Issues in Optimization

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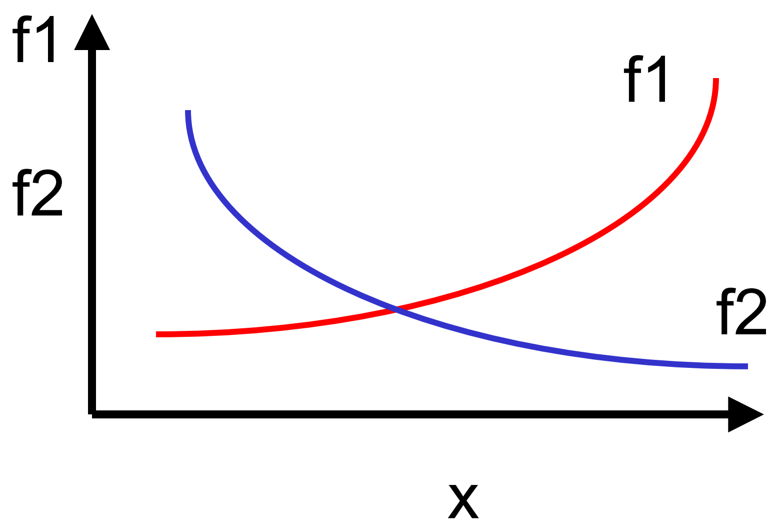
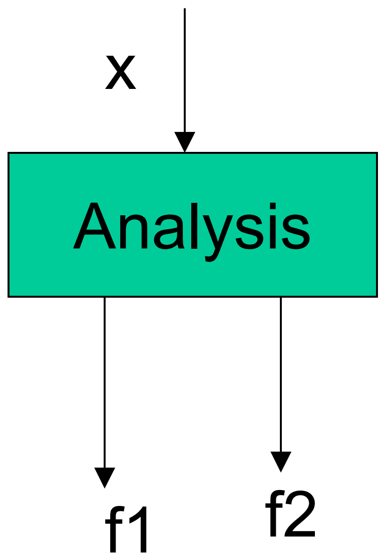
How to know whether optimization is needed

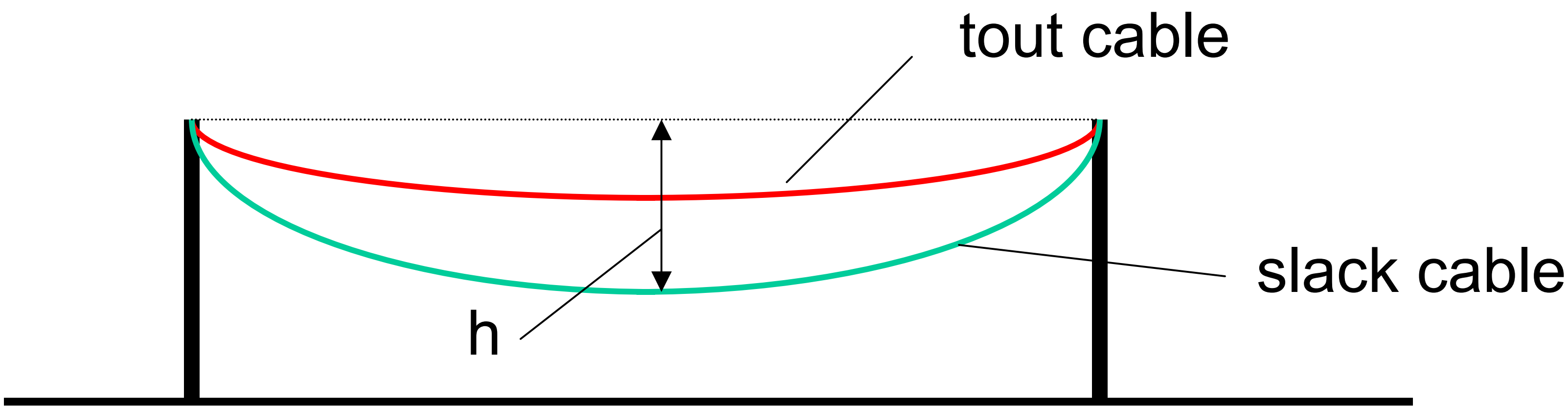
How to recognize that the problem at hand needs optimization.

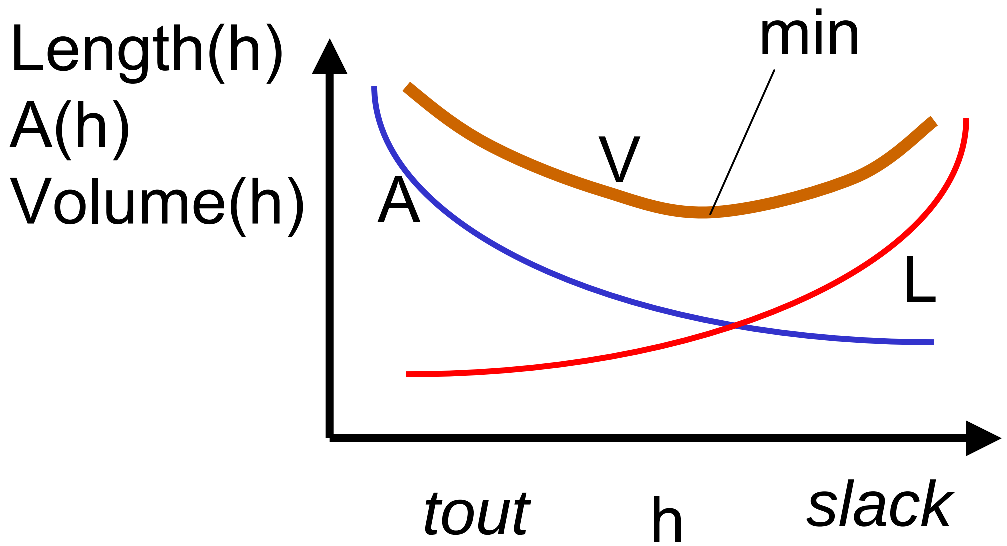
• General Rule of the Thumb:
there must be at least two opposing trends as functions of a design variable





