Handout 8: Lead compensation

Eric Feron

March 1, 2004

**Lead Compensation goals:** Raise phase (and gain) at high frequencies while not touching low-frequency system’s characteristics: Can extend bandwidth of system.

**Canonical lead element:**

\[ K_{lead}(s) = \frac{s/a + 1}{s/b + 1}, \quad 0 \leq a < b. \]

Typical lead Bode Plot:
Table of maximum phase lead for lead compensator:

<table>
<thead>
<tr>
<th>( b/a )</th>
<th>Phase lead (deg)</th>
<th>Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Plant under study:**

\[
G(s) = \frac{1/10}{(s + 1)(s/10 + 1)^2}
\]

**Requirements:** Want to increase BW beyond 4\( rad/sec \), must beat \( p(j\omega) \).

**Compensation Scheme:** We first adjust the gain \( K \) in the feedback loop to 150.

Phase Margin is

Gain Margin is

BW is
\[ G = \frac{1}{10(5+1)(5/10+1)^2} \]

\[ K_p = 150 \]
The lead compensation goals: raise phase and gain at high frequencies while not touching low-frequency system's characteristics. Can extend ('am band-width) of system.

Typical lead pole plot:

<table>
<thead>
<tr>
<th>w</th>
<th>Phase lead (deg) &amp; Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>45</td>
</tr>
<tr>
<td>2</td>
<td>90</td>
</tr>
<tr>
<td>3</td>
<td>135</td>
</tr>
</tbody>
</table>

Table of maximum phase lead for lead compensator.

\begin{tabular}{|c|c|}
\hline
\text{Frequency} & \text{Phase Lead} \\
\hline
\text{10 Hz}     & \text{45 deg}  \\
\text{100 Hz}    & \text{90 deg}  \\
\hline
\end{tabular}

\text{(IF Plant under study)}

\begin{align}
\phi (s) &= \frac{1}{101(s+1)(s+10)^2}
\end{align}

Requirements: Want to increase BW beyond 10 rad/sec, must boost \text{Sp(j\omega)}.

\begin{align}
\text{(IF Compensation Scheme)}: \text{We first adjust the gain \text{Sp} in the feedback loop to 150.}
\end{align}

\begin{align}
\text{Phase Margin} &= \\
\text{Gain Margin} &= \\
\text{Ny} &= \\
\text{ed compensation:}
\end{align}

\begin{align}
K_{\text{lead}}(s) &= \frac{1}{s+1}\frac{1}{s+10}
\end{align}

Final design: Bode plot

Final design: Root locus

\text{Root locus for Proportional compensator}
\[ V_{lead} = \frac{1}{s+1} \]

\[ V_0 = 150 \]

\[ \frac{1}{1.25 \cdot s + 1} \]

\[ \varepsilon = \frac{1}{10(s+1)(3.6 + 1)^2} \]

Bode Diagram

- Compensated
- Uncompensated
Lead compensation:

\[ K_{\text{lead}}(s) = \frac{s/a + 1}{s/b + 1} \]
Final design: Bode plot