Airline Fleet Assignment

Cynthia Barnhart
16.75 Airline Management

Outline:
- Problem Definition and Objective
- Fleet Assignment Network Representation
- Fleet Assignment Models and Algorithms
- Extension of Fleet Assignment to Schedule Design
- Conclusions
Airline Schedule Planning

Schedule Design

Fleet Assignment

Aircraft Routing

Crew Scheduling

Select optimal set of flight legs in a schedule

A flight specifies origin, destination, and departure time

Contribution = Revenue - Costs

Assign crew (pilots and/or flight attendants) to flight legs
Fleet Assignment

Problem Definition

• Given:
  - Flight Schedule: a set of (daily) flight legs;
  - Aircraft fleet: consisting of different fleet types;
  - Passenger demand pattern;
  - Revenue and operating cost data;

• Find:
  A feasible fleet assignment, i.e. a mapping from flight legs to fleet types that maximizes
  profit = total revenue - total operating costs.
Fleet Assignment

- **Class Exercise:** Flight Network

Diagram:
- **ORD** (Chicago O'Hare)
  - CL50x (2 flights)
  - CL55x (2 flights)

- **LGA** (New York LaGuardia)
  - CL33x (3 flights)
  - CL50x (2 flights)

- **BOS** (Boston Logan)
  - CL30x (3 flights)
## Fleet Assignment

### Class Exercise: Flight Schedule, Fares, & Demand

<table>
<thead>
<tr>
<th>Flight #</th>
<th>From</th>
<th>To</th>
<th>Dept Time (EST)</th>
<th>Arr Time (EST)</th>
<th>Fare [$]</th>
<th>Demand [passengers]</th>
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</thead>
<tbody>
<tr>
<td>CL301</td>
<td>LGA</td>
<td>BOS</td>
<td>1000</td>
<td>1100</td>
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<td>250</td>
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<tr>
<td>CL302</td>
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<td>1200</td>
<td>150</td>
<td>250</td>
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<tr>
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<td>1900</td>
<td>150</td>
<td>100</td>
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<td>BOS</td>
<td>LGA</td>
<td>0700</td>
<td>0800</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>CL332</td>
<td>BOS</td>
<td>LGA</td>
<td>1030</td>
<td>1130</td>
<td>150</td>
<td>300</td>
</tr>
<tr>
<td>CL333</td>
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<td>LGA</td>
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<td>1900</td>
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<td>1400</td>
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<td>1800</td>
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<td>0700</td>
<td>1000</td>
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<tr>
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<td>LGA</td>
<td>0830</td>
<td>1130</td>
<td>400</td>
<td>150</td>
</tr>
</tbody>
</table>
Fleet Assignment

**Class Exercise: Fleet Information**

<table>
<thead>
<tr>
<th>Fleet type</th>
<th>Number of aircraft owned</th>
<th>Capacity [seats]</th>
<th>Per flight operating cost [$000]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>LGA - BOS</td>
</tr>
<tr>
<td>DC-9</td>
<td>1</td>
<td>120</td>
<td>10</td>
</tr>
<tr>
<td>B737</td>
<td>2</td>
<td>150</td>
<td>12</td>
</tr>
<tr>
<td>A300</td>
<td>2</td>
<td>250</td>
<td>15</td>
</tr>
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<td></td>
<td></td>
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<td>15</td>
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</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>20</td>
</tr>
</tbody>
</table>
Fleet Assignment

• Class Exercise:

Find:

An assignment of fleet types to the flights in this network that maximizes net profit.
Evaluating assignment profits:

\[ c_{l,f} := fare_l \times \min(D_l, Cap_f) - OpCost_{l,f} \]

where:

- \( c_{l,f} \): profitability of assigning fleet type \( f \) to flight leg \( l \);
- \( fare_l \): fare of flight leg \( l \);
- \( D_l \): demand of flight leg \( l \);
- \( Cap_f \): capacity of fleet type \( f \);
- \( OpCost_{l,f} \): operating cost of assigning fleet type \( f \) to flight leg \( l \).
Fleet Assignment

- **Class Exercise**: Evaluating assignment profitabilities...