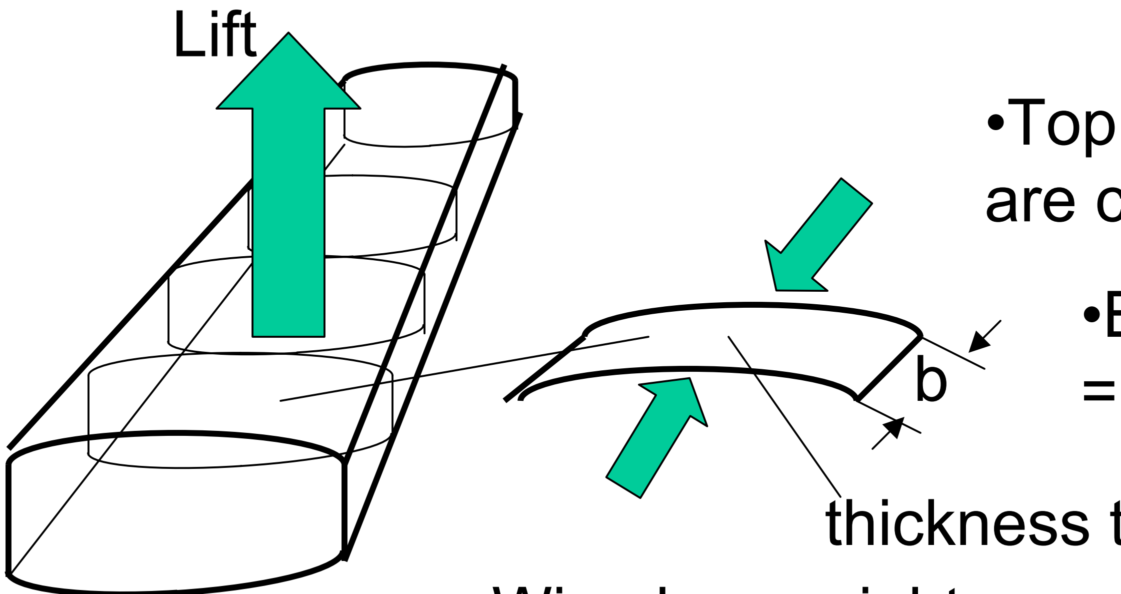
Power Line Cable



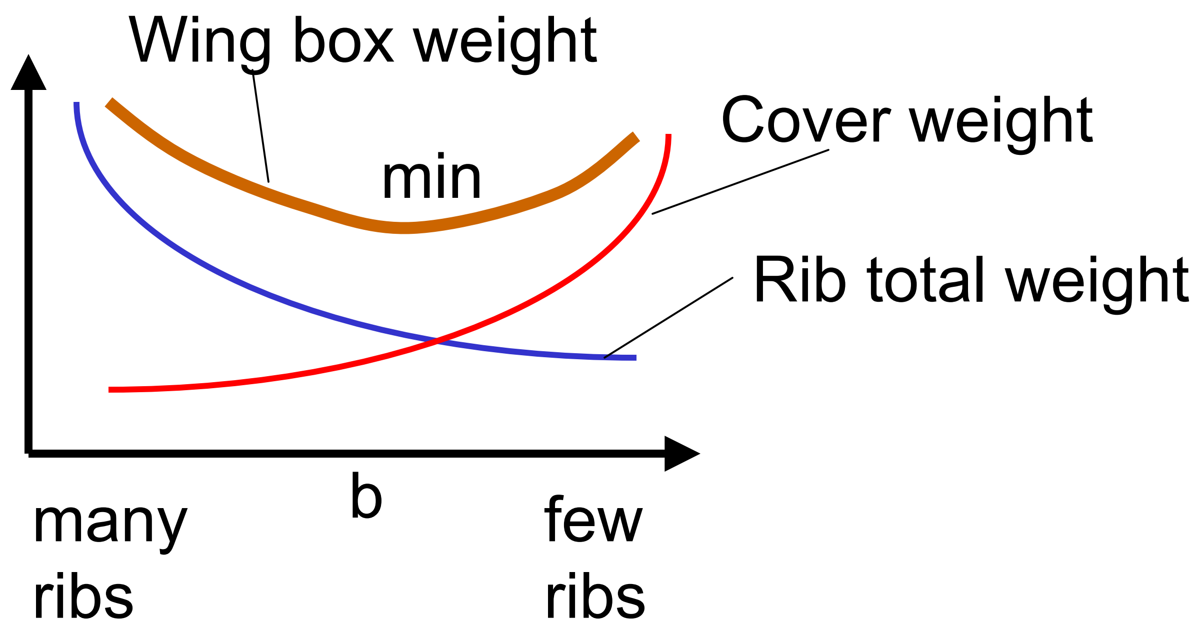


- Given:
  - Ice load
  - self-weight small
  - h/span small

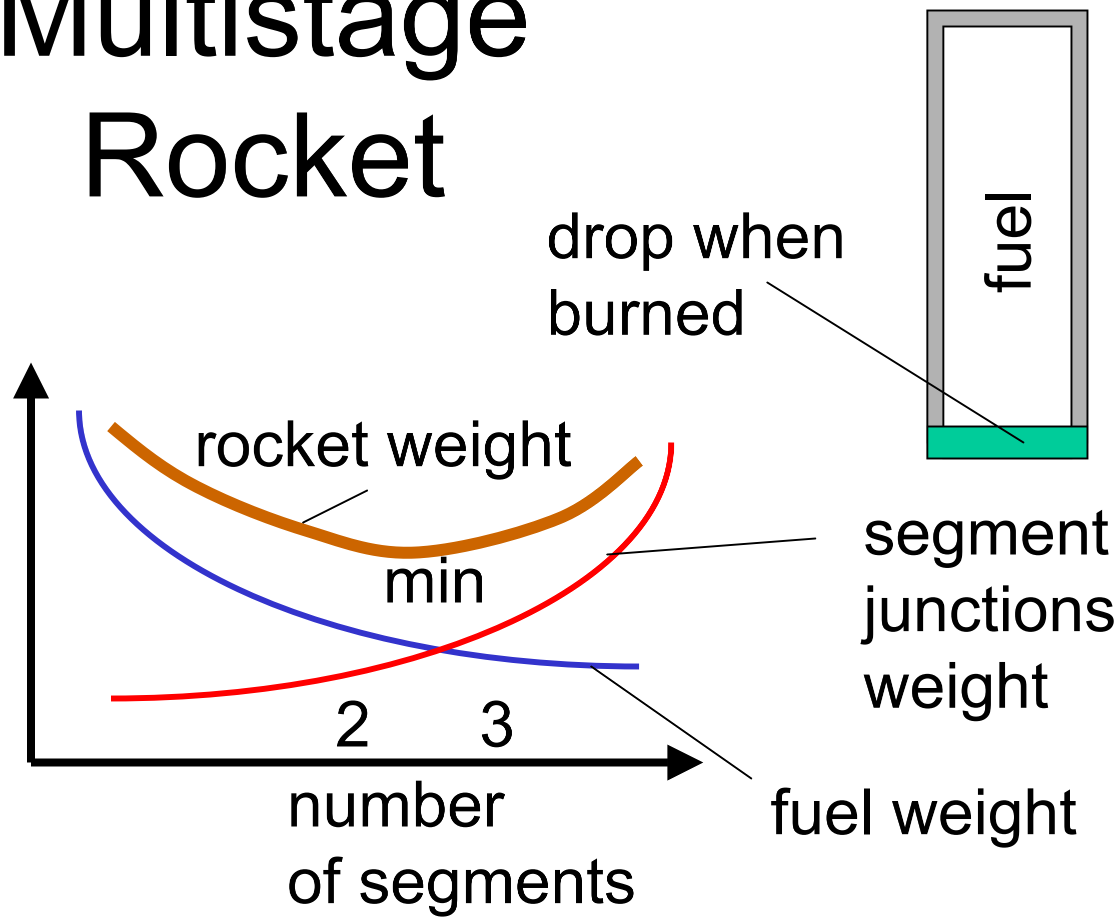
Wing Thin-Walled Box



- Top cover panels are compressed
- Buckling stress = $f(t/b)^2$



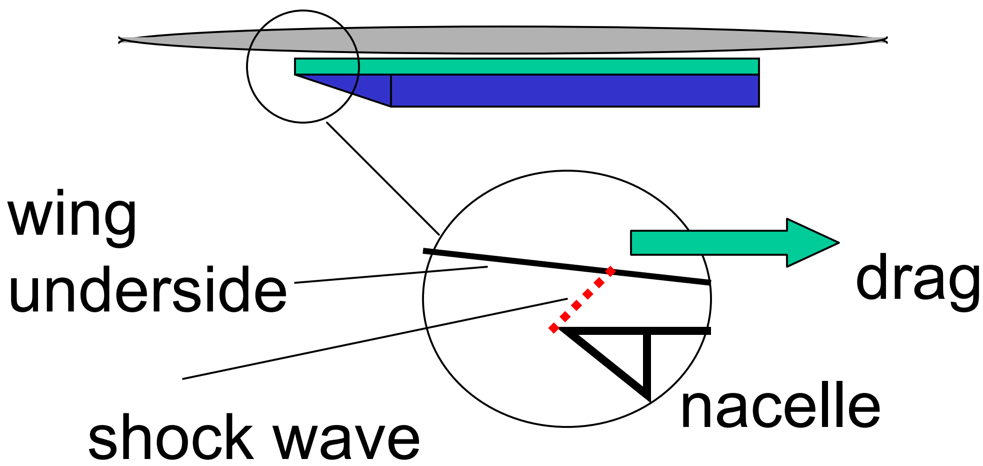
Multistage Rocket

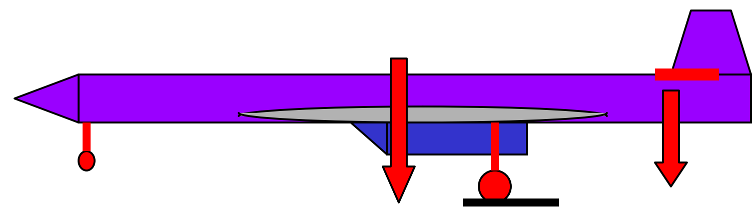


- More segments (stages) = less weight to carry up = less fuel
- More segments = more junctions = more weight to carry up
- Typical optimum: 2 to 4.

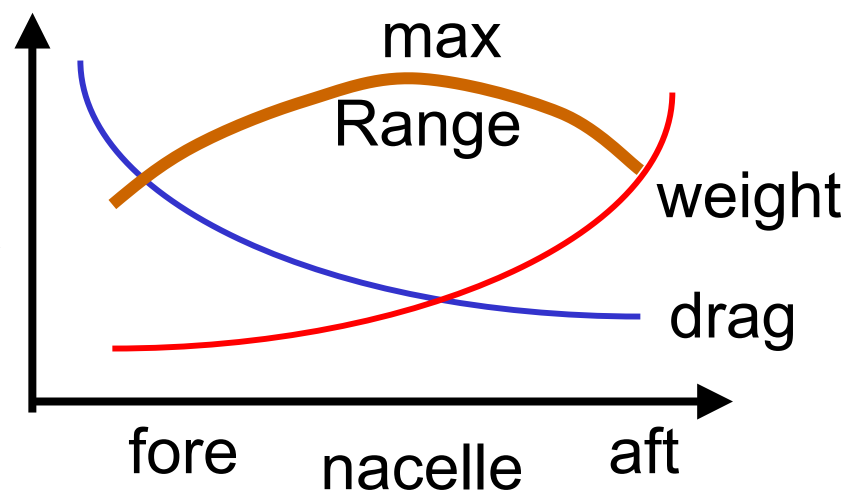
Saturn V

Under-wing Nacelle Placement

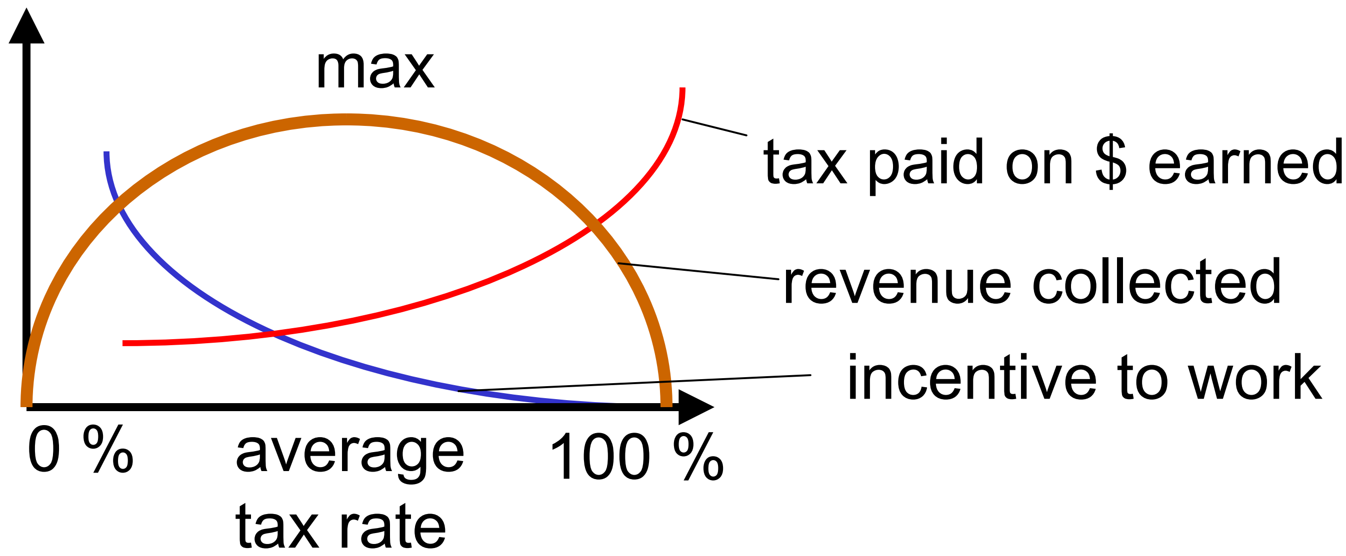




- Inlet ahead of wing max. depth = shock wave impinges on forward slope = drag
- Nacelle moved aft = landing gear moves with it = larger tail (or longer body to rotate for take-off = more weight

