16.36 Communication Systems Engineering
Spring 2009

For information about citing these materials or our Terms of Use, visit: http://ocw.mit.edu/terms.
Part 1: Quick Questions (50 points; 5 points each)

Please provide brief explanations for your answers in order to receive full credit.

1. Time-shifting a signal changes its frequency composition.
   a. True
   b. False

2. A signal $x(t)$ with bandwidth $W$, is sampled at a sampling rate $2W$. Can the following be used to reconstruct the original signal from its samples?

   $x(t) = \sum_{n=-\infty}^{\infty} x(nT_S)k(\frac{t}{T_S} - n)$ where, $k(t) = \begin{cases} 
   \text{Sinc}(t) & t \in [-1,1] \\
   0 & \text{otherwise}
   \end{cases}$

   Explain your answer.

3. When conditioning on some event $Y$, the entropy of a random variable $X$ will
   a. Increase
   b. Decrease
   c. Remain unchanged

   Please circle all possible answers and explain your choices briefly.
4. When designing a uniform quantizer for a Gaussian source, the mid-point of the quantization regions are the optimal quantization levels to minimize distortion.
   a. True
   b. False

5. The entropy of a uniform quantizer is always equal to the logarithm of the number of quantization levels.
   a. True
   b. False

6. Which of the following lengths of codewords are feasible for prefix-free codes:
   a. \{2,2,3,3,3,3,4\}
   b. \{1,2,4,4,4,5\}
   c. \{2,2,2,3,4,4\}

   Circle all that apply and briefly explain your choices.
7. For which of the following source alphabet probabilities can one construct a Huffman code with average codeword length that is equal to the source entropy
   a. \{1/4, 1/4, 1/8, 1/8, 1/8, 1/8\}
   b. \{1/2, 1/8, 3/8\}
   c. \{1/4, 1/4, 1/4, 1/4\}

   Circle all that apply and briefly explain your choices.

8. Given a band limited signal, \(x(t)\), with Bandwidth \(W\), that is modulated by a carrier of frequency \(f_c\) (i.e., \(u(t) = x(t)\cos(2\pi f_c t)\)). At what frequency must the signal, \(u(t)\), be sampled for perfect reconstruction?
   a. \(2f_c\)
   b. \(2f_c+2W\)
   c. \(2f_c+W\)
   d. \(2W\)

9. A Gaussian source with zero mean and variance 10, is to be quantized using a uniform quantizer with 8 quantization levels. (See attached table).
   a. What is the resulting distortion?
   b. What is the spacing between quantization levels?
   c. What is the resulting entropy?
10. A source is encoded using the Lempel-Ziv algorithm with four bit code words. The encoded sequence is given by:

\[ \text{0001 0000 0101 0110} \]

Using the above information reconstruct the dictionary and find the original bit sequence.

**Question 2: Modulation (30 points)**

Suppose you want to construct a 16-PAM modulator, using the basic pulse, \( g(t) \) given below.

\[
g(t) = \begin{cases} 
A & \text{if } t \in [0, T] \\
0 & \text{otherwise}
\end{cases}
\]

Frequency planning regulations require that the two-sided null-to-null bandwidth of \( g(t) \) does not exceed 8 kHz.

a) What is the maximum information rate \( R_B \) that can be supported with 16-ary PAM (M=16)?
b) The baseband signal, $g(t)$ is modulated by a sinusoidal carrier at frequency 2.4 MHz. Sketch an approximate frequency representation of the modulated signal. Clearly label magnitude and frequencies.

c) Consider next a symmetric 16-QAM modulator.
   i. Sketch the signal constellation plot, and label the points with their amplitude levels.
   ii. For each signal point give the associated signal energy level
**Question 3: Quantization (20 points)**

Suppose that the individual samples form independent, identically distributed random variables are distributed between -1 and 1 according to the pdf \( f_x(x) \) given below:

\[
f_x(x) = \begin{cases} 
| x | & -1 \leq x \leq 1 \\
0 & otherwise
\end{cases}
\]

Suppose that you quantize this source using a three level quantizer (three quantization regions). Also assume that the quantization regions are chosen to be: \([-1, -1/3]\), \([-1/3, 1/3]\), and \([1/3, 1]\).

a) What would you choose as the quantized values for each of the regions in order to minimize average distortion?

b) What is the resulting binary entropy of the quantized source?