In this table we employ the probability function (cf. Sec. 7.2) denoted by

$$PF(x) = \frac{1}{\sqrt{2\pi}} \exp \left(-\frac{x^2}{2}\right)$$

and its integral, the probability integral, denoted by

$$PI(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp \left(-\frac{v^2}{2}\right) dv$$

These functions are plotted in Fig. E.2-1.

This table is given in three sections:

E.1 Gaussian-input RIDFs
E.2 Gaussian-plus-bias-input RIDFs
E.3 Gaussian-plus-bias-plus-sinusoid-input RIDFs

E.1 GAUSSIAN-INPUT RIDFs

$$x(t) = r(t)$$ an unbiased Gaussian process

$$N_B(\sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{\infty} y(r) \exp \left(-\frac{r^2}{2\sigma^2}\right) dr$$
### Table of Random-Input Describing Functions (RIDs) (Continued)

<table>
<thead>
<tr>
<th>Nonlinearity</th>
<th>Comments</th>
<th>$N_R(\sigma)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Diagram 1]</td>
<td>$D_0 = 0$&lt;br&gt;$N$ is index of last quantizer level</td>
<td>$\frac{2}{\sigma} \sum_{i=1}^{N} (D_i - D_{i-1}) PF\left(\frac{\delta_i}{\sigma}\right)$</td>
</tr>
<tr>
<td>1. General odd quantizer</td>
<td></td>
<td></td>
</tr>
<tr>
<td>![Diagram 2]</td>
<td>$N$ is index of last quantizer level</td>
<td>$\frac{2D}{\sigma} \sum_{i=1}^{N} PF\left(\frac{2i - 1}{\sigma}\right)$</td>
</tr>
<tr>
<td>2. Uniform quantizer</td>
<td>See Fig. E.1-1</td>
<td></td>
</tr>
<tr>
<td>![Diagram 3]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Relay with dead zone</td>
<td>See Fig. E.1-1</td>
<td>$\frac{2}{\sigma} PF\left(\frac{\delta}{\sigma}\right)$</td>
</tr>
</tbody>
</table>
4. Ideal relay

\[ \sqrt{\frac{2D}{\pi \sigma}} \]

5. Preload

\[ \sqrt{\frac{2D}{\pi \sigma}} + m \]

6. General piecewise-linear odd memoryless nonlinearity

\[ \sqrt{\frac{2D}{\pi \sigma}} + 2 \sum_{i=1}^{N-1} (m_i - m_{i+1}) P_l \left( \frac{\delta_i}{\sigma} \right) + 2m_N - m_1 \]

N is index of last linear segment
<table>
<thead>
<tr>
<th>Nonlinearity</th>
<th>Comments</th>
<th>$N_p(\sigma)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Nonlinearity Diagram]</td>
<td></td>
<td>$m \left[ 2PI\left(\frac{\delta}{\sigma}\right) - 1 \right]$</td>
</tr>
<tr>
<td>7. Sharp saturation or limiter</td>
<td>See Fig. E.1-2</td>
<td></td>
</tr>
<tr>
<td>![Nonlinearity Diagram]</td>
<td></td>
<td>$2m \left[ 1 - PI\left(\frac{\delta}{\sigma}\right) \right]$</td>
</tr>
<tr>
<td>8. Dead zone or threshold</td>
<td>See Fig. E.1-2</td>
<td></td>
</tr>
<tr>
<td>![Nonlinearity Diagram]</td>
<td></td>
<td>$m_1 + 2(m_2 - m_1)\left[ 1 - PI\left(\frac{\delta}{\sigma}\right) \right]$</td>
</tr>
<tr>
<td>9. Gain-changing nonlinearity</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
10. Limiter with dead zone

\[ 2m \left[ Pl \left( \frac{\delta_2}{\sigma} \right) - Pl \left( \frac{\delta_1}{\sigma} \right) \right] \]

11. Gain-changing nonlinearity with dead zone

\[ 2(m_1 - m_2) Pl \left( \frac{\delta_2}{\sigma} \right) - 2m_1 Pl \left( \frac{\delta_1}{\sigma} \right) + 2m_2 \]

12.