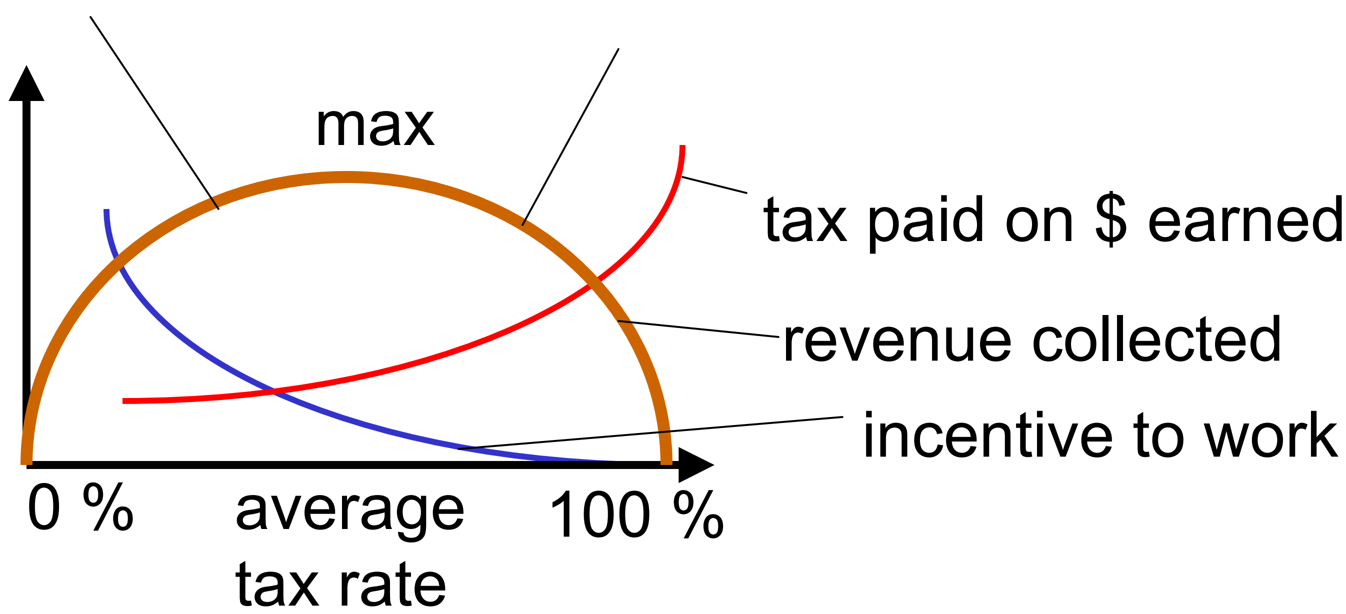


National Taxation



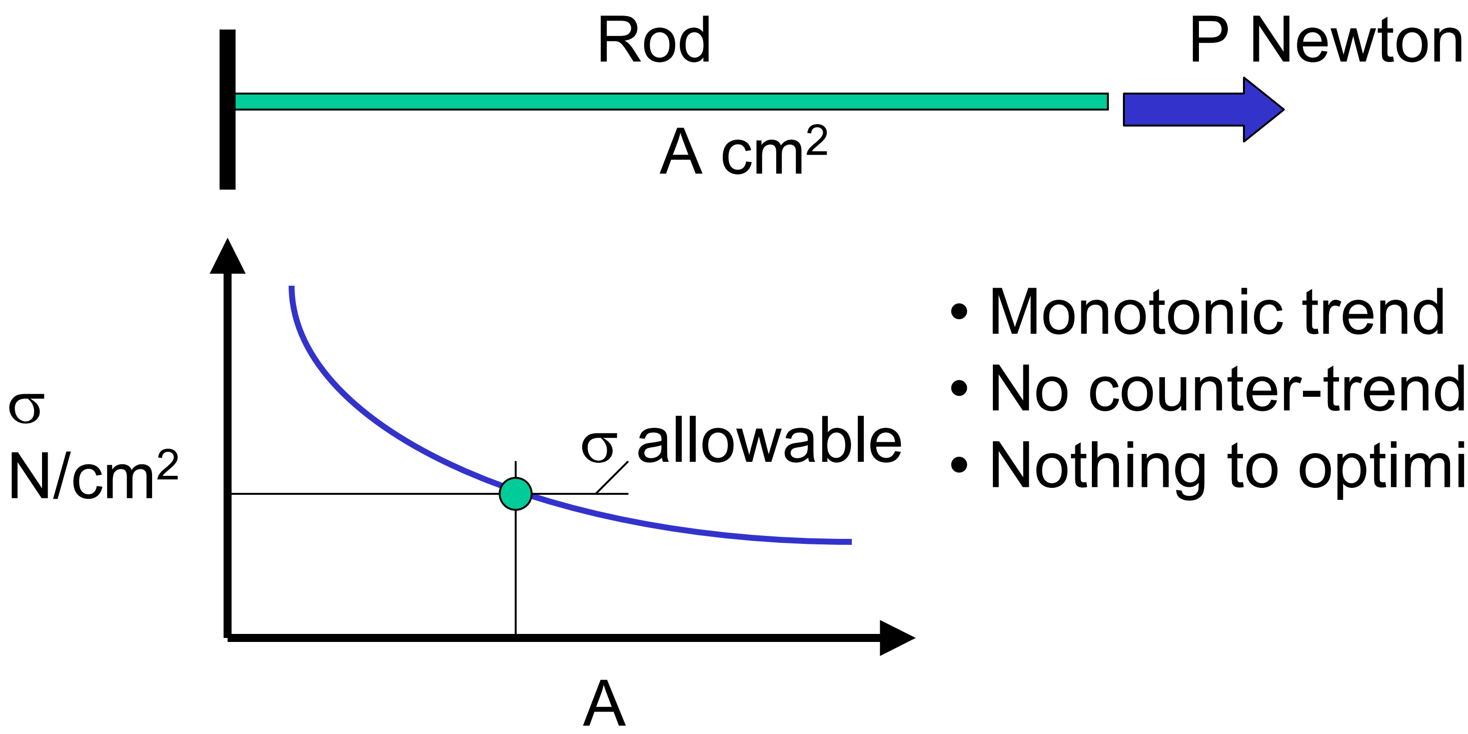
- More tax/last $ = less reason to strive to earn
- More tax/$ = more $ collected per “unit of economic activity”

National Taxation



- More tax/last $ = less reason to strive to earn
- More tax/$ = more $ collected per “unit of economic activity”
- What to do:
    - If we are left of max = increase taxes
    - If we are right of max = cut taxes

