Introduction to Revenue Management:
Flight Leg Revenue Optimization

16.75 Airline Management

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1. Airline Revenue Maximization
   - Pricing vs. Yield (Revenue) Management

2. Computerized RM Systems
   - RM System in ePODS

3. Single-leg Fare Class Seat Allocation Problem
   - Partitioned vs. Serial Nesting of Booking Classes
   - Deterministic vs. Probabilistic Demand

4. EMSRb Model for Seat Protection
   - Example of Calculations
1. Airline Revenue Maximization

- Two components of airline revenue maximization:

  **Differential Pricing:**
  - Various “fare products” offered at different prices for travel in the same O-D market

  **Yield Management (YM):**
  - Determines the number of seats to be made available to each “fare class” on a flight, by setting booking limits on low fare seats

- Typically, YM takes a set of differentiated prices/products and flight capacity as given:
  - With high proportion of fixed operating costs for a committed flight schedule, revenue maximization to maximize profits
Why Call it “Yield Management”? 

• Main objective of YM is to **protect seats** for later-booking, high-fare business passengers.

• YM involves tactical control of airline’s seat inventory:
  – But too much emphasis on **yield** (revenue per RPM) can lead to overly severe limits on low fares, and lower overall load factors
  – Too many seats sold at lower fares will increase load factors but reduce yield, adversely affective total revenues

• **Revenue maximization** is proper goal:
  – Requires proper balance of load factor and yield

• Many airlines now refer to “Revenue Management” (RM) instead of “Yield Management”
# Seat Inventory Control Approaches

**EXAMPLE: 2100 MILE FLIGHT LEG**

**CAPACITY = 200**

<table>
<thead>
<tr>
<th>FARE CLASS</th>
<th>AVERAGE REVENUE</th>
<th>YIELD EMPHASIS</th>
<th>LOAD FACTOR EMPHASIS</th>
<th>REVENUE EMPHASIS</th>
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<tbody>
<tr>
<td>Y</td>
<td>$420</td>
<td>20</td>
<td>10</td>
<td>17</td>
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<td>B</td>
<td>$360</td>
<td>23</td>
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<td>H</td>
<td>$230</td>
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<tr>
<td>V</td>
<td>$180</td>
<td>30</td>
<td>55</td>
<td>37</td>
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<tr>
<td>Q</td>
<td>$120</td>
<td>15</td>
<td>68</td>
<td>40</td>
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<table>
<thead>
<tr>
<th></th>
<th>TOTAL PASSENGERS</th>
<th>LOAD FACTOR</th>
<th>TOTAL REVENUE</th>
<th>AVERAGE FARE</th>
<th>YIELD (CENTS/RPM)</th>
</tr>
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<tr>
<td></td>
<td>110</td>
<td>55%</td>
<td>$28,940</td>
<td>$263</td>
<td>12.53</td>
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<tr>
<td></td>
<td></td>
<td>80%</td>
<td>$30,160</td>
<td>$189</td>
<td>8.98</td>
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<tr>
<td></td>
<td></td>
<td>68%</td>
<td>$31,250</td>
<td>$230</td>
<td>10.94</td>
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Revenue Management Techniques

- **Overbooking**
  - Accept reservations in excess of aircraft capacity to overcome loss of revenues due to passenger “no-show” effects

- **Fare Class Mix (Flight Leg Optimization)**
  - Determine revenue-maximizing mix of seats available to each booking (fare) class on each flight departure

- **Traffic Flow (O-D) Control (Network Optimization)**
  - Further distinguish between seats available to short-haul (one-leg) vs. long-haul (connecting) passengers, to maximize total network revenues
  - Currently under development by some airlines
2. Computerized RM Systems

- Size and complexity of a typical airline’s seat inventory control problem requires a computerized RM system.