Profitability [\$000 per day]

<table>
<thead>
<tr>
<th>Flight #</th>
<th>DC-9</th>
<th>B737</th>
<th>A300</th>
</tr>
</thead>
<tbody>
<tr>
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<td>10.5</td>
<td>22.5</td>
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<td>8</td>
<td>10.5</td>
<td>22.5</td>
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<td>22.5</td>
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<td>40</td>
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<td>60</td>
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<tr>
<td>CL552</td>
<td>33</td>
<td>43</td>
<td>40</td>
</tr>
</tbody>
</table>
Fleet Assignment

Time-Line Network:

- CL331
- CL301
- CL332
- CL302
- CL333
- CL303
- CL551
- CL552
- CL501
- CL502

BOS

LGA

ORD

Timescale
Fleet Assignment

Optimal solution:

- Leg
- \Delta capacity

<table>
<thead>
<tr>
<th>Leg</th>
<th>\Delta capacity</th>
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<tbody>
<tr>
<td>CL301</td>
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<tr>
<td>CL332</td>
<td>-50</td>
</tr>
<tr>
<td>CL333</td>
<td>-30</td>
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<tr>
<td>CL501</td>
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<tr>
<td>CL551</td>
<td>+50</td>
</tr>
<tr>
<td>CL552</td>
<td>0</td>
</tr>
</tbody>
</table>

Revenue = $428,500
Cost = $148,000
Profit = $280,500
Fleet Assignment

- Time-Line Network:

Airport A

Airport B

Flight arc

Ground arc

$y_v^-$

$v$

$y_v^+$

$w_{i,f}$

count line
Notations

• Decision Variables
  – $f_{k,i}$ equals 1 if fleet type $k$ is assigned to flight leg $i$, and 0 otherwise
  – $y_{k,o,t}$ is the number of aircraft of fleet type $k$, on the ground at station $o$, and time $t$

• Parameters
  – $C_{k,i}$ is the cost of assigning fleet $k$ to flight leg $i$
  – $N_k$ is the number of available aircraft of fleet type $k$
  – $t_n$ is the “count time”

• Sets
  – $L$ is the set of all flight legs $i$
  – $K$ is the set of all fleet types $k$
  – $O$ is the set of all stations $o$
  – $CL(k)$ is the set of all flight arcs for fleet type $k$ crossing the count time
Fleet Assignment Model (FAM)

\[
\text{Min} \sum_{k \in K} \sum_{i \in L} c_{k,i} f_{k,i}
\]

Subject to:

\[
\sum_{k \in K} f_{k,i} = 1 \quad \forall i \in L
\]

\[
y_{k,o,t^-} + \sum_{i \in I(k,o,t)} f_{k,i} - y_{k,o,t^+} - \sum_{i \in O(k,o,t)} f_{k,i} = 0 \quad \forall k,o,t
\]

\[
\sum_{o \in O} y_{k,o,t_n} + \sum_{i \in CI(k)} f_{k,i} \leq N_k \quad \forall k \in K
\]

\[
f_{k,i} \in \{0,1\} \quad y_{k,o,t} \geq 0
\]

Hane et al. (1995), Abara (1989), and Jacobs, Smith and Johnson (2000)
Constraints

• Cover Constraints
  – Each flight must be assigned to exactly one fleet

• Balance Constraints
  – Number of aircraft of a fleet type arriving at a station must equal the number of aircraft of that fleet type departing

• Aircraft Count Constraints
  – Number of aircraft of a fleet type used cannot exceed the number available
Objective Function

- For each fleet - flight combination: Cost \equiv \text{Operating cost} + \text{Spill cost} - \text{Recaptured revenue}

- Operating cost associated with assigning a fleet type \( k \) to a flight leg \( j \) is relatively straightforward to compute
  - Can capture range restrictions, noise restrictions, water restrictions, etc. by assigning “infinite” costs

- Spill cost for flight leg \( j \) and fleet assignment \( k \) = average revenue per passenger on \( j \) \* MAX(0, unconstrained demand for \( j \) – number of seats on \( k \))
  - Unclear how to compute revenue for flight legs, given revenue is associated with itineraries

- Recaptured revenue
  - Revenue from passengers that are recaptured back to the airline after being spilled from another flight leg
## FAM Example: Spill

Demand = 100  
Fare = $100

<table>
<thead>
<tr>
<th>Fleet Type</th>
<th>Capacity</th>
<th>Spill Cost</th>
<th>Op. Cost</th>
<th>Assignment Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>80</td>
<td>$2,000</td>
<td>$5,000</td>
<td>$7,000</td>
</tr>
<tr>
<td>ii</td>
<td>100</td>
<td>$0</td>
<td>$6,000</td>
<td>$6,000</td>
</tr>
<tr>
<td>iii</td>
<td>120</td>
<td>$0</td>
<td>$7,000</td>
<td>$7,000</td>
</tr>
<tr>
<td>iv</td>
<td>150</td>
<td>$0</td>
<td>$8,000</td>
<td>$8,000</td>
</tr>
</tbody>
</table>
FAM Example: Recapture

