\[ G(s) = \frac{k}{(s + 1)^2} \quad k = y \]

<table>
<thead>
<tr>
<th>( v )</th>
<th>( M_v )</th>
<th>( \frac{y}{v^2 + 1} )</th>
<th>Phase</th>
<th>( -2 \tan^{-1} \left( \frac{\omega}{v} \right) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>3.2</td>
<td></td>
<td>-53.13</td>
<td>-0.5273</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td></td>
<td>-90</td>
<td>-1.5708</td>
</tr>
<tr>
<td>2</td>
<td>0.8</td>
<td></td>
<td>-126.87</td>
<td>-2.2143</td>
</tr>
<tr>
<td>4</td>
<td>0.2353</td>
<td></td>
<td>-151.93</td>
<td>-2.6576</td>
</tr>
</tbody>
</table>
2. \( \frac{G(s)H(s)}{s^2 + 16s + 100} \)

   a) Corner freq \( w = 1, 10 \)

   b) At low freq, slope = 70 (0 dB/dec)

   c) At high freq, slope = 72 (-40 dB/dec)

   c) Bode Plot

   -20 dB/dec
   -40 dB/dec
GM 9.33 18 0 7.7 m/s
PM 51.2° 0 1.57 m/s
4) \( C(s) = \frac{0.130 \cdot (s + 0.05) \cdot (s^2 + 1600)}{(s^2 + 0.05^2 + 16) \cdot (s + 70)} \)

a) \( K(s) = 2 \quad H(s) = 0.5 \)

\[ g_m = \infty \]
\[ q_m = 79.3 \quad 13.2 \text{ rad/sec} \]

b) \( K(s) = k_1 + k_2 \quad k_2/k_1 = 0.5 \)
\[ = k_2 \]
\[ = \frac{k_2}{\left( \frac{k_1}{k_2} \cdot s + 1 \right)} \]

\[ 10^{-16} = \frac{0.30 \cdot (\omega^2 + 0.105^2)^{1/2} \cdot (1600 - \omega^2)^{1/2}}{\omega \left( (16 - \omega^2)^2 + (0.105 \omega)^2 \right)^{1/2}} \]

\[ w = 0.16 \quad K_2 = 0.669 \quad K = 2K_2 \]

\[ w = 4 \quad K_2 = 0.011 \quad K_1 = 0.02314 \]
Bode Diagram

Frequency (rad/sec): 40
Phase (deg): 180

System: untitled1

Magnitude (dB)

Phase (deg)

Frequency (rad/sec)
#5

Need integrator. Need zero to get phase back. Lead gives better overall response, but not necessary.
Final solutions

Problem 6

a) Assume the velocity is zero. If the external applied force has amplitude less than $D$, then (say positive), then the switch in the feedback path reacts first with a force of amplitude $-D$ (backwards) so any forward move of the mass then the mass cannot move forward. It cannot move backwards either, because as soon as it tries and does that the reaction force in the tested feedback path becomes $+D$ and stop the mass from trying to move backwards.

Strictly:

If the velocity is now positive, then the reaction force in the feedback path is negative, of amplitude $-D$ as intuition suggests.

And vice versa if the velocity is strictly negative.
We can use describing function to see, indeed, if there are uncontrolled oscillations. This is not a rigorous argument, but it's off to use it since not much else is available to you.

As we have seen, in the case of a switch nonlinearity, there is a limit cycle if the root locus crosses the jω axis away from jω = 0. In our case, the root locus is:

So it never crosses the jω axis —

⇒ there is no limit cycle.
5. The system is subject to a forcing sinusoidal function of amplitude $2D$ velocity.