- Consider a US Major airline with:
  - 2000 flight legs per day
  - 10 booking classes
  - 300 days of bookings before departure

- At any point in time, this airline’s seat inventory consists of 6 million booking limits:
  - This inventory represents the airline’s potential for profitable operation, depending on the revenues obtained
  - Far too large a problem for human analysts to monitor alone
Typical 3rd Generation RM System

- Collects and maintains historical booking data by flight and fare class, for each past departure date.
- Forecasts future booking demand and no-show rates by flight departure date and fare class.
- Calculates limits to maximize total flight revenues:
  - Overbooking levels to minimize costs of spoilage/denied boardings
  - Booking class limits on low-value classes to protect high-fare seats
- Interactive decision support for RM analysts:
  - Can review, accept or reject recommendations
Example of Third Generation RM System

- REVENUE DATA
- HISTORICAL BOOKING DATA
- ACTUAL BOOKINGS
- NO-SHOW DATA

- FORECASTING MODEL
- OPTIMIZATION MODEL
- OVERBOOKING MODEL

- RECOMMENDED BOOKING LIMITS
Dynamic Revision and Intervention

- RM systems revise forecasts and re-optimize booking limits at numerous “checkpoints” of the booking process:
  - Monitor actual bookings vs. previously forecasted demand
  - Re-forecast demand and re-optimize at fixed checkpoints or when unexpected booking activity occurs
  - Can mean substantial changes in fare class availability from one day to the next, even for the same flight departure

- Substantial proportion of fare mix revenue gain comes from dynamic revision of booking limits:
  - Human intervention is important in unusual circumstances, such as “unexplained” surges in demand due to special events
Current State of RM Practice

- Most of the top 25 world airlines (in terms of revenue) have implemented 3rd generation RM systems.

- Many smaller carriers are still trying to make effective use of leg/fare class RM
  - Lack of company-wide understanding of RM principles
  - Historical emphasis on load factor or yield, not revenue
  - Excessive influence and/or RM abuse by dominant sales and marketing departments
  - Issues of regulation, organization and culture

- About a dozen leading airlines are looking toward network O-D control development and implementation
  - These carriers could achieve a 2-5 year competitive advantage with advanced revenue management systems
“Vanilla” RM System in ePODS

- Airlines’ RM systems forecast fare class demand for each flight leg departure:
  - Simple “pick-up” forecasts of bookings still to come
  - Unconstraining of closed observations based on booking curve probabilities.

- Optimization is leg-based EMSRb seat protection algorithm:
  - Booking limits set for each fare class on each flight leg departure, revised 16 times during booking process.

- No overbooking or no-shows in ePODS.
ePODS replicates airline RM system actions over time, taking into account previous interventions:

- Previously applied booking limits affect actual passenger loads and, in turn, future demand forecasts

“Historical” booking data is used to generate forecasts for “future” departures.

RM system only uses data available from past observations.
PODS Simulation: Basic Schematic

- PASSENGER DECISION MODEL
- REVENUE MANAGEMENT OPTIMIZER
- FORECASTER
- HISTORICAL BOOKING DATA BASE

Path/Class Availability
Path/Class Bookings/Cancellations
Current Bookings
Future Bookings
Update
Historical Bookings

- **Given for a future flight leg departure:**
  - Total booking capacity of (typically) the coach compartment
  - Several fare (booking) classes that share the same inventory of seats in the compartment
  - Forecasts of future booking demand by fare class
  - Revenue estimates for each fare (booking) class

- **Objective is to maximize total expected revenue:**
  - Allocate seats to each fare class based on value
In a partitioned CRS inventory structure, allocations to each booking class are made separately from all the other classes.

**EXAMPLE (assuming uncertain demand):**
- Given the following allocations for each of 3 classes—Y = 30, B = 40, M = 70 for an aircraft coach cabin with booking capacity = 140.
- If 31 Y customers request a seat, the airline would reject the 31st request because it exceeds the allocation for the Y class.
- It is possible that airline would reject the 31st Y class customer, even though it might not have sold all of the (lower-valued) B or M seats yet!

