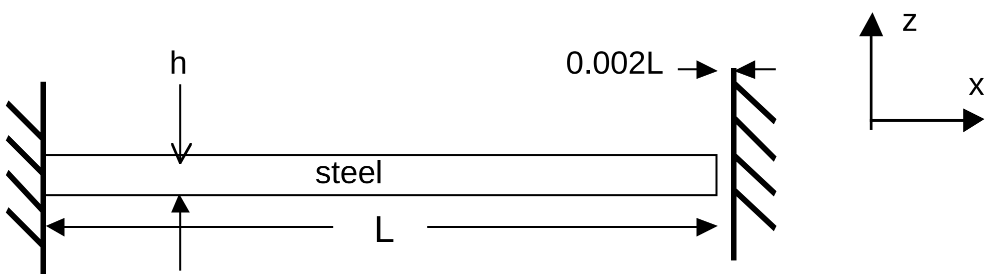
16.20

Due: Lecture 25

HOME ASSIGNMENT #5

Warm-Up Exercises

Let's consider a steel bar that is rigidly clamped at one end and free at the other end with a gap of 0.002L between that end and a wall. The bar is of length L, thickness h, and width b. Bending and instability issues are not of concern. The bar is subjected to a uniform temperature change of ΔT from room temperature of 70°F. Properties of steel are modulus of 30 Msi, Poisson's ratio of 0.3, and coefficient of thermal expansion of 6 μstrain/°F.



NOTE: For the following, determine the total strain and the two components of thermal and mechanical.

1. Determine the stress and strain state in the x-direction for temperatures where the bar end does not contact the wall.

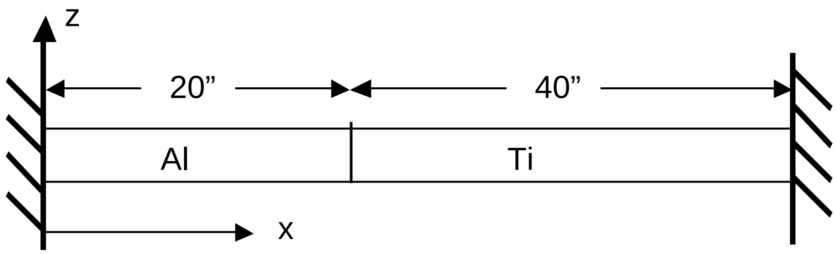
2. Determine the temperature at which the bar first contacts the wall and the corresponding stress and strain state in the x-direction.

3. Determine the stress and strain state in the x-direction for temperatures where the bar end does contact the wall.

Practice Problems

4. A bar is 60″ long, 2.5″ thick, and 2.5″ wide and is held between two rigid walls. The first 20″ of the bar is made of aluminum while the second part is made of titanium. The bar is subject to a constant temperature change, ΔT, of 100°F. Buckling is not a concern.

	Aluminum	Titanium
E	10.0 Msi	15.5 Msi
ν	0.30	0.34
α	12.0 μstrain/°F	5.0 μstrain/°F



(a) Determine the axial stress, σ_x, and the total strain, ε_x, distribution in the bar as a function of x.

(b) Plot the three components of the strain (total, thermal, and mechanical) as a function of x.

(c) What is the displacement of the material junction point?

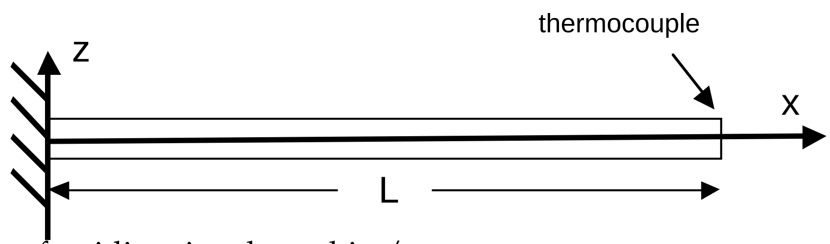
(d) Where would you need to apply St. Venant's principle? Describe what would happen at these points.

(e) Assuming the three-dimensional effects do not change the answer to part (c), determine the full three-dimensional state of stress and strain (total, thermal, mechanical) at the junction point.

5. A 3-meter long unidirectional graphite/epoxy structural bar/beam is part of a satellite solar panel. This structure has a rectangular cross section and is 50 mm wide and 5 mm thick. The temperature distribution on it is linear in the x-direction and can be expressed as:

$$T(x) = T_{ref} + T_o (x/L)$$

where T_{ref} is the temperature at which the structure was assembled. This graphite/epoxy structure can be modeled as being cantilevered. A thermocouple is placed at the tip to determine the temperature. Determine the tip deflection of this structure as a function of the tip temperature.



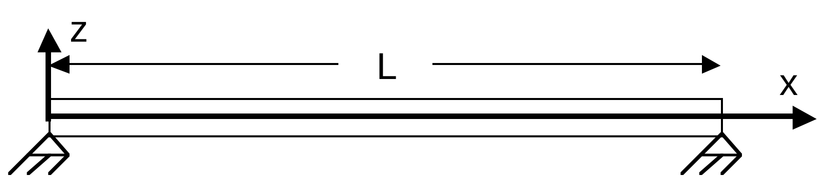
Properties of unidirectional graphite/epoxy are:

E_L = 140 GPa ν_{LT} = 0.28 α_L = – 0.11 μstrain/°C
E_T = 20.0 GPa ν_{TL} = 0.040 α_T = 6.7 μstrain/°C

Application Tasks

6. A structure of length L is held between two rigid pinned supports. The structure is made of steel with a modulus of 29 Msi, a Poisson's ratio of 0.3, and a coefficient of thermal expansion of 6.5 μstrain/°F. The structure has a square cross section with the sides being 1/25 of the length of the structure. The bar is subject to a temperature differential of the form:

$$\Delta T(x) = A\, x\, (3x^2 - 2L^2)$$



(a) Determine the axial stress, σ_x, and the total strain, ε_x, distributions in the bar as a function of x (and the bar length, constant A, and the material parameters).

(b) Plot the three components of the strain (total, thermal, and mechanical) in the x-direction as a function of x for a 6-foot long structure. The value of A is $0.2°F/ft^3$.

(c) Neglecting end effects, what would be the general state of strain for a bar of this configuration (not just of steel, but of any isotropic material)?

(d) For the case of part (b), there is now a constant thermal differential of 35°F between the top surface and bottom surface that is linear in z. Describe what will change about the behavior of the structure. Determine, as best as possible, the stress and strain states in the x-direction.