1. Problem 6–28  The Lagrange parameters $\alpha_m$ and $\beta_m$ for the minimum-energy orbit are

$$\alpha_m = \pi \quad \cos \beta_m = \frac{3c - r_1 - r_2}{r_1 + r_2 + c}$$

and the transfer time is

$$\sqrt{\frac{\mu}{a_m^3}}(t_2 - t_1) = \pi - (\beta_m - \sin \beta_m)$$

where $\alpha$ and $\beta$ are angles in the upper half-plane.

2. Problem 6–30  The transfer time for an elliptic arc with vacant focus $F^*$ connecting points $P_1$ and $P_2$, after it has been transformed to a rectilinear ellipse, can be calculated from the one-dimensional form of the vis-viva integral

$$v^2 = \left(\frac{dr}{dt}\right)^2 = \mu \left(\frac{2}{r} - \frac{1}{a}\right)$$

Derive the equation

$$\sqrt{\mu}(t_2 - t_1) = \int_{s-c}^{s} \frac{r \, dr}{\sqrt{2r - r^2/a}}$$

and carry out the integration using the following change of variable

$$r = a(1 - \cos \theta)$$

to obtain Lagrange’s equation

$$\sqrt{\mu}(t_2 - t_1) = a^3[(\alpha - \sin \alpha) - (\beta - \sin \beta)]$$

where $\alpha$ and $\beta$ are angles in the upper half-plane.

In a two-body orbital boundary-value problem, the initial and terminal position vectors are $\mathbf{r}_1 = 4\mathbf{i}_x$ and $\mathbf{r}_2 = 4\mathbf{i}_x + 3\mathbf{i}_y$.

3. Calculate
   a. The minimum possible eccentricity for an elliptic orbit.
   b. The semimajor axis of the minimum energy orbit.
   c. The eccentricity vector for the minimum energy orbit and its eccentricity.
   d. The eccentricity vector the minimum energy orbit.
   e. The parameter of the minimum energy orbit.

4. The velocity vector at the initial point is $\mathbf{v}_1 = 4\mathbf{i}_x + 3\mathbf{i}_y$. Calculate
   a. The velocity vector $\mathbf{v}_2$ at the terminal point.
   b. The gravitational constant $\mu$.

5. For the minimum energy orbit, calculate
   a. The time of flight from $\mathbf{r}_1$ to $\mathbf{r}_2$.
   b. The velocity vector at the initial point.