Nothing to Optimize

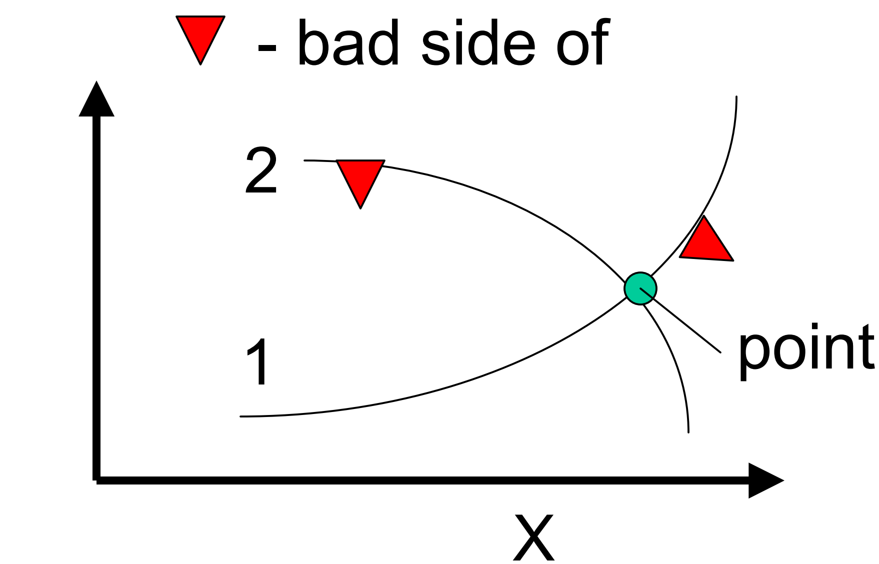


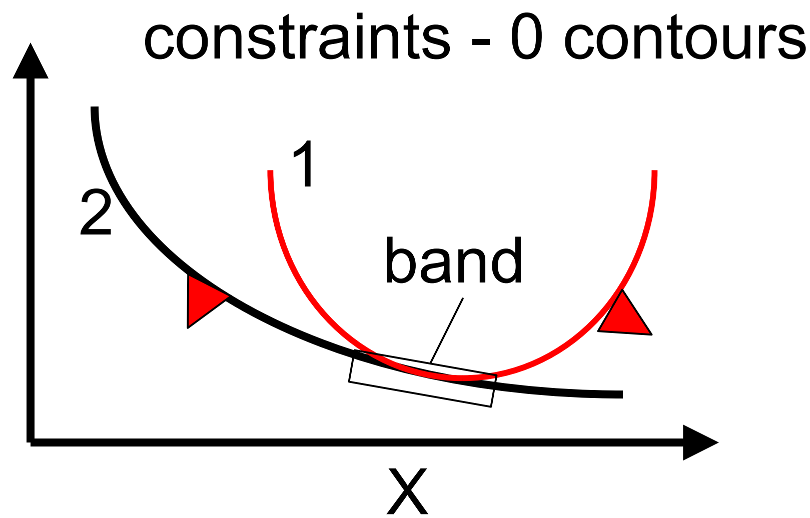
- Monotonic trend
- No counter-trend
- Nothing to optimize

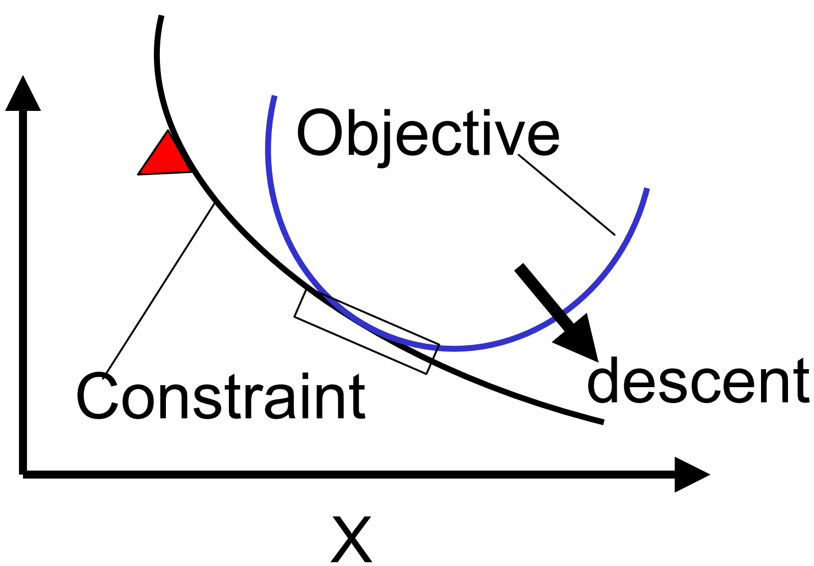
Various types of design optima

Design Definition: Sharp vs. Shallow

constraints - 0 contours



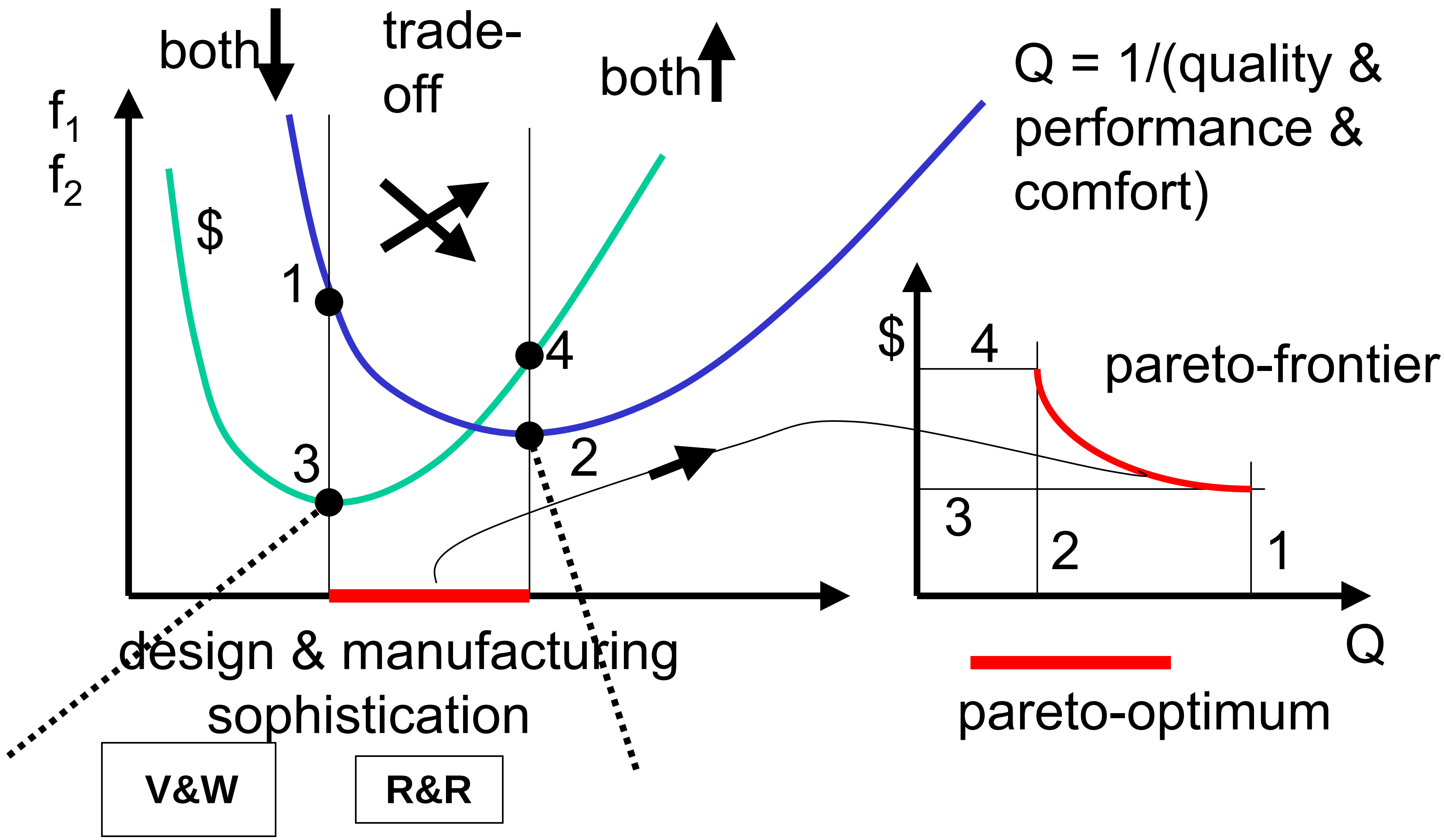




- Near-orthogonal intersection defines a design point

- Tangential definition identifies a band of of designs

Multiobjective Optimization



A Few Pareto-Optimization Techniques

- Reduce to a single objective: $F = \Sigma_i w_i f_i$

where w's are judgmental weighting factors

- Optimize for f_1; Get $f^*_{1;}$;
  - Set a floor $f_1 >= f^*_i$; Optimize for f_2; get f_2 ;
  - Keep floor f_1, add floor f_2 ; Optimize for f_3 ;
  - Repeat in this pattern to exhaust all f's;
- The order of f's matters and is judgmental

- Optimize for each f_i independently; Get n optimal designs; Find a compromise design equidistant from all the above.

- Pareto-optimization intrinsically depends on judgmental preferences

Imparting Attributes by Optimization

- Changing w_i in $F = \Sigma_i w_i f_i$ modifies the design within broad range

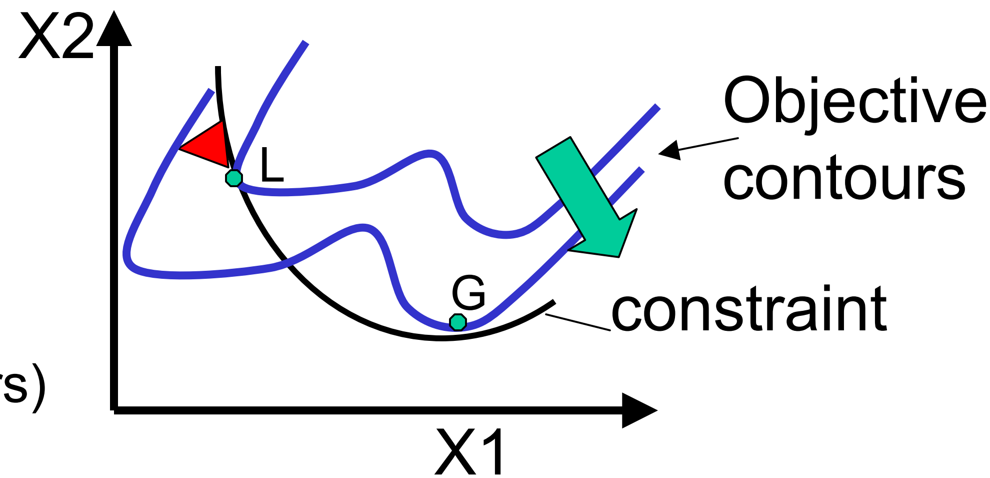
- Example: Two objectives
  - setting $w_1 = 1$; $w_2 = 0$ produces design whose $F = f_1$
  - setting $w_1 = 0$; $w_2 = 1$ produces design whose $F = f_2$
  - setting $w_1 = 0.5$; $w_2 = 0.5$ produces design whose F is in between.

