PS2 Solutions

1: Engine Tuning

As shown in class, the ideal Otto cycle is depicted in this diagram:

![Ideal Otto cycle diagram]

Image by MIT OpenCourseWare.

The actual 4-stroke Otto cycle (if you were to put a pressure sensor in the cylinder) looks more like this:

![Actual Otto cycle diagram]

Image by MIT OpenCourseWare.

Label the actions that are occurring in the engine at points

A) Compression
B) Power stroke
C) Exhaust valves close / intake valves open

The amount of energy that an engine outputs is proportional to the area of this “box” in the ideal engine cycle. Given what you know about engine cycles, draw the modified PV diagrams (sketch which way each part of the graph would move) for how each of these features impact the cycle:
1. Supercharging (forced induction)

Pressure in the cylinder will always be above ambient \( P_0 \) during periods of boost - increases amount of air available for combustion.

![Graph](Image by MIT OpenCourseWare.)

2. Optimal valve timing, intake and free-flow exhaust (reducing engine “breathing”)

Reduces the amount of energy wasted during "breathing" at the bottom of the cycle curve - brings the cycle closer to the ideal otto curve shape.

![Graph](Image by MIT OpenCourseWare.)

3. Boring out an engine (increasing its displacement)

Expands the curve laterally; the power taken out of this heat engine is the area in the top 'loop' - by making this larger we get more power.

![Graph](Image by MIT OpenCourseWare.)
Problem 2
\[ S.A = R(1)^2 = 3.14 \text{ m}^2 \]
\[ 250 \text{ lb. to kg} = 113.4 \text{ kg} \cdot 9.8 = 1111.32 \text{ N} \]
\[ = 353.923 \text{ Pa} \]

A) mass of contained N\textsubscript{2}:
- nitrogen gas is modeled as an ideal gas in this problem.
- initial temperature: 298K
- press: 101,679 Pa
- volume = 1.57m\textsuperscript{3}
- \[ PV = nRT \]
- individual gas constant for N\textsubscript{2} = 297 J/(kg·K)

\[ m = \frac{(101,697)(1.57)}{(297)(298)} = 1.8 \text{ kg} \]

B) Constant-pressure heat capacity:
\[ C_p = M_g C_p = 1.8 \text{ kg} \cdot \frac{1.04ks}{kg \cdot k} = 1.87 \text{ kJ/k} \]

C) New volume: \[ 3.14m^2 \cdot 4.5m = 14.14m^3 \]
Pressure remains constant:
\[ \frac{P_1V_1}{RT_1} = m = \frac{P_2V_2}{RT_2} \Rightarrow \frac{V_1}{T_1} = m = \frac{V_2}{T_2} \]
\[ \frac{1.57m^3}{298K} = \frac{14.14}{T_2} \]
\[ T_2 = 2,682K \] (Tungsten melts at 3,680K)

D) Temperature Change:
\[ 2,682 - 298 = 2,384K. \]
\[ 2,384 \cdot 1.87 = 4.458 \text{ kJ} \]
\[ PE = mgh \rightarrow (113.4)(9.8)(4) = 4,445.28J \]
- Energies are off by 3 orders of magnitude

E) 1 Watt = 1 J/s
\[ 1500W = 1500J/s = 1.5 \text{ kJ/s} \]
\[ 4,458 \text{ kJ} \cdot 1s/1.5kJ = 2,972 \text{ sec} = 49.5\text{min} \]

F) Anything that sounds like "no".
G) Entropy Change:
"Assume that the piston is adiabatic and that the cylinder is perfectly insulated on all sides."

→ There is no entropy transfer to the outside.

