Subject: Ablative cooling

By ablation we mean the recession of a surface due to heating, usually by a hot gas. It is the key process for

a) Re-entry heat shields  
b) Solid propellant nozzles  
c) Rocket case insulation  
d) Fire-proofing skyscrapers’ structures

Consider a hot gas flowing over a surface which can

1) evaporate, and  
2) whose vapor can react with the external flow

A heat balance at the surface gives

\[ q_s = q_w - (\rho v)_w \Delta h_w \]  \hspace{1cm} (1)

where the heat of ablation, \( \Delta h_w = h_w - h_s \), in J/kg, can include a heat of vaporization and decomposition and \( q_w \) is the heat flux from the fluid boundary layer.

Let us assume that it is useful to write

\[ q_w = \rho_e u_e St (H_e + \frac{u_e^2}{2} - H_w) \]

where \( H_w \) is the total enthalpy for the wall material at the wall temperature, and \( H_e = c_p g T_{ge} \) is the gas specific enthalpy at the temperature \( T_{ge} \) just outside the boundary layer. Notice that this generalizes our previous expression for heat transfer by replacing the total enthalpy for the static enthalpy; this is of general validity for high-speed flows. By energy conservation in the core flow,

\[ H_e + \frac{u_e^2}{2} = c_p g T_c. \]
Then the heat transfer to the solid is

$$q_s = \rho_e u_e St \left( H_e + \frac{u_e}{2} - H_w \right) - (\rho v)_w (h_w - h_s)$$

Since $q_s$ is the heat flux available to heat the wall, it is clear that the evaporation reduces the wall heat flux, i.e. the heat flux into the solid.

Now what determines $(\rho v)_w$? First note that $(\rho v)_w = r_s$ where $r$ is our "recession rate". Consider a "thermal wave" propagating into the solid. We will now work in the receding frame, in which we can assume a steady situation. If the recession rate is $r(m/s)$, we see solid material moving at $v_y = -r$, and convecting a heat flux. Then, if there is no local heat generation,

$$\frac{d}{dy}\left(-k_s \frac{dT}{dy} - r \rho_s c_s T\right) = 0$$

which integrates to

$$-k_s \frac{dT}{dy} + r \rho_s c_s T = \text{const.} = r \rho_s c_s T_{w_{\infty}},$$

or

$$\frac{d\left(T - T_{w_{\infty}}\right)}{dy} + \frac{r}{\alpha} (T - T_{w_{\infty}}) = 0 \quad (\alpha = \frac{k_s}{\rho_s c_s}, \text{ heat diffusivity, in m}^2/\text{s})$$

$$T - T_{w_{\infty}} = (T_w - T_{w_{\infty}}) e^{-\frac{r}{\alpha}}$$

The heat flux at at $y = 0$ is

$$q_s = -k_s \frac{dT}{dy} \bigg|_{y=0} = k_s \frac{r}{\alpha} (T_w - T_{w_{\infty}}),$$

giving

$$q_s = \rho c_r r (T_w - T_{w_{\infty}}),$$

or

$$\rho r = \frac{q_s}{c_r (T_w - T_{w_{\infty}})}$$

So returning to our expression for the heat flux to the wall,

$$q_s = \rho_e u_e S_{pg} \left( T_e - T_w \right) - \rho_s r \Delta h_w$$

$$\rho r c_r (T_w - T_{w_{\infty}})$$

$$\Rightarrow \quad \rho r = \frac{\rho_e u_e c_{pg} \left( T_e - T_w \right)}{c_r (T_w - T_{w_{\infty}}) + \Delta h_w} S_{s_{fg}}$$

$$\frac{\rho r}{\rho_e u_e S_{s_{fg}}} = \frac{c_{pg} \left( T_e - T_w \right)}{c_r (T_w - T_{w_{\infty}}) + \Delta h_w} \equiv \psi$$