- Using w_i as control, optimization serves as a tool to “steer” the design toward a desired behavior or having pre-determined, desired attributes.

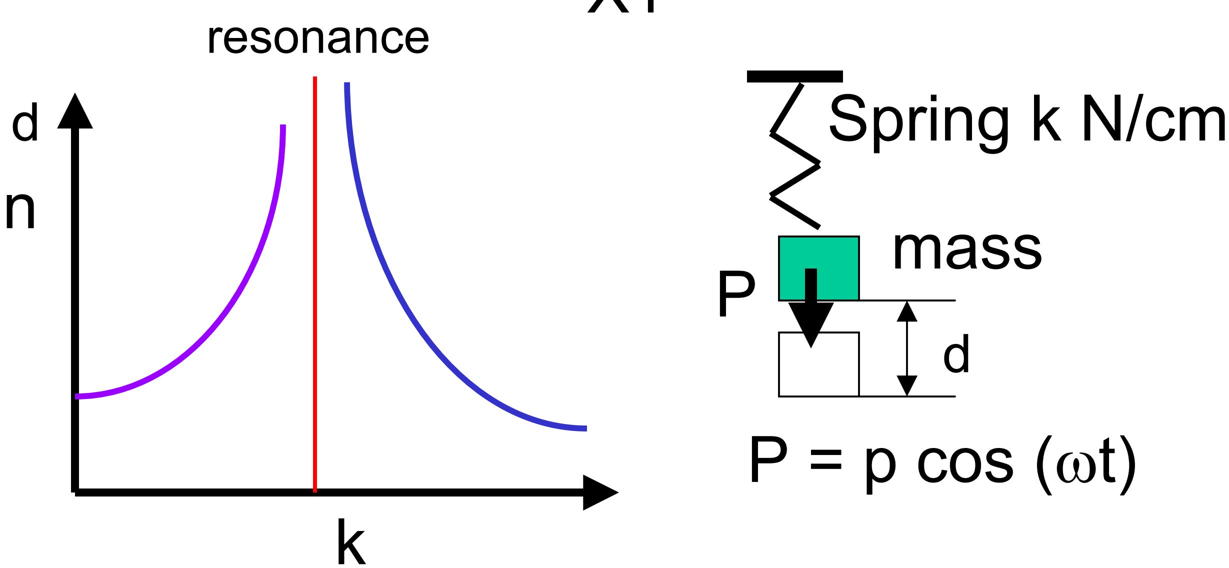
Optimum: Global vs. Local

Why the problem:

- Nonconvex objective or constraints (wiggly contours)



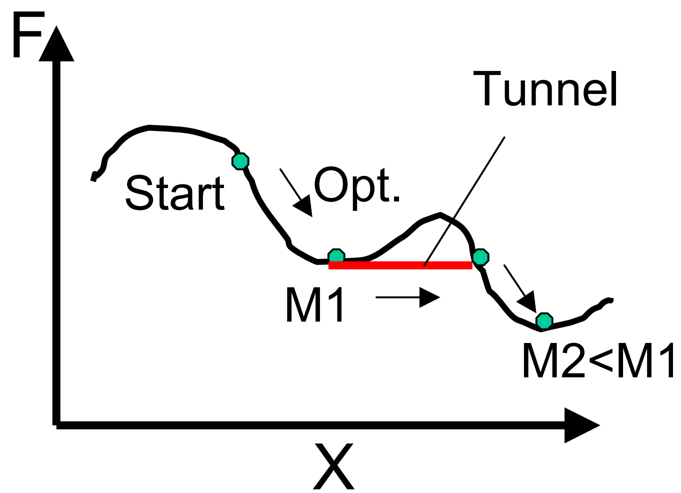
- Disjoint design space



- Local information, e.g., derivatives, does not distinguish local from global optima - the Grand Unsolved Problem in Analysis

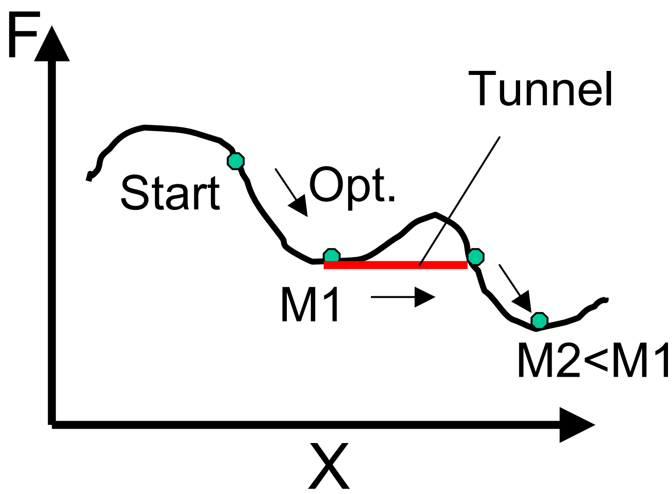
What to do about it

A “shotgun” approach:



•“Tunneling” algorithm finds a better minimum

What to do about it



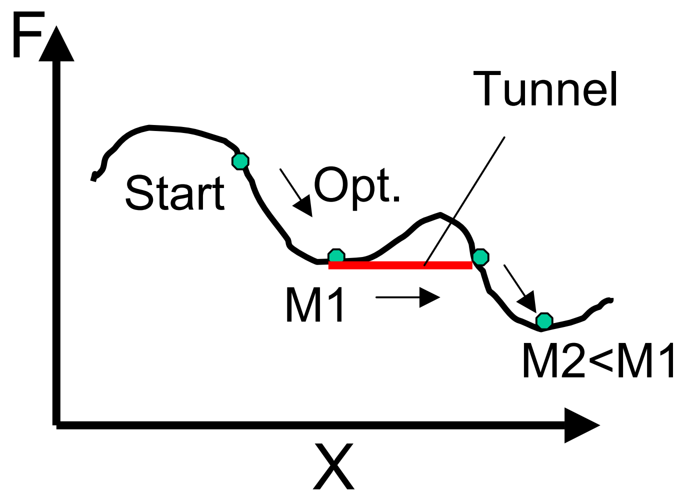
•"Tunneling" algorithm finds a better minimum

shotgun

Multiprocessor computer

What to do about it

A "shotgun" approach:



•"Tunneling" algorithm finds a better minimum

- Use a multiprocessor computer
- Start from many initial designs
- Execute multipath optimization
- Increase probability of locating global minimum
- Probability, no certainty
- Multiprocessor computing = analyze many in time of one = new situation = can do what could not be done before.

Using Optimization to Impart Desired Attributes

Larger scale example: EDOF = 11400;

Des. Var. = 126; Constraints = 24048;

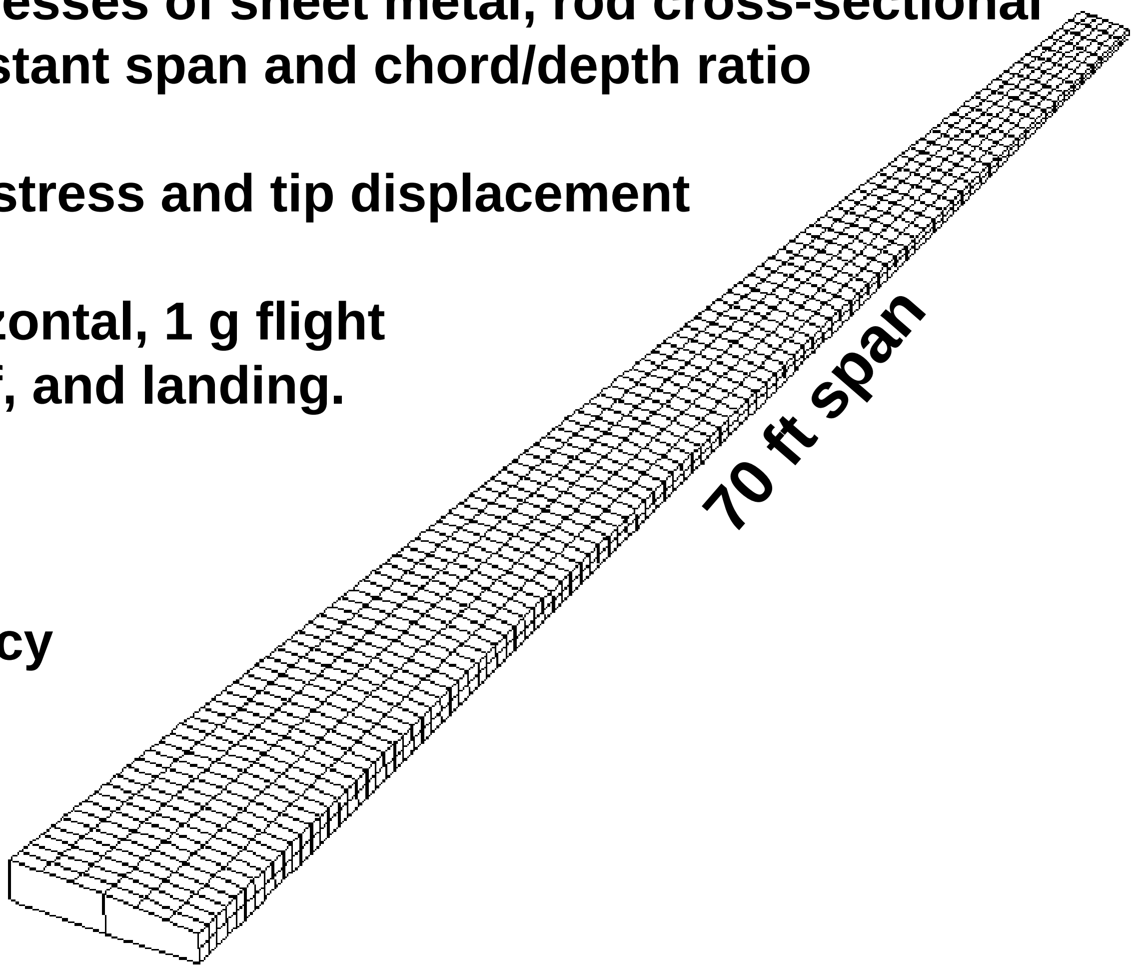
Built-up, trapezoidal, slender transport aircraft wing