100 seats

A

( 80, $200 )

B

( 90, $250 )

100 seats

C

9AM

( 50, $400 )

20

30 × 0.3 = 9 recaptured passengers

10AM

29

( 20, $400 )

(Demand, Fare)

Recapture Rate
Fleet Assignment

- A few observations on FAM:
  - Nodes can be consolidated to reduce model size;
  - Fleet-specific time-line networks are possible;
  - Fleet assignment not aircraft assignment!
  - Note that feasibility of FAM implies that aircraft rotations exist (takes only a little bit of thinking);
  - However, these rotations might not be maintenance feasible...
Fleet Assignment

- Solvability:
  - FAM can be solved using standard branch-and-bound software;
  - Solution times are FAST, thanks to FAM’s small LP gaps...
Fleet Assignment

Computational Sample: 2,044 flight legs, 9 fleet types

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<tr>
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<tr>
<td># of non-zero entries</td>
<td>50,034</td>
</tr>
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<tr>
<th>Strength of formulation</th>
<th></th>
</tr>
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<tbody>
<tr>
<td>Root node LP solution</td>
<td>21,401,658</td>
</tr>
<tr>
<td>Best IP solution</td>
<td>21,401,622</td>
</tr>
<tr>
<td>Difference</td>
<td>36</td>
</tr>
</tbody>
</table>

| Solution time [sec]           | 974   |
Fleet Assignment

• FAM suffers from a significant drawback in its modeling of the revenue side...
• Passengers book itineraries not flight legs...
• Capacity decisions on one leg will affect passenger spill on other legs...
• This phenomenon is known as network effects.
Fleet Assignment Model (FAM)

\[ \text{Min} \sum_{k \in K} \sum_{i \in L} c_{k,i} f_{k,i} \]

Subject to:

\[ \sum_{k \in K} f_{k,i} = 1 \quad \forall i \in L \]

\[ y_{k,o,t}^- + \sum_{i \in I(k,o,t)} f_{k,i} - y_{k,o,t}^+ - \sum_{i \in O(k,o,t)} f_{k,i} = 0 \quad \forall k, o, t \]

\[ \sum_{o \in O} y_{o,i} + \sum_{i \in C(k)} f_{k,i} \leq N_k \quad \forall k \in K \]

\[ f_{k,i} \in \{0,1\} \quad y_{k,o,t} \geq 0 \]

Major Shortcoming:

FAM assumes leg independence

Hane et al. (1995), Abara (1989), and Jacobs, Smith and Johnson (2000)
FAM Example: Network Effects

Fleet Type | Capacity | Spill Cost
--- | --- | ---
i | 80 | ?
ii | 100 | ?
iii | 120 | ?
iv | 150 | $0

Leg Interdependence

Network Effects
Spill Cost Computation and Underlying Assumption

• Given:
  - Spill cost for flight leg $j$ and fleet assignment $k$
    $= \text{average revenue per passenger on } j \ast \text{MAX(0, unconstrained demand for } j - \text{number of seats on } k)$

• Implication:
  - A passenger might be spilled from some, but not all, of the flight legs in his/her itinerary
An Iterative Approach

• FAM Solver
  – Basic Fam
    • Possibly with minor modifications
• Spill Calculation:
  – Simulations
  – Passenger mix model
• Problem Modification:
  – Objective cost coefficient update
Passenger Mix

- Passenger Mix Model (PMIX)
  - Kniker (1998)
  - Given a fixed, fleeted schedule, unconstrained passenger demands by itinerary (requests), and recapture rates find maximum revenue for passengers on each flight leg

Network Effects and Recapture
Problem Modification

• Based on differences in expected spill from FAM and the Spill Calculator, we modify the FAM problem
  – Update objective cost coefficients

• Cost coefficient update, many heuristics possible
FAM Spill Calculation Heuristics

• Fare Allocation
  – Full fare - the full fare is assigned to each leg of the itinerary
  – Partial fare - the fare divided by the number of legs is assigned to each leg of the itinerary
  – Shared fare - the fare divided by the number of \textit{capacitated} legs is assigned to each \textit{capacitated} leg in the itinerary

