1. Behavior of the 3-node simplex element in the incompressible limit. Using the 3-node simplex element, repeat the plane-strain, plate-with-hole calculations of the previous assignment using traction boundary conditions for the case \(-h 0.125\) and for the following values of the Poisson ratio: \(\nu = 0.49, 0.499, 0.49999\). Comment on the results.

2. Behavior of the 3-node simplex element in problems with singularities. Consider the case of a semi-infinite crack coincident with the half axis \(x_2 = 0, x_1 < 0\) subjected to mode I remote loading. Restrict the analysis to the rectangular domain \([-L, L] \times [0, L]\). The boundary conditions on the line \(x_2 = 0\) are: traction-free for \(x_1 < 0\), \(u_2 = 0\) (symmetry) for \(x_1 \geq 0\). On the remainder of the boundary the displacements are constrained to equal the \(K\)-field:

\[
\begin{align*}
  u_1 &= \frac{K}{2\mu} \sqrt{\frac{r}{2\pi}} \cos(\theta/2)[\kappa - 1 + 2\sin^2(\theta/2)] \\
  u_2 &= \frac{K}{2\mu} \sqrt{\frac{r}{2\pi}} \sin(\theta/2)[\kappa + 1 - 2\cos^2(\theta/2)]
\end{align*}
\]

where \(\kappa = 3 - 4\nu\), \(\mu\) is the shear modulus, \(\nu\) Poisson’s ratio, \(r\) is the distance to the origin and the polar angle \(\theta\) is measured from the \(x_1\)-axis. Compute the finite element solution for meshes corresponding to
values of the mesh scaling factor $h = 1/2^p$ with $p = 0, \ldots, 5$. Plot the
dehomed mesh and the contours of $\sigma_{11}, \sigma_{22}$. Also plot the energy-norm
error $\| \mathbf{u}_h - \mathbf{u} \|_E$, computed by numerical quadrature, as a function
of the number of elements $E$ in $\log - \log$ scale. Comment on the
convergence properties of this finite element approximation.

3. Write a 6-node simplex element elmt01 to do plane stress/strain linear
elasticity. Verify that your element and, thus, the implementation,
satisfies the patch test.

4. Repeat the calculations in 1. using the 6-node element. Comment on
the results.

5. Repeat the calculations in 2. using the 6-node element. Comment on
the results.