- Design variables: thicknesses of sheet metal, rod cross-sectional areas, inner volume (constant span and chord/depth ratio

- Constraints: equivalent stress and tip displacement

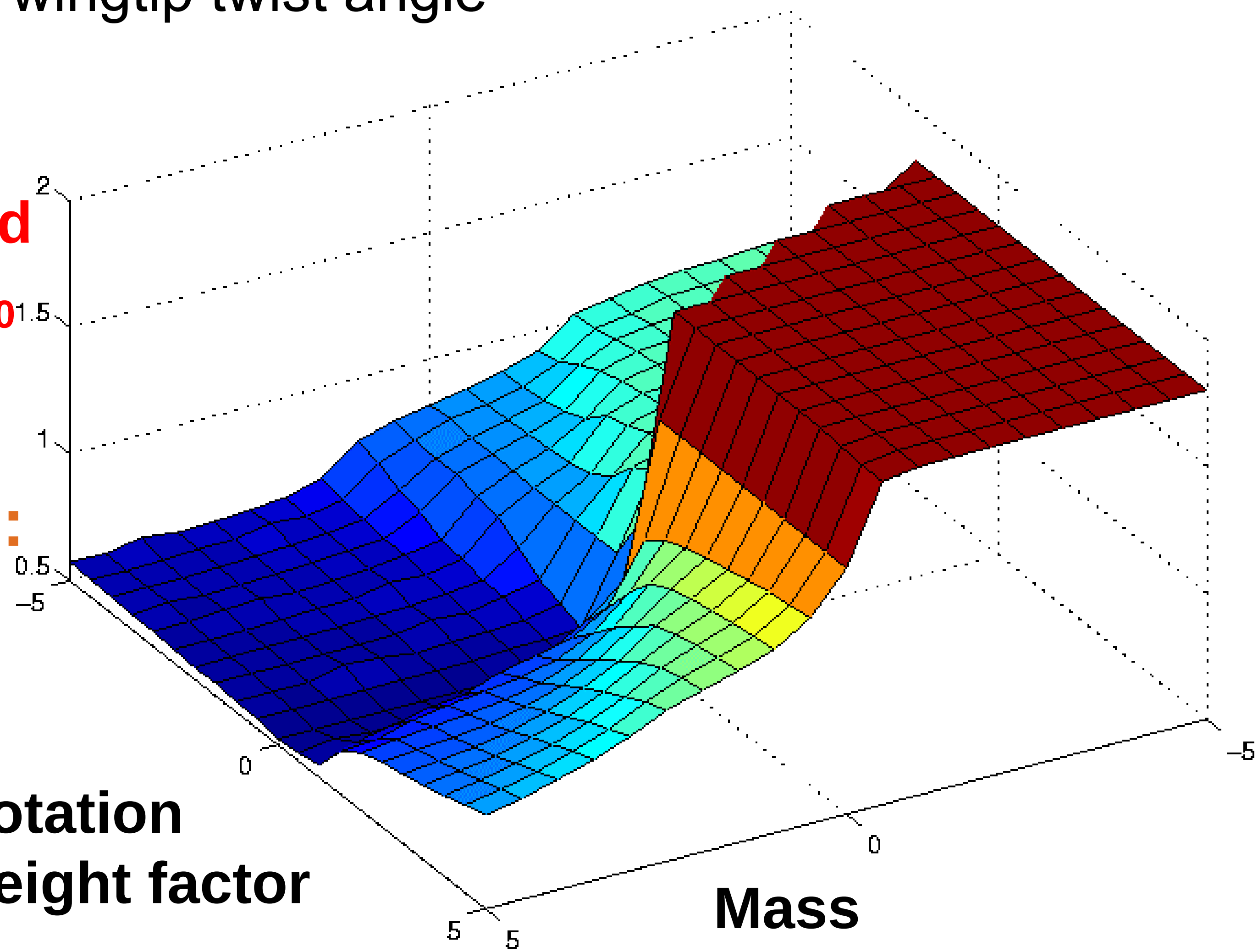
- Two loading cases: horizontal, 1 g flight with engine weight relief, and landing.

- Four attributes:
    - structural mass
    - 1st bending frequency
    - tip rotation
    - internal volume



Case : $F = w_1 (M/M_0) + w_2 (Rotat/Rotat_0)$

Rotat = wingtip twist angle

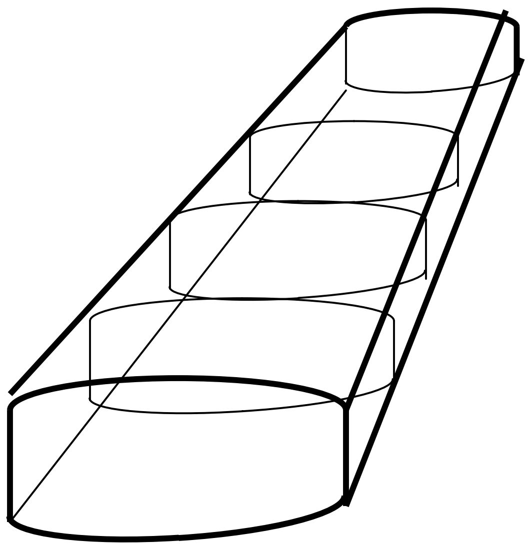


- **Broad variation: 52 % to 180 %**

Optimization Crossing the Traditional Walls of Separation

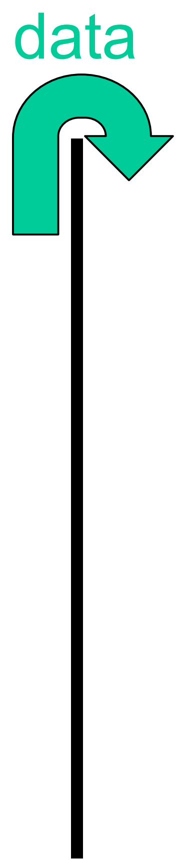
Optimization Across Conventional Barriers

Vehicle design

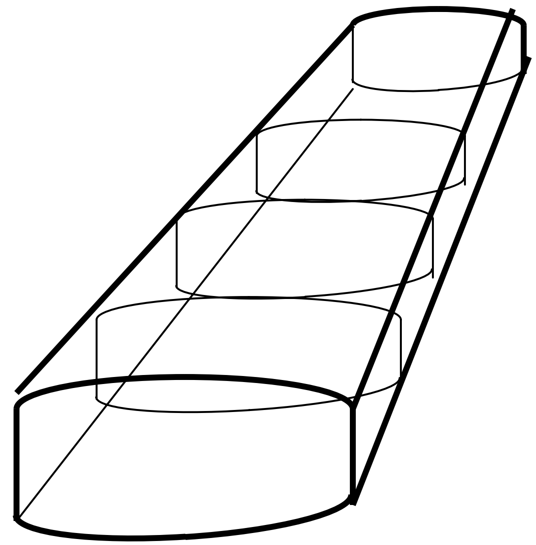


• Focus on vehicle physics and variables directly related to it

• E.g, range;
  wing aspect ratio



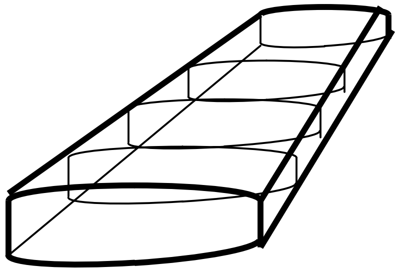
Fabrication



• Focus on manufacturing process and its variables

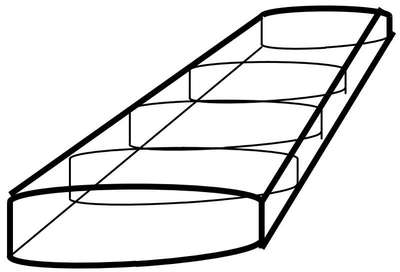
• E.g., cost;
  riveting head speed

Two Loosely Connected Optimizations



•Seek design variables
to maximize performance
under constraints of:

Physics
Cost
Manufacturing difficulty



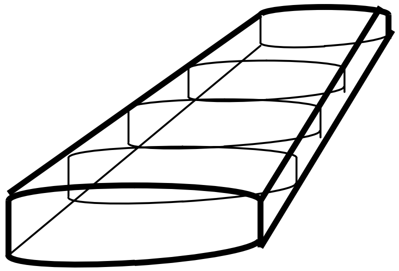
• Seek process variables
to reduce the fabrication cost.