\[ 2m \left[ 1 - Pl \left( \frac{\delta}{\sigma} \right) \right] + 2 \frac{D}{\sigma} Pf \left( \frac{\delta}{\sigma} \right) \]
<table>
<thead>
<tr>
<th>Nonlinearity</th>
<th>Comments</th>
<th>$N_R(\alpha)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.</td>
<td><img src="image" alt="Diagram" /></td>
<td>$m_1 + 2(m_2 - m_1) \left[ 1 - PI \left( \frac{\delta}{\sigma} \right) \right] + 2 \frac{D}{\sigma} PF \left( \frac{\delta}{\sigma} \right)$</td>
</tr>
<tr>
<td>14.</td>
<td><img src="image" alt="Diagram" /></td>
<td>$\sqrt{\frac{2D}{\pi \sigma}} \left[ 1 - \sqrt{2\pi} PF \left( \frac{\delta}{\sigma} \right) \right]$</td>
</tr>
<tr>
<td>$y = c$</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>15.</td>
<td>$y = x$</td>
<td>1</td>
</tr>
<tr>
<td>16. Linear gain</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>( y = x )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>17. Odd square law</td>
<td>( y = x^3 )</td>
<td>( \frac{\sqrt{2}}{\pi} 2\sigma )</td>
</tr>
<tr>
<td>18. Cubic characteristic</td>
<td>( y = x^3</td>
<td>x</td>
</tr>
<tr>
<td>19. Odd quartic characteristic</td>
<td>( y = x^5 )</td>
<td>( 15\sigma^4 )</td>
</tr>
<tr>
<td>20. Quintic characteristic</td>
<td>( y = x^5</td>
<td>x</td>
</tr>
<tr>
<td>21.</td>
<td>( y = x^7 )</td>
<td>( 105\sigma^6 )</td>
</tr>
<tr>
<td>22.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nonlinearity</td>
<td>Comments</td>
<td>$N_R(\sigma)$</td>
</tr>
<tr>
<td>-------------</td>
<td>---------------------------</td>
<td>-----------------------------------</td>
</tr>
<tr>
<td>$y = x^n$</td>
<td>$n = 3, 5, 7, \ldots$</td>
<td>$n(n - 2)(n - 4) \cdots (1)\sigma^{n-1}$</td>
</tr>
<tr>
<td>$y = x^{n-1}</td>
<td>x</td>
<td>$</td>
</tr>
<tr>
<td>$y = \sqrt{x}$</td>
<td>$(x \geq 0)$</td>
<td>$0.860\sigma^{-1/2}$</td>
</tr>
<tr>
<td></td>
<td>$= -\sqrt{-x}$</td>
<td>$(x &lt; 0)$</td>
</tr>
<tr>
<td>26. Odd square root</td>
<td>See Fig. E.1-3</td>
<td></td>
</tr>
<tr>
<td>$y = x^{1/3}$</td>
<td></td>
<td>$0.830\sigma^{-2/3}$</td>
</tr>
<tr>
<td>27. Cube root characteristic</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y = x^b$</td>
<td>$(x \geq 0)$</td>
<td>$\Gamma(x)$ is gamma function</td>
</tr>
<tr>
<td></td>
<td>$= -(−x)^b$</td>
<td>$(x &lt; 0)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\sqrt{\frac{2}{\pi}} 2^{b/2}\Gamma \left(1 + \frac{b}{2}\right)\sigma^{b+1}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>$y = M \sin mx$</td>
<td>$\sqrt{2}\pi MmPF(m\sigma) = Mme^{-m^2\sigma^2/2}$</td>
<td></td>
</tr>
<tr>
<td>29. Harmonic Nonlinearity</td>
<td>See Fig. E.1-4</td>
<td></td>
</tr>
<tr>
<td>$y = M \sinh mx$</td>
<td>$Mme^{-\sigma^2/2}$</td>
<td></td>
</tr>
<tr>
<td>30.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y = 1 - e^{-x}$ $(x \geq 0)$</td>
<td>$2ce^{x^2/2}[1 - PI(x)]$</td>
<td></td>
</tr>
<tr>
<td>$= -(1 - e^x)$ $(x &lt; 0)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>31. Exponential saturation</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure E.1-1 Quantizer RIDF.

