16.333: Lecture #1

Equilibrium States

Aircraft performance

Introduction to basic terms
Aircraft Performance

- Accelerated horizontal flight - balance of forces
  - Engine thrust $T$
  - Lift $L$ (⊥ to $V$)
  - Drag $D$ (|| to $V$)
  - Weight $W$

$$T - D = m \frac{dV}{dt} = 0 \text{ for steady flight}$$

and

$$L - W = 0$$

- Define $L = \frac{1}{2} \rho V^2 S C_L$ where
  - $\rho$ - air density (standard tables)
  - $S$ - gross wing area $= \bar{c} \times b$
  - $\bar{c}$ - mean chord
  - $b$ - wing span
  - $AR$ - wing aspect ratio $= b/\bar{c}$

$$Q = \frac{1}{2} \rho V^2 \text{ dynamic pressure}$$

$V$ = speed relative to the air
- $C_L$ lift coefficient – for low Mach number, $C_L = C_{L0}(\alpha - \alpha_0)$
  
  - $\alpha$ angle of incidence of wind to the wing
  - $\alpha_0$ is the angle associated with zero lift

- Back to the performance:

  \[ L = \frac{1}{2} \rho V^2 S C_L \quad \text{and} \quad L = mg \]

  which implies that $V = \sqrt{\frac{2mg}{\rho S C_L}}$ so that

  \[ V \propto C_L^{-1/2} \]

  and we can relate the effect of speed to wing lift

- A key number is stall speed, which is the lowest speed that an aircraft can fly steadily

  \[ V_s = \sqrt{\frac{2mg}{\rho S C_{L_{max}}}} \]

  where typically get $C_{L_{max}}$ at $\alpha_{max} = 10^\circ$
Steady Gliding Flight

- Aircraft at a steady glide angle of $\gamma$

- Assume forces are in equilibrium

\[
L - mg \cos \gamma = 0 \quad (1)
\]
\[
D + mg \sin \gamma = 0 \quad (2)
\]

Gives that

\[
\tan \gamma = \frac{D}{L} \equiv \frac{C_D}{C_L}
\]

$\Rightarrow$ Minimum gliding angle obtained when $C_D/C_L$ is a minimum

- High $L/D$ gives a low gliding angle

- Note: typically

\[
C_D = C_{D_{\text{min}}} + C_L^2 \frac{\pi}{\pi A Re}
\]

where

- $C_{D_{\text{min}}}$ is the zero lift (friction/parasitic) drag
- $C_L^2$ gives the lift induced drag
- $e$ is Oswald’s efficiency factor $\approx 0.7 - 0.85$
Total drag then given by

\[ D = \frac{1}{2} \rho V^2 S C_D = \frac{1}{2} \rho V^2 S (C_{D_{\text{min}}} + k C_L^2) \]  \hspace{1cm} (3)

\[ = \frac{1}{2} \rho V^2 S C_{D_{\text{min}}} + k \frac{(mg)^2}{\frac{1}{2} \rho V^2 S} \]  \hspace{1cm} (4)

So that the speed for minimum drag is

\[ V_{\text{min drag}} = \sqrt{\frac{2mg}{\rho S}} \left( \frac{k}{C_{D_{\text{min}}}} \right)^{1/4} \]
Steady Climb

- Equations:
  \[ T - D - W \sin \gamma = 0 \]  \( (5) \)
  \[ L - W \cos \gamma = 0 \]  \( (6) \)

  \( \Rightarrow \) which gives
  \[ T - D - \frac{L}{\sin \gamma} \cos \gamma = 0 \]

  so that
  \[ \tan \gamma = \frac{T - D}{L} \]

- Consistent with 1–3 if \( T = 0 \) since then \( \gamma \) as defined above is negative

- Note that for small \( \gamma \), \( \tan \gamma \approx \gamma \approx \sin \gamma \)

  \[ R/C = V \sin \gamma \approx V \gamma \approx \frac{(T - D)V}{L} \]

  so that the rate of climb is approximately equal to the excess power available (above that needed to maintain level flight)
Steady Turn

- Equations:

\[ L \sin \phi = \text{centrifugal force} \quad (7) \]
\[ = \frac{mV^2}{R} \quad (8) \]
\[ L \cos \phi = W = mg \quad (9) \]

\[ \Rightarrow \tan \phi = \frac{V^2}{Rg} \quad v=R_\omega \quad \frac{V_\omega}{g} \quad (10) \]

- Note: obtain \( R_{\text{min}} \) at \( C_{L_{\text{max}}} \)

\[ R_{\text{min}} \left( \frac{1}{2} \rho V^2 S C_{L_{\text{max}}} \right) \sin \phi = \frac{WV^2}{g} \]

\[ \Rightarrow R_{\text{min}} = \frac{W/S}{1/2 \rho g C_{L_{\text{max}}} \sin \phi_{\text{max}}} \]

where \( W/S \) is the wing loading and \( \phi_{\text{max}} < 30^\circ \)
• Define load factor \( N = L/mg \). i.e. ratio of lift in turn to weight

\[
N = \sec \phi = (1 + \tan^2 \phi)^{1/2} \\
\tan \phi = \sqrt{N^2 - 1}
\]

so that

\[
R = \frac{V^2}{g \tan \phi} = \frac{V^2}{g \sqrt{N^2 - 1}}
\]

• For a given load factor (wing strength)

\[
R \propto V^2
\]

• Compare straight level with turning flight
  
  – If same light coefficient

\[
C_L = \frac{L}{\frac{1}{2} \rho V^2 S} = \frac{mg}{\frac{1}{2} \rho V^2 S} \equiv Nmg \frac{1}{2} \rho V_t^2 S
\]

so that \( V_t = \sqrt{NV} \) gives the speed increase (more lift)

• Note that \( C_L \) constant \( \Rightarrow \) \( C_D \) constant \( \Rightarrow \) \( D \propto V^2 C_D \)
  
  \[
  \Rightarrow T_t \propto D_t \propto V_t^2 C_D \sim ND
  \]

so that must increase throttle or will descend in the turn