• Spill Cost for each variable
  – Representative Fare
    • A “spill fare” is calculated; each passenger spilled results in a loss of revenue equal to the spill fare
  – Integration
    • Sort each itinerary by fare, spill costs are sum of $x$ lowest fare passengers, where $x = \max\{0, \text{demand} - \text{capacity}\}$
An Illustrative Example

<table>
<thead>
<tr>
<th>Fleet Type</th>
<th>Seats</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>100</td>
</tr>
<tr>
<td>B</td>
<td>200</td>
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</table>

<table>
<thead>
<tr>
<th>Market</th>
<th>Itinerary</th>
<th>Average Fare</th>
<th>No. of Pax</th>
</tr>
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<tbody>
<tr>
<td>X-Y</td>
<td>1</td>
<td>$200</td>
<td>75</td>
</tr>
<tr>
<td>Y-Z</td>
<td>2</td>
<td>$225</td>
<td>150</td>
</tr>
<tr>
<td>X-Z</td>
<td>1-2</td>
<td>$300</td>
<td>75</td>
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<tbody>
<tr>
<td></td>
<td>Spill</td>
<td>Spill</td>
<td>Spill</td>
</tr>
<tr>
<td>Fl. 1- Fl. 2</td>
<td>$30,000</td>
<td>$38,125</td>
<td>31,875</td>
</tr>
<tr>
<td>A-A</td>
<td>$11,250</td>
<td>$15,625</td>
<td>12,500</td>
</tr>
<tr>
<td>A-B</td>
<td>$22,500</td>
<td>$28,125</td>
<td>28,125</td>
</tr>
<tr>
<td>B-B</td>
<td>$3,750</td>
<td>$5,625</td>
<td>5,625</td>
</tr>
</tbody>
</table>
Spill Calculation: Results

- For a 3 fleet, 226 flights problem:
  - The best representative fare solution results in a gap with the optimal solution of $2,600/day
  - Using a shared fare scheme and integration approach, we found a solution with an $8/day gap.
- By simply modifying the basic spill model, significant gains can be achieved
Itinerary-Based Fleet Assignment

• Impossible to estimate airline profit exactly using link-based costs
• Enhance basic fleet assignment model to include passenger flow decision variables
  – Associate operating costs with fleet assignment variables
  – Associate revenues with passenger flow variables
Itinerary-based Fleet Assignment Definition

- Given
  - a fixed schedule,
  - number of available aircraft of different types,
  - unconstrained passenger demands by itinerary, and
  - recapture rates,

Find maximum contribution

- Network effects
Itinerary-Based FAM (IFAM)

Fleet Assignment

Consistent Spill + Recapture

\[ t^r_p \geq 0 \quad f_{k,i} \in \{0,1\} \quad y_{k,o,t} \geq 0 \]

Kniker (1998)
Itinerary-Based FAM (IFAM)

\[
\text{Min} \sum_{k \in K} \sum_{i \in L} \delta_{k,i} f_{k,i} + \sum_{p \in P} \sum_{r \in P} (\text{fare}_p - b_p \text{fare}_r) t_p^r
\]

Subject to:

\[
\sum_{k \in K} f_{k,i} = 1 \quad \forall i \in L
\]

\[
y_{k,o,t}^- + \sum_{i \in I(k,o,t)} f_{k,i} - y_{k,o,t}^+ - \sum_{i \in O(k,o,t)} f_{k,i} = 0 \quad \forall k,o,t
\]

\[
\sum_{o \in O} y_{k,o,t_n} + \sum_{i \in CL(k)} f_{k,i} \leq N_k \quad \forall k \in K
\]

\[
\sum_{k} f_{k,i} SEATS_k + \sum_{r \in P} \sum_{p \in P} \delta_i^p t_p^r - \sum_{r \in P} \sum_{p \in P} \delta_i^p b_p^r t_p^r \geq Q_i \quad \forall i \in L
\]

\[
\sum_{r \in P} t_p^r \leq D_p \quad \forall p \in P
\]

\[
t_p^r \geq 0 \quad f_{k,i} \in \{0,1\} \quad y_{k,o,t} \geq 0
\]

Kniker (1998)
Column and Constraint Generation

Original RMP

1

2

3

4

5

6

7

8
Implementation Details

• Computer
  – Workstation class computer
  – 2 GB RAM
  – CPLEX 6.5

• Full size schedule
  – ~2,000 legs
  – ~76,000 itineraries
  – ~21,000 markets
  – 9 fleet types