Figure E.1-2 RIDFs for limiter and threshold characteristics.
Figure E.1-3 RIDF for the simple polynomial nonlinearity $y = c_n x^n$ (n odd) or $y = c_n x^{n-1} |x|$ (n even).
Figure E.1-4 Harmonic nonlinearity RIDF.
E.2 GAUSSIAN-PLUS-BIAS-INPUT RIDFs

\[ x(t) = r(t) + B \]

The gain to the gaussian input component is given by:

\[ N_B(\sigma, B) = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{\infty} y(r + B) r \exp\left(-\frac{r^2}{2\sigma^2}\right) dr \]

and the corresponding gain to the bias input component is:

\[ N_B(\sigma, B) = \frac{1}{\sqrt{2\pi\sigma B}} \int_{-\infty}^{\infty} y(r + B) \exp\left(-\frac{r^2}{2\sigma^2}\right) dr \]

This section uses the additional function \( G(x) = xPI(x) + PF(x) \).

The functions \( PF(x), PI(x), \) and \( G(x) \) are plotted in Fig. E.2-1.
### Table of Random-Input Describing Functions (RIFs) (Continued)

<table>
<thead>
<tr>
<th>Nonlinearity</th>
<th>Comments</th>
<th>$N_R(\sigma, B)$ and $N_B(\sigma, B)$</th>
</tr>
</thead>
</table>
| ![Diagram 1](image1) | $D_0 = 0$  
$N$ is index of last quantizer level. | 
$N_R = \frac{1}{\sigma} \sum_{i=1}^{N} (D_i - D_{i-1}) \left[ Pf \left( \frac{\delta_i + B}{\sigma} \right) + Pf \left( \frac{\delta_i - B}{\sigma} \right) \right]$  
$N_B = \frac{1}{B} \sum_{i=1}^{N} (D_i - D_{i-1}) \left[ Pf \left( \frac{\delta_i + B}{\sigma} \right) - Pf \left( \frac{\delta_i - B}{\sigma} \right) \right]$ |
| 1. General odd quantizer | ![Diagram 2](image2) | $N_R = \frac{D}{\sigma} \sum_{i=1}^{N} \left[ Pf \left( \frac{2i - 1}{2} \frac{h}{\sigma} + \frac{B}{\sigma} \right) + Pf \left( \frac{2i - 1}{2} \frac{h}{\sigma} - \frac{B}{\sigma} \right) \right]$  
$N_B = \frac{D}{B} \sum_{i=1}^{N} \left[ Pf \left( \frac{2i - 1}{2} \frac{h}{\sigma} + \frac{B}{\sigma} \right) - Pf \left( \frac{2i - 1}{2} \frac{h}{\sigma} - \frac{B}{\sigma} \right) \right]$ |
| 2. Uniform quantizer | ![Diagram 3](image3) | $N_R = \frac{D}{\sigma} \left[ Pf \left( \frac{\delta + B}{\sigma} \right) + Pf \left( \frac{\delta - B}{\sigma} \right) \right]$  
$N_B = \frac{D}{B} \left[ Pf \left( \frac{\delta + B}{\sigma} \right) - Pf \left( \frac{\delta - B}{\sigma} \right) \right]$ |
| 3. Relay with dead zone | ![Diagram 4](image4) | 

---

For further details and explanations, please refer to the original document.
4. Ideal relay

\[ N_B = 2 \frac{D}{\sigma} PF \left( \frac{B}{\sigma} \right) \]

\[ N_B = \frac{D}{B} \left[ 2PL \left( \frac{B}{\sigma} \right) - 1 \right] + m \]

5. Preload

\[ N_R = 2 \frac{D}{\sigma} PF \left( \frac{B}{\sigma} \right) + m \]

\[ N_B = \frac{D}{B} \left[ 2PL \left( \frac{B}{\sigma} \right) - 1 \right] + m \]

6. General piecewise-linear odd memoryless nonlinearity

\[ N_R = 2 \frac{D}{\sigma} PF \left( \frac{B}{\sigma} \right) + 2m_N - m_t + \sum_{i=1}^{N-1} (m_t - m_{t+1}) \left[ PL \left( \frac{\delta_i + B}{\sigma} \right) + PI \left( \frac{\delta_i - B}{\sigma} \right) \right] \]

\[ N_B = \frac{D}{B} \left[ 2PL \left( \frac{B}{\sigma} \right) - 1 \right] + 2m_N - m_t + \frac{\sigma}{B} \sum_{i=1}^{N-1} (m_t - m_{t+1}) \left[ G \left( \frac{\delta_i + B}{\sigma} \right) - G \left( \frac{\delta_i - B}{\sigma} \right) \right] \]

\( N \) is index of last linear segment.
<table>
<thead>
<tr>
<th>Nonlinearity</th>
<th>Comments</th>
<th>$N_R(\sigma,B)$ and $N_B(\sigma,B)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$y = m \delta$</td>
<td>$N_R = m \left[ P(I \left( \frac{\delta + B}{\sigma} \right) + P(I \left( \frac{\delta - B}{\sigma} \right) - 1 \right]$</td>
</tr>
<tr>
<td></td>
<td>$x$</td>
<td>$N_B = m \left{ \frac{\sigma}{B} \left[ G \left( \frac{\delta + B}{\sigma} \right) - G \left( \frac{\delta - B}{\sigma} \right) \right] - 1 \right}$</td>
</tr>
<tr>
<td>7. Sharp saturation or limiter</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$y = m \frac{\delta}{\delta}$</td>
<td>$N_R = m \left[ 2 - P(I \left( \frac{\delta + B}{\sigma} \right) - P(I \left( \frac{\delta - B}{\sigma} \right) \right]$</td>
</tr>
<tr>
<td></td>
<td>$x$</td>
<td>$N_B = m \left{ 2 - \frac{\sigma}{B} \left[ G \left( \frac{\delta + B}{\sigma} \right) - G \left( \frac{\delta - B}{\sigma} \right) \right] \right}$</td>
</tr>
<tr>
<td>8. Dead zone or threshold</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$y = m_{1} + (m_{2} - m_{1}) \frac{\delta}{\delta}$</td>
<td>$N_R = m_{1} + (m_{2} - m_{1}) \left[ 2 - P(I \left( \frac{\delta + B}{\sigma} \right) - P(I \left( \frac{\delta - B}{\sigma} \right) \right]$</td>
</tr>
<tr>
<td></td>
<td>$x$</td>
<td>$N_B = m_{1} + (m_{2} - m_{1}) \left{ 2 - \frac{\sigma}{B} \left[ G \left( \frac{\delta + B}{\sigma} \right) - G \left( \frac{\delta - B}{\sigma} \right) \right] \right}$</td>
</tr>
<tr>
<td>9. Gain-changing nonlinearity</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
10. Limiter with dead zone

\[ N_R = m \left[ PI \left( \frac{\delta_1 + B}{\sigma} \right) + PI \left( \frac{\delta_1 - B}{\sigma} \right) \right] - PI \left( \frac{\delta_1 + B}{\sigma} \right) - PI \left( \frac{\delta_1 - B}{\sigma} \right) \]

\[ N_B = m \sigma \left[ G \left( \frac{\delta_1 + B}{\sigma} \right) - G \left( \frac{\delta_1 - B}{\sigma} \right) - G \left( \frac{\delta_1 + B}{\sigma} \right) + G \left( \frac{\delta_1 - B}{\sigma} \right) \right] \]

11. Gain-changing nonlinearity with dead zone

\[ N_R = 2m_2 - m_1 \left[ PI \left( \frac{\delta_1 + B}{\sigma} \right) + PI \left( \frac{\delta_1 - B}{\sigma} \right) \right] + \]

\[ (m_1 - m_2) \left[ PI \left( \frac{\delta_2 + B}{\sigma} \right) + PI \left( \frac{\delta_2 - B}{\sigma} \right) \right] \]

\[ N_B = 2m_2 + \frac{\sigma}{B} \left[ - m_1 \left[ G \left( \frac{\delta_1 + B}{\sigma} \right) - G \left( \frac{\delta_1 - B}{\sigma} \right) \right] + \right. \]