→ However, entropy generation due to shift in equilibrium states (gas).

\[(S_2 - S_1)_{\text{system}} = S_{\text{transfer}} + S_{\text{gen}}\]

\[(S_2 - S_1)_{N_2} = m(C_p \cdot ln(T_2,a/T_1,g) + R \cdot ln(V_2/V_1))\]

\[= 1.8(1.87ln(\frac{2,384}{298}) + 297ln(\frac{14.13}{1.57}))\]

\[= 1,181 \text{ J/K}\]

H) Image by MIT OpenCourseWare.

Problem 3

A) a) 14.7 : 1
   b) 14.6 : 1
   c) 9:1

All well-known, can do math by hand or find this online.

B) a) mixture too lean, not enough fuel: air.
   b) engine is always on the verge of being starved for fuel. Although the mixture in the red run is slightly rich, it ensures there's always complete combustion of the air.

C) peak torque. Same for all IC engines.

D) → 432.5 cc of air burned every revolution.

   Air = 1.184 kg/m³ at 25°C .

   432.5 cc = 0.0004325 m³ → mass = 0.000512 kg

   fuel(@ 13.58 A/F ratio): \[\frac{0.000512}{13.58} = 0.000038 \text{ kg gasoline.}\]

   At 4,600 RPM, this means 0.173 kg of gasoline burned every minute.

   1 gal. gasoline = 3.78L ⇒ 6.073 lb/gal (wikipedia/other sources) → 2.754 kg/gal.

   → 15.9 min to burn 1 gal. @60mph → [ 15.9 mpg]
F) 49 ft-lbs of torque for one revolution does:

\[ 49.5 \text{ ft-bl} = 67.112 \text{ N-m} \]

work = f \cdot \text{distance}.

distance = \text{circumference of 1m radius circle} = 2R(1) = 6.28m.

one revolution = 67.112N \cdot 6.28m = 421.6 \text{ J/rev}.

Gasoline = 44 \text{ MJ/kg} \rightarrow @ \text{0.000038 kg/rev}:

   Engine takes 1,672J to make 421J \rightarrow 0.25 \text{ efficiency}
In this problem, we will investigate basic vehicle traction principles. The most important vehicle components when it comes to traction are the tires, since they are the only actual interface between the road and the car.

The figure above is a tire performance curve, which illustrates the nonlinear relationship between vertical load on a tire and available traction. As the vertical load on this tire increases, its cornering efficiency decreases. Assume all vehicles in this problem are using tires that can be represented by the above curve.

a) We are entering a ¼ mile drag race in our small, home built racecar. Assume our vehicle is 1800 lbs and has a single driver who weighs 200 lbs. The vehicle has an even weight distribution between front and the back, and the driver’s presence does not change it. What is the max acceleration our vehicle can attain without losing traction (in terms of g’s)? What would this translate to in terms of a ¼ mile time (assume traction is limiting factor, not top speed)?

Doing some metric conversions ensures that our calculations carry the right units:

<table>
<thead>
<tr>
<th>Metric</th>
<th>Customary</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vehicle_weight = 2000 lbs = 907.18 kg</td>
<td>Vehicle_weight = 2000 lbs</td>
</tr>
<tr>
<td>Tire_weight = 500 lbs = 226.80 kg</td>
<td>Tire_weight = 500 lbs</td>
</tr>
<tr>
<td>Tire_traction_force = 700 lbs = 3,112.5 N</td>
<td>Tire_traction_force = 700 lbs</td>
</tr>
<tr>
<td>Vehicle_traction_force = 12,450 N</td>
<td>Vehicle_traction_force = 2800 lbs</td>
</tr>
</tbody>
</table>

\[ F_{veh} = M_{veh} \times A \]
\[ A = \frac{M_{veh}}{F_{veh}} = \frac{12450 \text{ N}}{907.18 \text{ kg}} \]
\[ A = 13.72 \text{ m/s}^2 \Rightarrow 1.4 \text{ g} \]

\[ X_{final} = \frac{1}{4} \text{ mile} = 402.34 \text{ m} \]
\[ X_{final} = 0.5 \times A \times T^2 \]
\[ T = \sqrt{\frac{2 \times 402.34 \text{ m}}{13.72 \text{ m/s}^2}} \]
\[ T = \sqrt{ \frac{2 \times 402.34}{13.72} } = 7.66 \text{ s} \]

\[ V_f = A \times T = (13.72) \times (7.66) = 105.08 \text{ m/s} \quad \rightarrow \quad V_f = 235 \text{ mph} \]

