Practice Problems

5. For this problem, we will make use of the membrane analogy that was discussed in class and Unit 11 in the notes. Specifically, the membrane analogy for the torsion of narrow rectangular cross-sections will be used because each section of the channel cross-section and I cross-section beam can be represented as narrow rectangular cross-sections (see pp. 11-21 in Unit 11).

i) The angle of twist, \( \theta \), can be obtained from

\[
\frac{d\theta}{dz} = \frac{T}{GJ}
\]

\[\Rightarrow \theta = \int_0^z \frac{T}{GJ} \, dz = \frac{Tz}{GJ} \quad \text{(1)}
\]

Since \( T \) and \( J \) are constants.

So, we need to find the torsional constant \( J \). Let's consider the channel cross-section first.

\[
J_c = J_A + J_B + J_O \quad \text{(2)}
\]
From the membrane analogy of a narrow rectangular cross-section, we know that $J$ for a rectangle of length $b$ and height $h$,

$$J = \frac{bh^3}{3}$$

Thus,

$$J_0 = J_{\theta} = \frac{(55\text{mm})(5\text{mm})^3}{3} = 1.5 \times 10^{-4} \text{m}^4$$

$$J_\theta = \frac{(80\text{mm})(5\text{mm})^3}{3} = 3.3 \times 10^{-4} \text{m}^4$$

Plugging the values for $J_0$, $J_\theta$, and $J_{\theta}$ in equation (3), we get

$$J_c = (1.5 \times 10^{-4}) + (3.3 \times 10^{-4}) + (1.5 \times 10^{-4})$$

$$\Rightarrow J_c = 6.3 \times 10^{-4} \text{m}^4$$

Next, let's consider the I-beam.

$$J_I = J_0 + J_\theta + J_{\theta}$$
The I-beam is made from the same three pieces as the channel section, so the two sections must have the same torsional constants

\[ \bar{J}_1 = \bar{J}_c = 6.3 \times 10^{-9} \text{m}^4 \]

To find the angle of twist, \( \phi \), plug the given values into equation (3). \[ C \cdot \frac{I}{\bar{J}(1+\nu)} = \frac{\pi}{4} \]

For channel cross section: \[ \phi_c = \frac{(40)(2m)(2m)}{(177)(6.3 \times 10^{-3} \text{m}^4)} = 0.47 \text{ rad.} \]

\[ \phi_c = 27^\circ \]

Since the I-beam has the same torsional constant as the channel section, \( \phi_1 = \phi_c \)

\[ \phi_1 = 0.47 \text{ rad} = 27^\circ \]

b) The maximum resultant shear, \( T_{max} \), is

\[ T_{max} = \sqrt{T_{y2}^2 + T_{x2}^2} \]
From the derivation of the torsion of a narrow rectangular cross-section, we found,

\[ T_{g2} = \frac{2T}{J} x \]

\[ T_{r2} = 0 \]

\[ T_{max} = \frac{2T}{J} \alpha_{max} \]

We will find \( \alpha_{max} \) in the widest part of the cross-section, where the local coordinate \( x \) is maximized. In the widest part,

\[ \alpha_{max} = \frac{i}{2} t \]

For both the channel and the I cross-section beams, the thickness, \( t = 5 \text{mm} \), so,

\[ T_{max_{I}} = T_{max_{C}} = \frac{2T}{J} \cdot \frac{1}{2} t = \frac{(40 \text{N.m})}{(6.3 \times 10^{-6} \text{m}^3)} \cdot (0.005 \text{m}) \]

\[ T_{max_{I}} = T_{max_{C}} = 32 \text{ HPA} \]
1) In order for the resistance to be the same, the torsional constant needs to be the same. The torsional constant for the T-section is

\[ J = J_0 + J_\theta \]

\[ = \frac{(85\text{mm})(5\text{mm})^3}{3} + \frac{7(5\text{mm})^4}{3} \]  

(4)

Note that \( J_\theta \) = \( J_\perp \) and

\[ J_\theta = J_\perp = \frac{(80\text{mm})(5\text{mm})^3}{3} + \frac{2(35\text{mm})(5\text{mm})^3}{3} \]  

(5)

Comparing equations (4) and (5), we get,

\[ \frac{(85\text{mm})(5\text{mm})^3}{3} + \frac{7(5\text{mm})^4}{3} = \frac{(80\text{mm})(5\text{mm})^3}{3} + \frac{2(35\text{mm})(5\text{mm})^3}{3} \]

\[ \frac{7(5\text{mm})^4}{3} = \frac{(10\text{mm})(5\text{mm})^3}{3} + \frac{2(35\text{mm})(5\text{mm})^3}{3} \]

\[ \Rightarrow \quad \tau = (10\text{mm}) - (5\text{mm}) \]

\[ \tau = 65\text{mm} \]