Under serial nesting of booking classes, the airline would never turn down a Y fare request, as long as there are any seats (Y, B or M) left for sale.
Serially Nested Buckets

- Q1
  - Protected for class 1 from 2,3,...,l

- Q2
  - Protected for class 2 from 3,4,...,l

- Q3
If we assume that demand is deterministic (or known with certainty), it would be simple to determine the fare class seat allocations

- Start with highest fare class and allocate/protect exactly the number of seats predicted for that class, and continue with the next lower fare class until capacity is reached.

**EXAMPLE:** 3 fare classes (Y, B, M)

- Demand for Y = 30, B = 40, M = 85
- Capacity = 140

**Deterministic decision:** Protect 30 for Y, 40 for B, and allocated 70 for M (i.e., spill 15 M requests)

**Nested booking limits** Y=140  B=110  M=70
EMSРb Model for Seat Protection: Assumptions

• Basic modeling assumptions for serially nested classes:
  a) demand for each class is separate and independent of demand in other classes.
  b) demand for each class is stochastic and can be represented by a probability distribution
  c) lowest class books first, in its entirety, followed by the next lowest class, etc.
  d) booking limits are only determined once (i.e., static optimization model)
• Because higher classes have access to unused lower class seats, the problem is to find seat protection levels for higher classes, and booking limits on lower classes.

• To calculate the optimal protection levels:

  Define $P_i(S_i) = \text{probability that } X_i \geq S_i,$

  where $S_i$ is the number of seats made available to class $i$, $X_i$ is the random demand for class $i$. 
EMSRb Calculations (cont’d)

• The expected marginal revenue of making the $S$th seat available to class $i$ is:
  \[ \text{EMSR}_i(S_i) = R_i \times P_i(S_i) \]
  where $R_i$ is the average revenue (or fare) from class $i$

• The optimal protection level, $\pi_1$ for class 1 from class 2 satisfies:
  \[ \text{EMSR}_1(\pi_1) = R_1 \times P_1(\pi_1) = R_2 \]

• Once $\pi_1$ is found, set $BL_2 = \text{Capacity} - \pi_1$. Of course, $BL_1 = \text{Capacity}$ (authorized capacity if overbooking)
Consider the following flight leg example:

<table>
<thead>
<tr>
<th>Class</th>
<th>Mean Fcst.</th>
<th>Std. Dev.</th>
<th>Fare</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>10</td>
<td>3</td>
<td>1000</td>
</tr>
<tr>
<td>B</td>
<td>15</td>
<td>5</td>
<td>700</td>
</tr>
<tr>
<td>M</td>
<td>20</td>
<td>7</td>
<td>500</td>
</tr>
<tr>
<td>Q</td>
<td>30</td>
<td>10</td>
<td>350</td>
</tr>
</tbody>
</table>

To find the protection for the Y fare class, we want to find the largest value of \( \pi_Y \) for which

\[
EMSR_Y(\pi_Y) = R_Y \times P_Y(\pi_Y) \geq R_B
\]
Example (cont’d)

\[
\text{EMSR}_Y(\pi_Y) = 1000 \times P_Y(\pi_Y) \geq 700 \\
P_Y(\pi_Y) \geq 0.70
\]

where \( P_Y(\pi_Y) \) = probability that \( X_Y \geq \pi_Y \).

• If we assume demand in Y class is *normally* distributed with mean, standard deviation given earlier, then we can create a standardized normal random variable as \((X_Y - 10)/3\).
• Next, we use Excel or go to the Standard Normal Cumulative Probability Table for different “guesses” for $\pi_Y$. For example,

- for $\pi_Y = 7$, $\text{Prob} \{ (X_Y - 10)/3 \geq (7 - 10)/3 \} = 0.841$
- for $\pi_Y = 8$, $\text{Prob} \{ (X_Y - 10)/3 \geq (8 - 10)/3 \} = 0.747$
- for $\pi_Y = 9$, $\text{Prob} \{ (X_Y - 10)/3 \geq (9 - 10)/3 \} = 0.63$

• So, we can see that $\pi_Y = 8$ is the largest integer value of $\pi_Y$ that gives a probability $\geq 0.7$ and therefore we will protect 8 seats for Y class!
Joint Protection for Classes 1 and 2

- How many seats to protect jointly for classes 1 and 2 from class 3?