\[ (m_1 - m_2) \left[ G \left( \frac{\delta_2 + B}{\sigma} \right) - G \left( \frac{\delta_2 - B}{\sigma} \right) \right] \]
<table>
<thead>
<tr>
<th>Nonlinearity</th>
<th>Comments</th>
<th>$N_R(\sigma,B)$ and $N_B(\sigma,B)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = x$</td>
<td>$N_R = 1$</td>
<td>$N_B = 1$</td>
</tr>
<tr>
<td>$y = c$</td>
<td>$N_R = 0$</td>
<td>$N_B = \frac{c}{B}$</td>
</tr>
<tr>
<td>$y = x$</td>
<td>$N_R = 1$</td>
<td>$N_B = 1$</td>
</tr>
</tbody>
</table>

TABLE OF RANDOM-INPUT DESCRIBING FUNCTIONS (RIDFs) (Continued)
<table>
<thead>
<tr>
<th>17. Odd-square law</th>
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</tr>
</thead>
<tbody>
<tr>
<td>( y = x</td>
<td>x</td>
</tr>
<tr>
<td></td>
<td>( N_B = 2 \sigma PF \left( \frac{B}{\sigma} \right) + B \left[ 1 + \left( \frac{\sigma}{B} \right)^2 \right] \left[ 2PF \left( \frac{B}{\sigma} \right) - 1 \right] )</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>18. Cubic characteristic</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = x^3 )</td>
<td>( N_R = 3 \sigma^3 + 3B^2 )</td>
</tr>
<tr>
<td></td>
<td>( N_B = 3 \sigma^3 + B^2 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>20. Quintic characteristic</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = x^5 )</td>
<td>( N_R = 15 \sigma^5 + 30 \sigma^2 B^2 + 5B^4 )</td>
</tr>
<tr>
<td></td>
<td>( N_B = 15 \sigma^5 + 10 \sigma^2 B^2 + B^4 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>22.</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = x^n )</td>
<td>( n = 3, 5, 7, \ldots )</td>
</tr>
<tr>
<td></td>
<td>( N_R = \sum_{k(\text{even})=0}^{n-1} \frac{n!}{k! (n-k)!} \sigma^{n-k-1} B^k (1)(3) \cdots (n-k) )</td>
</tr>
<tr>
<td></td>
<td>( N_B = B^{n-1} + \sum_{k(\text{odd})=1}^{n-2} \frac{n!}{k! (n-k)!} \sigma^{n-k-1} B^{k-1} (1)(3) \cdots (n-k-1) )</td>
</tr>
</tbody>
</table>

<p>| 24. | See Sec. 7.2 |</p>
<table>
<thead>
<tr>
<th>Nonlinearity</th>
<th>Comments</th>
<th>$N_R(\sigma, B)$ and $N_B(\sigma, B)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = M \sin mx$</td>
<td></td>
<td>$N_R = M m \cos mB e^{-m^2\sigma^2/2}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$N_B = \frac{M}{B} \sin mB e^{-m^2\sigma^2/2}$</td>
</tr>
<tr>
<td>29. Harmonic nonlinearity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y = M \sinh mx$</td>
<td></td>
<td>$N_R = M m \cosh mB e^{m^2\sigma^2/2}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$N_B = \frac{M}{B} \sinh mB e^{m^2\sigma^2/2}$</td>
</tr>
<tr>
<td>30.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y = 1 - e^{-\alpha x}$</td>
<td>$(x \geq 0)$</td>
<td>$N_R = \frac{2}{\sigma} \left( \frac{B}{\sigma} \right) + \frac{1}{\sigma} e^{2\sigma^2/2} \left[ e^{\alpha \sigma} \left(c\sigma - c\sigma PI \left(c\sigma + \frac{B}{\sigma} \right) - PF \left(c\sigma + \frac{B}{\sigma} \right) \right] + \left( e^{-\alpha \sigma} - e^{-B/\sigma} \right) \left[ c\sigma - c\sigma PI \left(c\sigma - \frac{B}{\sigma} \right) - PF \left(c\sigma - \frac{B}{\sigma} \right) \right] \right]$</td>
</tr>
<tr>
<td></td>
<td>$-(1 - e^{\alpha x})$</td>
<td>$N_B = \frac{1}{B} \left[ 2PI \left(\frac{B}{\sigma} \right) - 1 \right] + \frac{1}{B} e^{2\sigma^2/2} \left[ 1 - PI \left(c\sigma + \frac{B}{\sigma} \right) \right] - e^{-\alpha \sigma} \left[ 1 - PI \left(c\sigma - \frac{B}{\sigma} \right) \right]$</td>
</tr>
<tr>
<td>31. Exponential saturation</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$N_R = m_2 + (m_1 - m_2)PI \left(\frac{B}{\sigma} \right)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$N_B = m_2 + \frac{\sigma}{B} (m_1 - m_2)G \left(\frac{B}{\sigma} \right)$</td>
</tr>
<tr>
<td>51.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
52. Absolute value

