1. Prove that the root-locus resulting from the combination of two poles and one zero to the left of both of them on the real axis is a circle centered at the zero with radius given by $\sqrt{|(p_1 - z)||(p_2 - z)|}$.

2. Consider the root-locus for the equation

$$1 + \frac{K}{s(s + 10)(s + 50)} = 0.$$

(a) Show the real-axis segments clearly.

(b) Sketch the asymptotes for $K \rightarrow \infty$.

(c) For what values of $K$ are the roots on the imaginary axis?

3. Sketch the complete root-locus (positive and negative gains) for the following systems by hand. Perform all steps indicated in the handout and detail your computations.

(a) $KG(s) = \frac{K(s + 4)}{s(s - 4)(s^2 + 2s + 1)}$

(b) $KG(s) = \frac{K(s + 2)}{s(s + 1)(s^2 + 2s + 20)}$

(c) $KG(s) = \frac{2K}{s(s^2 + 5s + 10)}$
(d) 

\[ KG(s) = \frac{2K(s^2 + s + 1)}{s(s^2 + 2s + 9)(s^2 + s + 2)} \]

(e) 

\[ KG(s) = \frac{K(s + 0.1)(s + 0.5)}{s(s^2 + 2s + 1)(s + 3)(s + 12)} \]

(f) 

\[ KG(s) = \frac{K(s + 1)(s - 0.4)}{s(s + 3)(s + 4)(s^2 + 6)} \]