- The following calculations are necessary:

\[
\bar{X}_{1,2} = \bar{X}_1 + \bar{X}_2
\]

\[
\hat{\sigma}_{1,2} = \sqrt{\hat{\sigma}^2_1 + \hat{\sigma}^2_2}
\]

\[
R_{1,2} = \frac{R_1 \times \bar{X}_1 + R_2 \times \bar{X}_2}{\bar{X}_{1,2}}
\]

\[
P_{1,2}(S) = \Pr \{ob(X_1 + X_2 > S)\}
Protection for Y+B Classes

• To find the protection for the Y and B fare classes from M, we want to find the largest value of $\pi_{YB}$ that makes

$$EMSR_{YB}(\pi_{YB}) = R_{YB} \cdot P_{YB}(\pi_{YB}) \geq R_M$$

• Intermediate Calculations:

\[ R_{YB} = \frac{(10 \times 1000 + 15 \times 700)}{(10 + 15)} = 820 \]

\[ \bar{X}_{Y,B} = \bar{X}_Y + \bar{X}_B = 10 + 15 = 25 \]

\[ \hat{\sigma}_{Y,B} = \sqrt{\hat{\sigma}^2_Y + \hat{\sigma}^2_B} = \sqrt{3^2 + 5^2} = \sqrt{34} = 5.83 \]
• The protection level for Y+B classes satisfies:
  \[ 820 \times \text{PYB}(\pi_{YB}) \geq 500 \]
  \[ \text{PYB}(\pi_{YB}) \geq 0.6098 \]

• Again, we can make different “guesses” for \( \pi_{YB} \).
  for \( \pi_{YB} = 20 \), Prob \{ (X_{YB} - 25)/5.83 \geq (20 - 25)/5.83 \} = 0.805
  for \( \pi_{YB} = 22 \), Prob \{ (X_{YB} - 25)/5.83 \geq (22 - 25)/5.83 \} = 0.697
  for \( \pi_{YB} = 23 \), Prob \{ (X_{YB} - 25)/5.83 \geq (23 - 25)/5.83 \} = 0.633
  for \( \pi_{YB} = 24 \), Prob \{ (X_{YB} - 25)/5.83 \geq (24 - 25)/5.83 \} = 0.5675
So, we can see that \( \pi_{YB} = 23 \) is the largest integer value of \( \pi_{YB} \) that gives a probability \( \geq 0.6098 \) and therefore we will jointly protect 23 seats for Y and B class from class M!

Suppose we had an aircraft with authorized booking capacity 80 seats, our Booking Limits would be:

- \( BL_Y = 80 \)
- \( BL_B = 80 - 8 = 72 \)
- \( BL_M = 80 - 23 = 57 \)
General Case for Class n

- How many seats to protect jointly for classes 1 through n from class n+1?
- The following calculations are necessary:

\[
\overline{X}_{1,n} = \sum_{i=1}^{n} \overline{X}_i
\]

\[
\hat{\sigma}_{1,n} = \sqrt{\sum_{i=1}^{n} \hat{\sigma}_i^2}
\]

\[
R_{1,n} = \frac{\sum_{i=1}^{n} R_i \cdot \overline{X}_i}{\overline{X}_{1,n}}
\]
General Case (cont’d)

- We then find the value of $\pi_n$ that makes

$$\text{EMSR}_{1,n}(\pi_n) = R_{1,n} \times P_{1,n}(\pi_n) = R_{n+1}$$

- Once $\pi_n$ is found, set $\text{BL}_{n+1} = \text{Capacity} - \pi_n$