• RMP constraint matrix size
  – ~77,000 columns
  – ~11,000 rows

• Final size
  – ~86,000 columns
  – ~19,800 rows

• Solution time
  – LP: > 1.5 hours
  – IP: > 4 hours

88% Saving from Row Generation
> 95% Saving from Column Generation
Fleet Assignment

Computational Sample: 2,044 flight legs, 9 fleet types

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<tr>
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<th>I FAM</th>
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<tbody>
<tr>
<td>Root node LP solution</td>
<td>21,401,658</td>
<td>21,302,460</td>
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<td>Best IP solution</td>
<td>21,401,622</td>
<td>21,066,811</td>
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<tr>
<td>Difference</td>
<td>36</td>
<td>235,649</td>
</tr>
</tbody>
</table>

| Solution time [sec]    | 974          | >100,000      |
| Contribution [$/day]   | 21,178,815   | 21,066,811    |
IFAM Contributions

• Annual improvements over basic FAM
  – Network Effects: $30 million
  – Recapture: $70 million
• These estimates are upper bounds on achievable improvements
Subnetwork-Based FAM

- IFAM has limited opportunity for expansion to include schedule design decisions
  - Fractionality of solution to LP relaxation is a big issue

- Need alternative fleet assignment kernel
  - Capture network effects
  - Maintains tractability
Basic Concept

- Isolate network effects
  - Spill occurs only on *constrained legs*

- < 30% of total legs are potentially constrained
- < 6% of total itineraries are potentially binding
Modeling Challenges

- Utilize composite variables (Armacost, 2000; Barnhart, Farahat and Lohatepanont, 2001)

<table>
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<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<th>7</th>
<th>8</th>
<th>9</th>
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<tbody>
<tr>
<td></td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>B</td>
<td>B</td>
<td>B</td>
<td>C</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td></td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>A</td>
<td>B</td>
<td>C</td>
</tr>
</tbody>
</table>

3 Fleet Types: A, B, and C

- **Challenges**
  - Efficient **column enumeration**
SFAM Formulation

\[
\text{Min} \sum_{m=1}^{M^s} \sum_{n=1}^{r_{n,s}^m} \left( C_{n,s}^m \right) \left( f_{n,s}^m \right) \\
\sum_{m=1}^{M^s} \sum_{n=1}^{r_{n,s}^m} \left( \delta_{n,s}^m \right) \left( f_{n,s}^m \right) = 1 \quad \forall i \in L \\
y_{k,o,t} + \sum_{i \in I(k,o,t)} \sum_{m=1}^{M^s} \sum_{n=1}^{r_{n,s}^m} \left( K_{n,s}^m \right) k,i \left( f_{n,s}^m \right) - y_{k,o,t} - \sum_{i \in O(k,o,t)} \sum_{m=1}^{M^s} \sum_{n=1}^{r_{n,s}^m} \left( K_{n,s}^m \right) k,i \left( f_{n,s}^m \right) = 0 \quad \forall k, o, t \\
\sum_{o \in A} y_{k,o,t} + \sum_{i \in CL(k)} \sum_{m=1}^{M^s} \sum_{n=1}^{r_{n,s}^m} \left( y_{n,s}^m \right) k \left( f_{n,s}^m \right) \leq N_k \quad \forall k \in K \\
\left( f_{n,s}^m \right) \in \{0,1\} \quad y_{k,o,t} \geq 0
\]

FAM solution algorithm applies
SFAM Results

• Testing performed on full size schedules
  – Runtimes similar to FAM, much faster than itinerary-based approaches
    • Tight LP relaxations
  – SFAM achieve improved solutions relative to FAM and itinerary-based approach

• SFAM has potential for further integration or extension
  – Time windows, stochastic demand, schedule design
Caveats

2. Deterministic Demand

4. Optimal Control of Paxs

3. Demand Forecast Errors

1. Recapture Rate Errors

Recapture Rate
Recapture Rate Sensitivity

Specified Recapture Rate → IFAM

Fleeting Decision

Solve PMix with varied recapture rates

Estimated Revenue

Fleeting Contribution

Operating Cost

PMix flows passengers on fleeted schedule assuming full knowledge of passenger choices
Recapture Rate Sensitivity

- Sensitivity of IFAM
- Improvement gained from network effects alone
- Improvement gained from network effects and recapture

Recapture Rate Multiplier ($\delta$)
IFAM Sensitivity Analysis

- **Simulations**
  - Simulate 500 realizations of demand based on Poisson distributions