This problem assumed that the vehicle is only limited by available traction, and that the available traction does not change as a function of speed or tire temperature. However, in real life, rolling resistance, aerodynamic drag, engine power limitations, and other system losses would level off the acceleration well before we reached this ridiculous top speed.

b) We are now racing a heavier, 3000 lb vehicle around a circular race track. As the car turns (going counterclockwise), there will be a lateral weight transfer from the left to the right side of the vehicle. Use the following simplified formula to determine the weight transfer we would expect with a 1 g cornering force, where ‘\( W_{\text{car}} \)’ is the weight of the car, ‘\( a \)’ is the cornering acceleration in g’s, ‘\( h_{cg} \)’ is the height of the center of gravity in inches (assume 20 inches), ‘\( g \)’ is gravity, and ‘\( l_{\text{track}} \)’ is the vehicle track width (assume 60 inches).

\[ W_T = \frac{W_{\text{car}} \times a \times h_{cg}}{g \times l_{\text{track}}} \]

Determine the traction that each wheel is able to maintain after the weight shift. What cornering force (in g’s) does this equate to? How does that compare to the traction available to this vehicle on a straight track?

First, determine the lateral weight transfer:

\[ W_t = \frac{(3000 \text{ lb} \times 1 \text{ g} \times 20 \text{ inches})}{(1 \text{ g} \times 60 \text{ inches})} = 1000 \text{ lb} \]

Now, find the tractive force available on each wheel:

<table>
<thead>
<tr>
<th>Wheel</th>
<th>Initial Weight</th>
<th>Initial Traction</th>
<th>Weight Shift</th>
<th>Shifted Weight</th>
<th>Available Traction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Front Left</td>
<td>750</td>
<td>850</td>
<td>-500</td>
<td>250</td>
<td>450</td>
</tr>
<tr>
<td>Back Left</td>
<td>750</td>
<td>850</td>
<td>-500</td>
<td>250</td>
<td>450</td>
</tr>
<tr>
<td>Front Right</td>
<td>750</td>
<td>850</td>
<td>+500</td>
<td>1250</td>
<td>1150</td>
</tr>
<tr>
<td>Back Right</td>
<td>750</td>
<td>850</td>
<td>+500</td>
<td>1250</td>
<td>1150</td>
</tr>
</tbody>
</table>

Initial Vehicle Traction Capability = 4*850 lbs = 3,400 lbs
Max Initial Acceleration = 3,400 lbs / 3,000 lbs = 1.13 g’s

1g Cornering Vehicle Traction Capability = 2*1150 lbs + 2*450 lbs = 3,200 lbs
Max Acceleration, 1g Cornering Vehicle = 3,200 lbs / 3,000 lbs = 1.07 g’s
So, the tractive capability of the vehicle that is cornering at 1 g is reduced from 1.13 g’s to 1.07 g’s. It won’t lose traction while cornering at this speed unless it also accelerates forward or backward such that the resultant acceleration vectors is greater than 1.07 g’s.

c) Understeer and oversteer are important concepts to understand, and they can be explained by vehicle weight distribution. Assume we have the same vehicle as in part (b), but this time with a 60-40 weight distribution front to back. Assume the vehicle enters a turn and experiences approximately the same weight transfer as in part (b). How much traction is available for the front and rear wheels now? Will the car understeer or oversteer? Why?