The steady-state response should be a periodic function.

\[ r = 2D e^{j\omega t} \]

\[ \begin{array}{c}
\text{Input} \\
\downarrow \\
\text{Block} \\
\downarrow \\
\text{Output}
\end{array} \]

5. Let us assume that the only signals of interest are sinusoids.

We have: \[ e = 2D e^{j\omega t} - f \]

We approximate \( f \) by \[ f = f_0 e^{j(\omega t + \phi)} \]

and \[ f_0 = \frac{4D}{\pi V_0} \]

\[ v = v_0 e^{-j\omega t} \]

Thus \( \phi_1 = \phi_2 = -90^\circ \)

\[ v_0 = \frac{|e|}{m\omega} \]
Thus:

\[ f_e = \frac{4D}{\pi} \frac{\omega}{\frac{1}{1/j\omega} - \frac{1}{j\omega}} = \frac{4D}{1/j\omega} \]

so:

\[ e = e^{2D} \left( 1 - \frac{2D}{j\omega} \right) e^{j\omega t} \]

or:

\[ \frac{e}{2D} = (1 - \frac{2}{1/j\omega}) e^{j\omega t} \]

So in general the DF for long fraction is from \( R \) to \( e \):

\[ (1 - \frac{2}{j\omega}) \]

So at the output, we have a "sinc-squared" of amplitude \( \left| \frac{1}{mj\omega} \right| (1 - \frac{2}{1/j\omega})^{2D} \).
In general, we have:

\[ e = R e^{-j\omega t} \]

with \( f = \frac{4D}{\pi j} \).

\[ v = \frac{1}{mj\omega} e^{-j\omega t} \]

so:

\[ v = \frac{1}{mj\omega} \left( e^{-j\omega t} \right) \cdot \frac{R - 4D}{m j \omega} \]

or:

\[ \frac{v}{R e^{j\omega t}} = \frac{1}{mj\omega R} \left( R - \frac{4D}{\pi f} \right) \]

This leads to:

\[ N(A, \omega) = \left( \frac{1}{m \omega} \right) (-f + \frac{4D}{\pi R}) \]
Problem 7

\[ \ddot{x} = -\dot{x} \text{ for } |x| < 1 \rightarrow \text{circle} \]
\[ \ddot{x} = -1 \text{ for } x > 1 \rightarrow \text{parabola} \]
\[ \ddot{x} = 1 \text{ for } x < -1 \rightarrow \text{parabola} \]
\( x = 0.1 \text{ rad/1 sec.} \)

The centripetal acceleration is
\( x \cdot 0.01 \text{ m/sec}^2 \)

and is equal to 1 when \( x = 100 \text{ m} \).

The equation of motion are:

\[
\ddot{x} = -0.01x \quad \text{for} \quad |x| < 1 \quad \rightarrow \text{cycle}
\]

\[
\ddot{x} = -1 + 0.01x \quad \text{for} \quad x > 1
\]

\[
\ddot{x} = 1 + 0.01x \quad \text{for} \quad x < -1
\]

Analyze: \( \dot{x} = -1 + 0.01x \) in more detail

we have an equilibrium at \( \dot{x} = 0, x = 100 \)

and this is the equation of an inverted pendulum.
d) Assume now \( \omega \) spins up. The generic equation of motion are:

\[
\ddot{x} = (-1 + \omega^2) x \quad \text{for} \quad |x| < 1
\]

\[
\ddot{x} = -1 + \omega^2 x \quad \text{for} \quad x > 1
\]

\[
\ddot{x} = 1 + \omega^2 x \quad \text{for} \quad x < -1.
\]
\[ x^2 < 1 \]

Then:
\[ \dot{x} = (-1 + x^2) x \]

is stable for \(|x| < 1\)

and:
\[ \dot{x} = -1 + x^2 x \]

has equilibrium at \( \dot{x} = 0 \)

\[ x = \frac{1}{\sqrt{2}} \]

The phase plane looks like:

[Sketch of phase plane with ellipses and arrows indicating flow]
Assume now $x^2 > 1$.

Then $\dot{x} = (-1 + x^2) \dot{x}$ is unstable too.

The phase plane now looks like: