16.20 HANDOUT #4B
Fall, 2002
Shell Beam Theory

Notes

- y, z axes at “modulus-weighted” c.g.
- \( Q_y, Q_z \) are static moments about y, z
- \( S_y, S_z \) are shear resultants at shear center

a) Axial Stresses

\[
\sigma_x = \frac{E}{E_I} \left[ \frac{F^{TOT}}{A^*} - \left( \frac{I_{yy} M_{z}^{TOT} - I_{yz} M_{y}^{TOT}}{I_{yy} I_{zz} - I_{yz}^2} \right) y - \left( \frac{I_{zz} M_{y}^{TOT} - I_{yz} M_{z}^{TOT}}{I_{yy} I_{zz} - I_{yz}^2} \right) z - E_1 \alpha \Delta T \right]
\]

b) Joint Equilibrium @ stringer \( A_s^* \)

\[
q_{out} - q_{in} = - \frac{A_s^*}{A^*} \frac{dF^{TOT}}{dx} + \left( \frac{I_{yy} S_{y}^{TOT} - I_{yz} S_{z}^{TOT}}{I_{yy} I_{zz} - I_{yz}^2} \right) Q_{zs}^* + \left( \frac{I_{zz} S_{z}^{TOT} - I_{yz} S_{y}^{TOT}}{I_{yy} I_{zz} - I_{yz}^2} \right) Q_{ys}^* - E_I A_s^* \frac{d}{dx} (\alpha \Delta T)
\]
Where:

\[ E_1 = \text{reference modulus} \]

\[ F^{\text{TOT}} = F + F^T \]

\[ M_z^{\text{TOT}} = M_z + M_z^T \]

\[ S_y^{\text{TOT}} = \frac{dM_z^{\text{TOT}}}{dx} = S_y + \frac{dM_z^T}{dx} \quad \text{etc.} \]

\[ Q_{ys}^* = \int z dA^* = z A_s^* \]

\[ I_{yy}^* = \int z^2 dA^* \]

\[ dA^* = \frac{E}{E_1} \, dA \]