Handout 4: Root-Locus Review

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Summary of Guidelines for plotting a root-locus

1. Mark Poles $X$ and Zeros $O$.

2. Draw the locus on the real axis to the left of an odd number of real poles plus zeros.

3. Draw $n - m$ asymptotes ($n$ is the number of poles, $m$ the number of zeros). The asymptotes are centered at $\alpha$ and leave at angles $\Phi_l$, where

\[
\alpha = \frac{\sum p_i - \sum z_i}{n - m} = \frac{-a_1 + b_1}{n - m},
\]

\[
\phi_l = \frac{180^\circ + l360^\circ}{n - m}, \quad l = 0, 1, 2, \ldots n - m - 1.
\]

4. Compute the loci departure angles from the poles and arrival angles at the zeros.

5. Assume $s_0 = j\omega_0$ and compute the point(s) where the locus crosses the imaginary axis for positive $K$.

6. The equation has multiple roots at points on the locus where

\[
\frac{b}{ds} \frac{da}{ds} - \frac{a}{ds} \frac{db}{ds} = 0.
\]

If $s_0$ is on the real axis, these points are points of breakaway or break-in. Compute the angles of arrival and the angles of departure for any points of multiple roots.

7. Complete the locus, using the previous steps and your experience.
$$G(s) = \frac{s + 1}{s^2(s + 4)}$$
\[ G(s) = \frac{s + 1}{s^2(s + 12)} \]
\[ G(s) = \frac{(s + 0.1)^2 + 16}{s((s + 0.1)^2 + 25)} \]