\[ N_R = m \left[ 2PF \left( \frac{B}{\sigma} \right) - 1 \right] \]
\[ N_B = m \left[ 2 \frac{\sigma}{B} G \left( \frac{B}{\sigma} \right) - 1 \right] \]

53. Square-law

\[ N_R = 2B \]
\[ N_B = \frac{1}{B} [a^2 + B^4] \]

54.

\[ N_R = \frac{D}{\sigma} \left[ - PF \left( \frac{\delta + B}{\sigma} \right) + PF \left( \frac{\delta - B}{\sigma} \right) \right] \]
\[ N_B = \frac{D}{B} \left[ 2 - PI \left( \frac{\delta + B}{\sigma} \right) - PI \left( \frac{\delta - B}{\sigma} \right) \right] \]
<table>
<thead>
<tr>
<th>Nonlinearity</th>
<th>Comments</th>
<th>$N_R(\sigma,B)$ and $N_B(\sigma,B)$</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Diagram" /></td>
<td>Biased ideal relay</td>
<td>$N_R = D \left[ \frac{B}{\sigma} \right] - 1 - PI \left( \frac{\delta + B}{\sigma} \right) + PI \left( \frac{\delta - B}{\sigma} \right)$</td>
</tr>
<tr>
<td><img src="image2" alt="Diagram" /></td>
<td>Biased ideal relay</td>
<td>$N_B = \frac{D}{B} \left( 2 - \frac{B}{\delta} + \frac{\sigma}{\delta} \left[ 2G \left( \frac{B}{\sigma} \right) - G \left( \frac{\delta + B}{\sigma} \right) - G \left( \frac{\delta - B}{\sigma} \right) \right] \right)$</td>
</tr>
</tbody>
</table>

55. Biased ideal relay

56. Biased ideal relay

$N_R = \frac{D_1 + D_2}{\sigma} \left( \frac{\delta - B}{\sigma} \right)$

$N_B = \frac{D_1}{B} - \frac{D_1 + D_2}{B} PI \left( \frac{\delta - B}{\sigma} \right)$
Figure E.2-I Graphs of $PF(x)$, $PI(x)$, and $G(x)$. 

$PF(x)$, $PI(x)$, and $G(x)$ are plotted against the variable $x$. The graph shows the asymptotes for $x \to 0$: 
- $PI(0) = 0.5$
- $G(0) = PF(0) = 0.3989$
E.3 GAUSSIAN-PLUS-BIAS-PLUS-SINUSOID-INPUT RIDFs

\[ x(t) = r(t) + B + A \sin(\omega t + \theta) \]

The gain to the gaussian input component is given by

\[ N_\text{g}(\sigma, B, A) = \frac{1}{(2\pi)^{3/2}} \int_0^{2\pi} d\theta \int_{-\infty}^{\infty} dr \, y(r + B + A \sin \theta) \exp \left(- \frac{r^2}{2\sigma^2} \right) \]

the gain to the bias input component is

\[ N_\text{b}(\sigma, B, A) = \frac{1}{(2\pi)^{1/2} \sigma B} \int_0^{2\pi} d\theta \int_{-\infty}^{\infty} dr \, y(r + B + A \sin \theta) \exp \left(- \frac{r^2}{2\sigma^2} \right) \]

and the corresponding gain to the sinusoid input component is

\[ N_\text{a}(\sigma, B, A) = \frac{2}{(2\pi)^{3/2} \sigma A} \int_0^{2\pi} d\theta \int_{-\infty}^{\infty} dr \, y(r + B + A \sin \theta) \sin \theta \exp \left(- \frac{r^2}{2\sigma^2} \right) \]
### TABLE OF RANDOM-INPUT DESCRIBING FUNCTIONS (RIDFs) (Continued)