![Diagram showing IFAM Sensitivity Analysis process](image-url)
### IFAM vs. FAM

#### Demand Stochasticity

Demand deviation ~14%

#### Realizations

<table>
<thead>
<tr>
<th>Problem 1N-3A</th>
<th>FAM</th>
<th>IFAM</th>
<th>Difference (IFAM-FAM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenue</td>
<td>$4,858,089</td>
<td>$4,918,691</td>
<td>$60,602</td>
</tr>
<tr>
<td>Operating Cost</td>
<td>$2,020,959</td>
<td>$2,021,300</td>
<td>$341</td>
</tr>
<tr>
<td>Contribution</td>
<td>$2,837,130</td>
<td>$2,897,391</td>
<td>$60,261</td>
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</table>

<table>
<thead>
<tr>
<th>Problem 2N-3A</th>
<th>FAM</th>
<th>IFAM</th>
<th>Difference (IFAM-FAM)</th>
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</thead>
<tbody>
<tr>
<td>Revenue</td>
<td>$3,526,622</td>
<td>$3,513,996</td>
<td>$(12,626)</td>
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<tr>
<td>Operating Cost</td>
<td>$2,255,254</td>
<td>$2,234,172</td>
<td>$(21,082)</td>
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<tr>
<td>Contribution</td>
<td>$1,271,368</td>
<td>$1,279,823</td>
<td>$8,455</td>
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</tbody>
</table>
IFAM vs. FAM

Demand Stochasticity
Forecast Errors

Data Quality Issue

Model Sensitivity to Demand Forecast Errors

Simulated Demands (% of Forecasted Demand)

Contributions ($ million/day)

-5% -4% -3% -2% -1% 0 +1% +2% +3% +4% +5%

$1.80 $1.70 $1.60 $1.50 $1.40 $1.30 $1.20 $1.10 $1.00
Extending Fleet Assignment Models to Include “Incremental” Schedule Design...
Airline Schedule Planning

Schedule Design

Fleet Assignment

Aircraft Routing

Crew Scheduling

Select optimal set of flight legs in a schedule

Assign aircraft types to flight legs such that contribution is maximized
Schedule Design: Fixed Flight Network, Flexible Schedule Approach

• Fleet assignment model with time windows
  – Allows flights to be re-timed slightly (plus/minus 10 minutes) to allow for improved utilization of aircraft and improved capacity assignments

➢ Initial step in integrating flight schedule design and fleet assignment decisions
**Example: Results**

*Aircraft Utilization*

Do time windows allow a reduction in the number of required aircraft?

<table>
<thead>
<tr>
<th>TW = 0</th>
<th></th>
<th>TW = 20</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a/c req'd</td>
<td>cost</td>
<td>a/c req'd</td>
<td>cost</td>
</tr>
<tr>
<td>P1</td>
<td>365</td>
<td>28,261,302</td>
<td>363</td>
</tr>
<tr>
<td>P2</td>
<td>428</td>
<td>29,000,175</td>
<td>426</td>
</tr>
</tbody>
</table>
Results

• Time windows can provide significant cost savings, as well as a potential for freeing aircraft
  – $50 million in operating costs alone for one U.S. airline
Schedule Design: Optional Flights, Flexible Schedule Approach

• Fleet assignment with “optional” flight legs
  – Additional flight legs representing varying flight departure times
  – Additional flight legs representing new flights
  – Option to eliminate existing flights from future flight network

Incremental Schedule Design
### Demand and Supply Interactions

#### Market Share

<table>
<thead>
<tr>
<th>Market Share</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>450</td>
<td>100</td>
<td>150</td>
</tr>
<tr>
<td>410</td>
<td>100</td>
<td>190</td>
</tr>
<tr>
<td>300</td>
<td>100</td>
<td>200</td>
</tr>
</tbody>
</table>

#### Non-Linear Interactions

- 40
- 150

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3/9/2006
Schedule Design: Results

• Demand and supply interactions
  – Tractability potentially a big issue

• Resulting schedules operate fewer flights
  – Lower operating costs
  – Fewer aircraft required

• Order of magnitude impact: \( \sim $100 - $350 \) million improvement annually for variable market demand
  – Rough estimates: sensitive to quality of data, spill and recapture assumptions, demand forecasts and stochasticity
  – Comparison to \textit{planners’ schedules}
  – Excludes benefits from saved aircraft