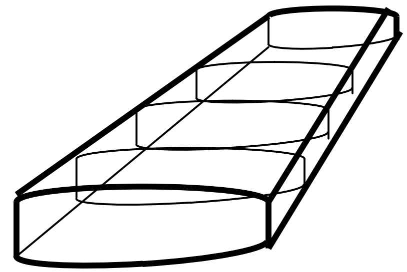
The return on investment (ROI) is a unifying factor

ROI = f(Performance, Cost of Fabrication)

Integrated Optimization

- Required: Sensitivity analysis on both sides





∂Range/ ∂(AspectRatio)

∂Cost/ ∂(Rivet head speed)

∂(Rivet head speed)/ ∂(AspectRatio)

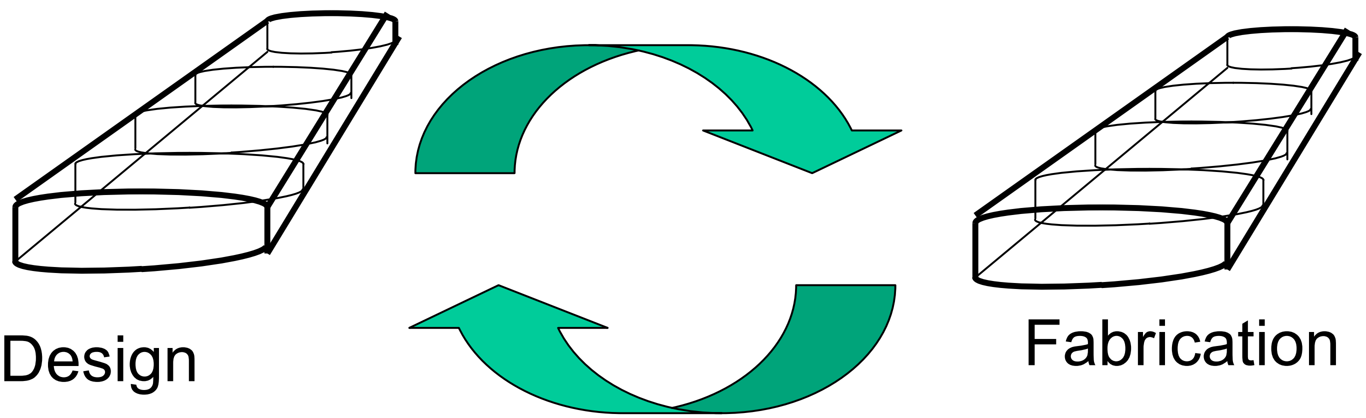
ROI = f(Range, Cost of Fabrication)

∂ROI/ ∂AspectRatio = ∂ROI/ ∂Cost ∂Cost/ ∂(Rivet h.s.) ∂(Rivet h.s)/ ∂(AspectRatio) +

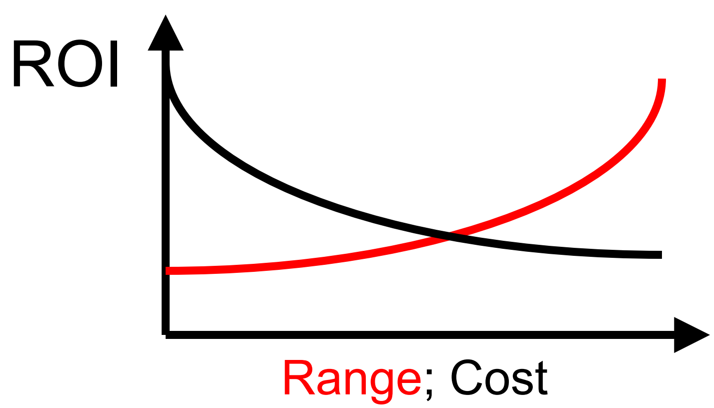
+ ∂(ROI)/∂Range ∂Range/∂(AspectRatio)

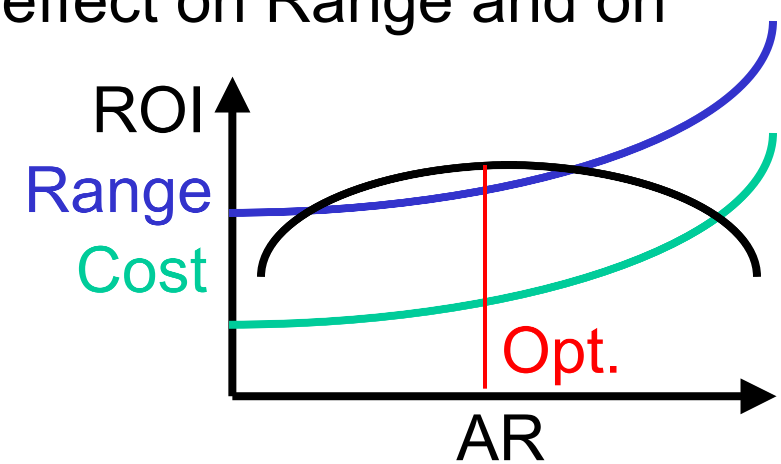
Integrated Optimization Design < --- > Fabrication

- Given the derivatives on both sides



- Unified optimization may be constructed to seek vehicle design variable, e.g., AspectRatio, for maximum ROI incorporating AR effect on Range and on fabrication cost.





Optimization Applied to Complex Multidisciplinary Systems

Multidisciplinary Optimization MDO

Coupling

Decomposition

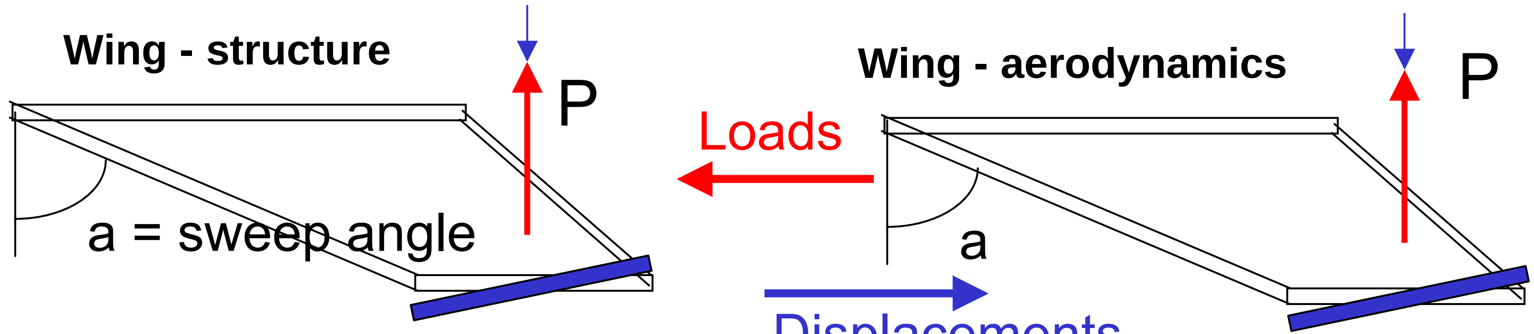
What to optimize for at the discipline level

Approximations

Sensitivity

Wing drag and weight both influence the flight range R.

R is the system objective



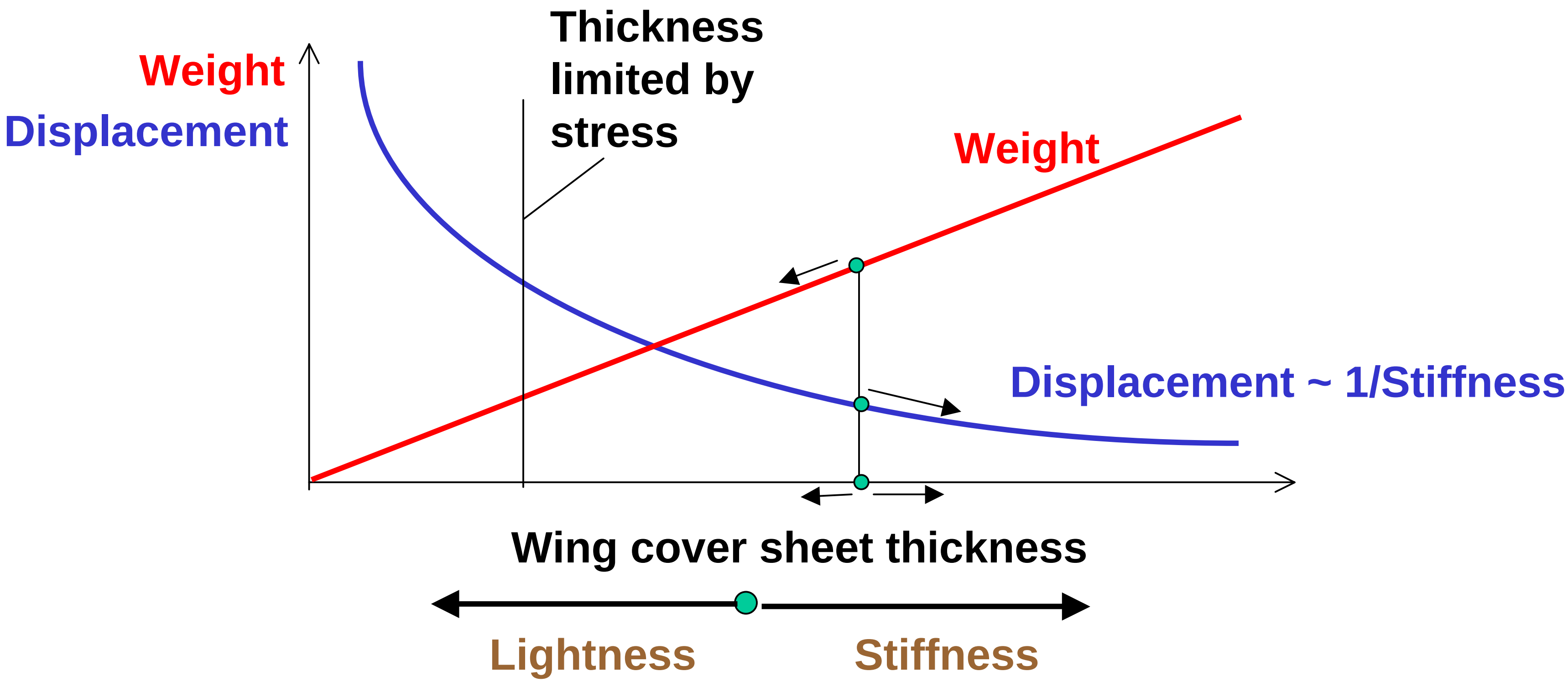
- Structure influences R by
  - directly by weight
  - indirectly by stiffness that affect displacements that affect drag

Loads & Displacements must be consistent

$$R = (k/Drag)\ LOG\ [(W_o + W_s + W_f)/ (W_o + W_s)]$$

- Dilemma: **What to optimize the structure for? Lightness?**
  Displacements = 1/Stiffness?
  An optimal mix of the two?