<table>
<thead>
<tr>
<th>Nonlinearity</th>
<th>Comments</th>
<th>$N_R(\sigma, B, A)$, $N_B(\sigma, B, A)$, and $N_A(\sigma, B, A)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = x^3$</td>
<td></td>
<td>$N_R = 3\sigma^2 + 3B^2 + \frac{3}{2}A^2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$N_B = 3\sigma^2 + B^2 + \frac{3}{2}A^2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$N_A = 3\sigma^2 + 3B^2 + \frac{3}{2}A^2$</td>
</tr>
<tr>
<td>18. Cubic characteristic</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y = M \sin mx$</td>
<td>$J_0$ and $J_1$ are the Bessel functions of orders 0 and 1, respectively.</td>
<td>$N_R = Mm \cos mB \exp \left( -\frac{m^2\sigma^2}{2} \right) J_0(mA)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$N_B = \frac{M}{B} \sin mB \exp \left( -\frac{m^2\sigma^2}{2} \right) J_0(mA)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$N_A = \frac{2M}{A} \cos mB \exp \left( -\frac{m^2\sigma^2}{2} \right) J_1(mA)$</td>
</tr>
<tr>
<td>29. Harmonic nonlinearity</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure E.3-1 Three-input RIDFs for the ideal-relay nonlinearity.

In 3 parts

Figure E.3-1a Gain to the gaussian input component. (ideal relay)
Figure E.3-1b  Gain to the bias input component. (ideal relay)

Figure E.3-1c  Gain to the sinusoid input component. (ideal relay)
Figure E.3-2  Three-input RIDFs for the limiter nonlinearity.

Figure E.3-2a  Gain to the gaussian and bias input components.  (limiter, $|B|/\delta = 0$)
Figure E.3-2b  Gain to the sinusoid input component. (limiter, $|B|/\delta = 0$)

Figure E.3-2c  Gain to the gaussian input component. (limiter, $|B|/\delta = 0.5$)
Figure E.3-2d  Gain to the bias input component. (limiter, $|B|/\delta = 0.5$)

Figure E.3-2e  Gain to the sinusoid input component. (limiter, $|B|/\delta = 0.5$)
Figure E.3-2h  Gain to the sinusoid input component.  (limiter, $|B|/\delta = 1$)
Figure E.3-21  Gain to the gaussian input component. (limiter, $|B|/\delta = 2$)
Figure E.3-2f  Gain to the bias input component. (limiter, $|B|/\delta = 2$)
Figure E.3-2k Gain to the sinusoid input component. (limiter, $|B|/\delta = 2$)
Three-input RIDFs for the relay with dead zone nonlinearity.

Gain to the gaussian and bias input components. (relay with dead zone, $|B|/\delta = 0$)
Figure E.3-3b  Gain to the sinusoid input component.  (relay with dead zone, $|B|/\delta = 0$)
Figure E.3-3c  Gain to the gaussian input component. (relay with dead zone, |B|/δ = 0.5)
Figure E.3-3d  Gain to the bias input component.  (relay with dead zone, $|B|/\delta = 0.5$)
Figure E.3-3e  Gain to the sinusoid input component.  (relay with dead zone, $|B|/\delta = 0.5$)
Figure E.3-3f  Gain to the gaussian input component. (relay with dead zone, $|B|/\delta = 1$)
Figure E.3-3g  Gain to the bias input component. (relay with dead zone, $|B|/\delta = 1$)
Figure E.3-3h  Gain to the sinusoid input component. (relay with dead zone, $|B|/\delta = 1$)
Figure E.3-3i  Gain to the gaussian input component. (relay with dead zone, $|B|/\delta = 2$)
Figure E.3-3j  Gain to the bias input component. (relay with dead zone, $|B|/\delta = 2$)
Figure E.3-3k  Gain to the sinusoid input component. (relay with dead zone, $|B|/\delta = 2$)