16.30/31 Homework Practice Problems #7

PRACTICE PROBLEMS ONLY - not to be submitted for credit.

Goals: Describing functions; Lyapunov stability analysis

1.

Problems 14.21 and 14.22 removed due to copyright restrictions.

2.
4. Prove using the Lyapunov Theorem that the origin is a stable equilibrium for each of the following systems:

(a) System 1:

\[
\begin{align*}
\dot{x} &= -x^3 - y^2 \\
\dot{y} &= xy - y^3
\end{align*}
\]

(b) System 2:

\[
\begin{align*}
\dot{x} &= y \\
\dot{y} &= -x^3
\end{align*}
\]

5. (Challenge problem) Consider the second-order nonlinear system

\[
\begin{align*}
\dot{x}_1 &= -x_2 + \epsilon x_1(x_1^2 + x_2^2)\sin(x_1^2 + x_2^2) \\
\dot{x}_2 &= x_1 + \epsilon x_2(x_1^2 + x_2^2)\sin(x_1^2 + x_2^2)
\end{align*}
\]

Study the stability of the equilibrium at the origin, for \(\epsilon \in [-1, 1]\). Is linearization sufficient? Find a Lyapunov function \(V\) that proves/disproves stability.

*Hint:* In order to find \(V\), it may be helpful to draw the trajectories of the system in the phase plane \((x_1, x_2)\).
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