Exercises 02

1. The angular momentum and eccentricity vectors of an orbit are

\[ h = 2 \sqrt{\frac{\mu}{3}} i_z \quad e = -\frac{1}{3}(2i_x + i_y) \]

Find the position and velocity vectors \( \mathbf{r} \) and \( \mathbf{v} \) when the direction of the position vector is \( i_r = i_x \). (Use \( \mu = 4\pi^2 \))

**Answer:** \( r = 4i_x \) and \( v = \frac{\pi}{\sqrt{3}}(i_x + i_y) \)

2. **Prob. 4–8** To derive the polar equation of an ellipse with the origin of coordinates at the center of the ellipse (See Lecture 2, Page 3), we may consider the triangle \( CPF \) where \( r \) is the radius from the center \( C \) to a point \( P \) on the ellipse and \( F \) is the focus of the ellipse.

The sides of the triangle are

\[ PF = a - ex = a - er \cos\theta \quad CF = ae \quad CP = r \]

We can use the Law of Cosines for the triangle

\[ (a - ex)^2 = (a - er \cos\theta)^2 = r^2 + a^2e^2 - 2ae r \cos\theta \]

which gives

\[ r^2(1 - e^2 \cos^2\theta) = a^2(1 - e^2) = b^2 \quad \text{or} \quad r = \frac{b}{\sqrt{1 - e^2 \cos^2\theta}} \]

1 mile = 1.609347221 km
1 au = 149,597,870.00 km
1 au = 92,955,620.79 miles
1 au/day = 1078.822025 miles/sec
1 au/day = 5,696,180.29 feet/sec