You may compare this to Sutton pg. 510 Eq. 15-6

Now how does $(\rho v)_w$ influence $S$? Physically, we know that the vapor comes off the wall with $u=0$, so it will tend to retard the flow near the wall.
Quantitatively, Lees says, for small surface blowoff effect,

\[
\frac{St}{St_o} = \frac{B}{e^B - 1} c_{ps} = \frac{1.2 \times 320}{0.2} \approx 2500 J / Kg / K , \quad B = \frac{(pv)_w}{\rho e u S_t}
\]

while Sutton quotes Lees as giving for the larger blowoffs

\[
\frac{St}{St_o} = 1.27 B^{-77} \quad 5 < B < 100
\]

These give the same trends, but somewhat different numbers. We adopt here Lees’ formulation: We had

\[
\frac{\rho f}{\rho e u S_t} = \psi, \quad B = \psi \frac{S_t}{S_{to}} = \psi \frac{B}{e^B - 1}
\]

or \(e^B - 1 = \psi \Rightarrow B = \ell n(1 + \psi)\)

so finally,

\[
\rho f = \left\{ \ell n \left[ 1 + \frac{c_{ps}(T_c - T_w)}{c_s(T_w - T_{w0}) + \Delta h_w} \right] \right\} \rho e u S_{to} \tag{1}
\]

and then

\[
q_s = (\rho f) c_s(T_w - T_{w0}) = \rho e u S_{to} c_s(T_w - T_{w0}) \ell n \left[ 1 + \frac{c_{ps}(T_c - T_w)}{c_s(T_w - T_{w0}) + \Delta h_w} \right] \tag{1'}
\]

At the throat, \( (\rho f) = \frac{\rho e u}{c} \). This is where \(q_{wo}\) and hence \(r,\) is maximum.

Numerically, one often sees \(\Delta h_w >> c_s(T_w - T_{w0})\), and also \(\psi << 1\). This would leave us with the approximations

\[
\rho f \approx \frac{c_{ps}(T_c - T_w)}{\Delta h_w - \rho e u S_{wo} S_t} \tag{2}
\]

and going back to the surface heat balance, \(q_{w} \approx \rho f \), i.e., to the first order, the heat does not penetrate below the ablating layer, which is as intended.
Equations (1) or (2) can be used to select the proper thickness of the ablative (sacrificial) layer, once the burn time is given: $\delta_{\text{w}} = r \cdot t_b$.

Now let us look at some numerical examples, to show the magnitudes of the terms:

A. For carbon, $\Delta H_{fg} = 172 \text{ kcal/g mole} = 60 \times 10^6 \text{ Joule/kg}$

$$\approx h_w - h_s = \frac{172000}{12} = 15,000 \text{ cal/g}$$

$$c_{ps} = \frac{1.2 \times 8.320}{0.2} \approx 2500 J/Kg/K$$

and assume $T_c - T_w = 1000 K$.

Now $\rho_{e u_e} = \Gamma \frac{p}{\sqrt{RT}} \approx 0.65 \frac{p}{\sqrt{RT}}$

$$\approx \frac{10^5 (p \text{ atm})}{8.32 \times 10^3 (3000)} \approx 58 (p_c \text{ atm}) \frac{kg}{m^2s}$$

Taking $p_c = 100 \text{ atm}$ and $S_t = 0.001$, we then calculate

$$r \approx \frac{2500 \times 1000}{2000 \times 60 \times 10^6} \times 58 \times 100 \times 0.001 = 1.2 \times 10^{-4} \text{ m/s} = 0.12 \text{ mm/s}$$

So in 100 sec., the change in the surface is $\approx 12 \text{ mm} = 1.2 \text{ cm}$.

This is of the right magnitude, but most important is to see what it depends on.

B. For rubber $h_w - h_s \approx 10,000 \text{ cal/gm mole}$. For an approximate empirical formula $C_{10}H_{20}$, this gives per unit mass

$$\frac{10000}{120 + 20} = \frac{10000}{140} \approx 70 \frac{\text{ cal}}{g} \approx 280,000 \frac{\text{ Joule}}{kg}$$

Also $c_p \approx 1 \text{ cal/g} = 4,000 \text{ Joule/kg K}$

So now we would get about 100 times the regression rate for carbon. In this case though, if the rubber is used in the casing of a solid rocket engine then $\rho_{e u_e} \ll (\rho_{e u_e})\text{throat}$, maybe by $\frac{1}{100}$, so the net regression may end up being similar.