<table>
<thead>
<tr>
<th>Wheel</th>
<th>Initial Weight</th>
<th>Initial Traction</th>
<th>Weight Shift</th>
<th>Shifted Weight</th>
<th>Available Traction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Front Left</td>
<td>900</td>
<td>950</td>
<td>-600</td>
<td>300</td>
<td>500</td>
</tr>
<tr>
<td>Back Left</td>
<td>600</td>
<td>800</td>
<td>-400</td>
<td>200</td>
<td>380</td>
</tr>
<tr>
<td>Front Right</td>
<td>900</td>
<td>950</td>
<td>+600</td>
<td>1500</td>
<td>1250</td>
</tr>
<tr>
<td>Back Right</td>
<td>600</td>
<td>800</td>
<td>+400</td>
<td>1000</td>
<td>1000</td>
</tr>
</tbody>
</table>

Front Weight Initial = 1800 lbs  Front Traction Initial = 1900 lbs
Front Max Accelerations = 1900 lbs / 1800 lbs = 1.06 g’s
Rear Weight Initial = 1200 lbs  Rear Traction Initial = 1600 lbs
Rear Max Acceleration = 1600 lbs / 1200 lbs = 1.33 g’s

Front Weight, 1g cornering = 1800 lbs  Front Traction, 1g cornering = 1750 lbs
Front Max Accel, 1g cornering = 0.97 g’s
Rear Weight, 1g cornering = 1200 lbs  Rear Traction, 1g cornering = 1380 lbs
Rear Max Acceleration = 1600 lbs / 1380 lbs = 1.16 g’s

So, while cornering on a counterclockwise turn, the available traction on the front tires is reduced compared to the available traction on the rear tires. This will result in understeer, as the front wheels want to come out of the turn while the rear wheels maintain traction. Moreover, because the front end of the car can’t corer at over 0.97 g’s, the vehicle won’t be able to corner at over 0.97 g’s.

d) If you were a racecar driver (circular track) and you had a fixed 60-40 front-rear weight distribution but could add a lateral weight bias of up to 20%, what would you do to maximize the cornering performance of your vehicle? What would your max cornering acceleration be in this case?

Since a racecar driver on a circular track only experiences weight transfers in one direction, it would make sense to establish a lateral weight bias on the left side to limit the traction reducing effects of cornering with an understeer.

<table>
<thead>
<tr>
<th>Wheel</th>
<th>Initial Weight (no bias)</th>
<th>Initial Weight (bias)</th>
<th>Initial Traction</th>
<th>Weight Shift</th>
<th>Shifted Weight</th>
<th>Available Traction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Front Left</td>
<td>900</td>
<td>1200</td>
<td>1100</td>
<td>-600</td>
<td>600</td>
<td>750</td>
</tr>
<tr>
<td>Back Left</td>
<td>600</td>
<td>900</td>
<td>950</td>
<td>-400</td>
<td>500</td>
<td>700</td>
</tr>
</tbody>
</table>
Front Weight Initial (bias) = 1800 lbs  Front Traction Initial = 1900 lbs
Front Max Accelerations = 1900 lbs / 1800 lbs = 1.06 g’s
Rear Weight Initial = 1200 lbs  Rear Traction Initial = 1450 lbs
Rear Max Acceleration = 1450 lbs / 1200 lbs = 1.21 g’s

Front Weight, 1g cornering = 1800 lbs  Front Traction, 1g cornering = 1850 lbs
Front Max Accel, 1g cornering = 1.03 g’s
Rear Weight, 1g cornering = 1200 lbs  Rear Traction, 1g cornering = 1320 lbs
Rear Max Acceleration = 1320 lbs / 1200 lbs = 1.1 g’s

So, the problem of understeer in part c is overcome by adding a lateral weight bias to the left around the circular racetrack. Max acceleration on the front wheels is now up to 1.03 g’s from .97 g’s, and on the rear wheels it is down from 1.16 g’s to 1.1 g’s. By sacrificing some of the traction available to us on the rear wheels, then, we have mitigated the understeer issue.