{ GPa} \\
\nu_{12} &= 0.28 \\
G_{12} &= 6.0 \text{ GPa}
\end{align*}
\]

The part is thin compared to its in-plane dimensions. Determine all the non-zero strain components for the following stress state:
Express strains in \([\text{microstrain}] = 10^{-6}\)

4. A piece of aluminum of the same shape as the graphite/epoxy part of the previous problem is loaded in the same manner:

\[
\sigma_{11} = 60 \text{ MPa} \\
\sigma_{22} = 30 \text{ MPa}
\]

The elastic constants of the aluminum are:

\[
E = 10.3 \text{ Msi} \\
\nu = 0.30
\]

Determine all the nonzero strain components for this case.

**Application Tasks**

5. Beginning with the plane stress (i.e. \(\sigma_{33} = \sigma_{13} = \sigma_{23} = 0\)) form of the stress-strain equations:

\[
\sigma_{\alpha\beta} = E_{\alpha\beta\gamma\gamma} \varepsilon_{\gamma\gamma}
\]

find the relations between the five in-plane engineering constants \((E_L, E_T, \nu_{LT}, \nu_{TL}, \text{ and } G_{LT})\) and the \(E_{\alpha\beta\gamma\gamma}^*\) for an orthotropic material.

*(HINT: Think compliances)*

6. Repeat the previous problem for an isotropic material.