Trade-off between opposing objectives of lightness and stiffness

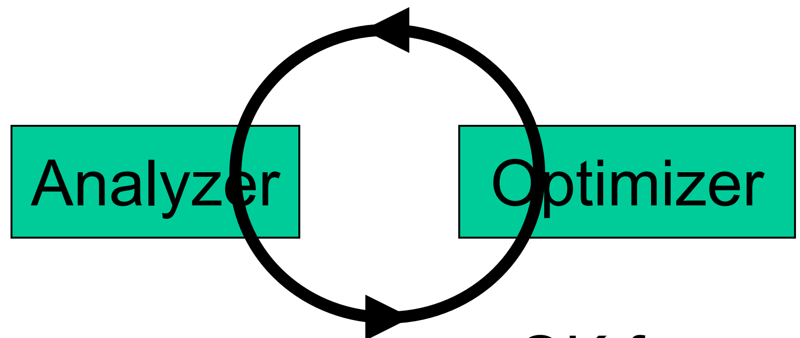


- What to optimize for?
- Answer: minimum of f = w1 Weight + w2 Displacement
- vary w1, w2 to generate a population of wings of diverse Weight/Displacement ratios
- Let system choose w1, w2.

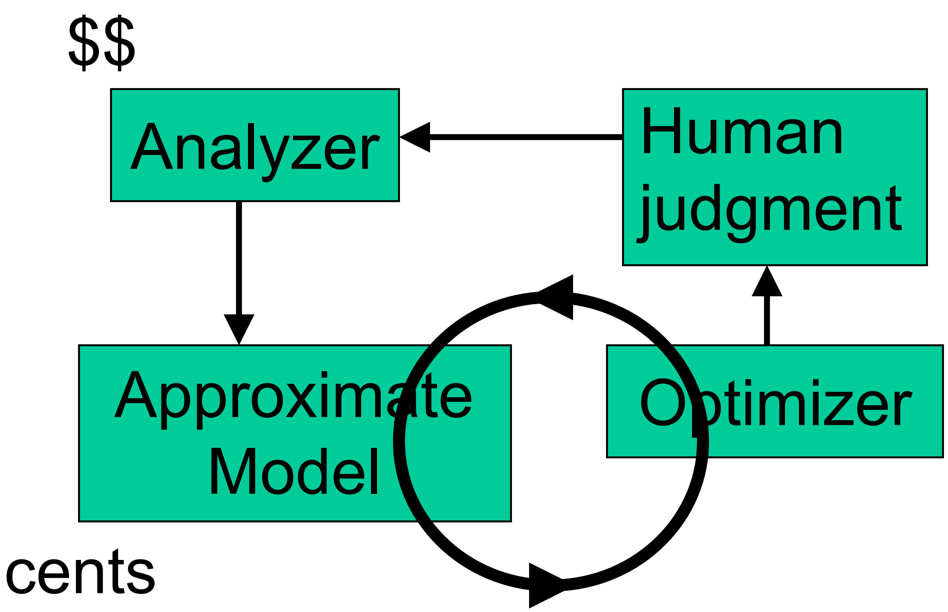
Approximations

- a.k.a. Surrogate Models

•Why Approximations:



- OK for small problems



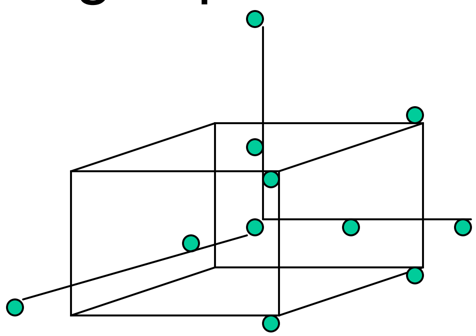
- Now-standard practice for large problems to reduce and control cost

Design of Experiments(DOE) & Response Surfaces (RS)

• RS provides a "domain guidance", rather than local guidance, to system optimizer

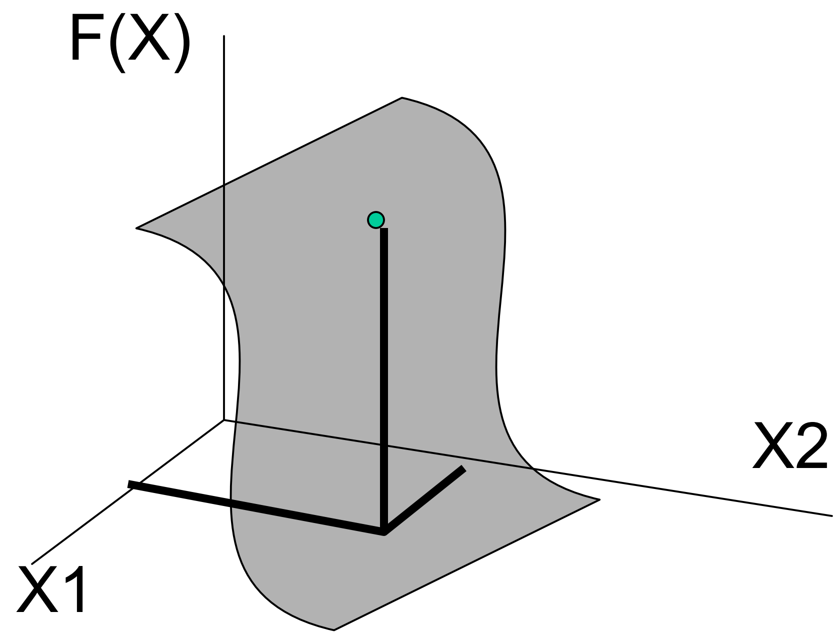
DOE

•Placing design points in design space in a pattern



•Example: Star pattern (shown incomplete)

RS



$F(X) = a + \{b\}'\{X\} + \{X\}'[c]X$

•quadratic polynomial

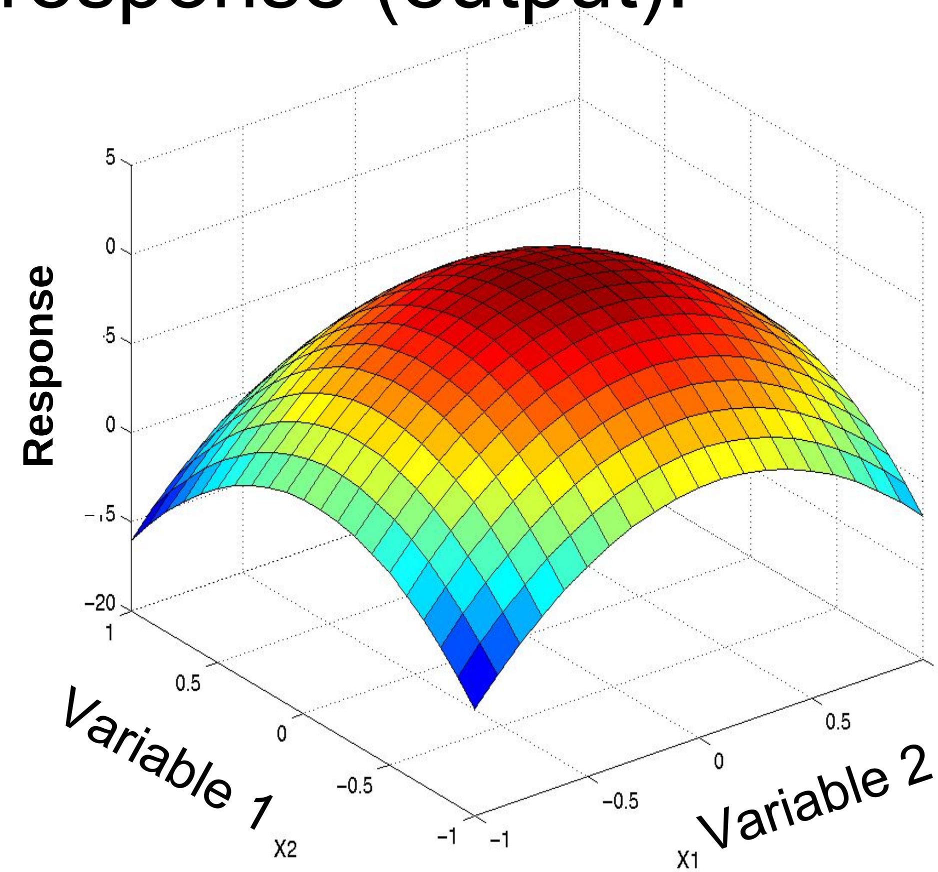
•hundreds of variables

Response Surface Approximation

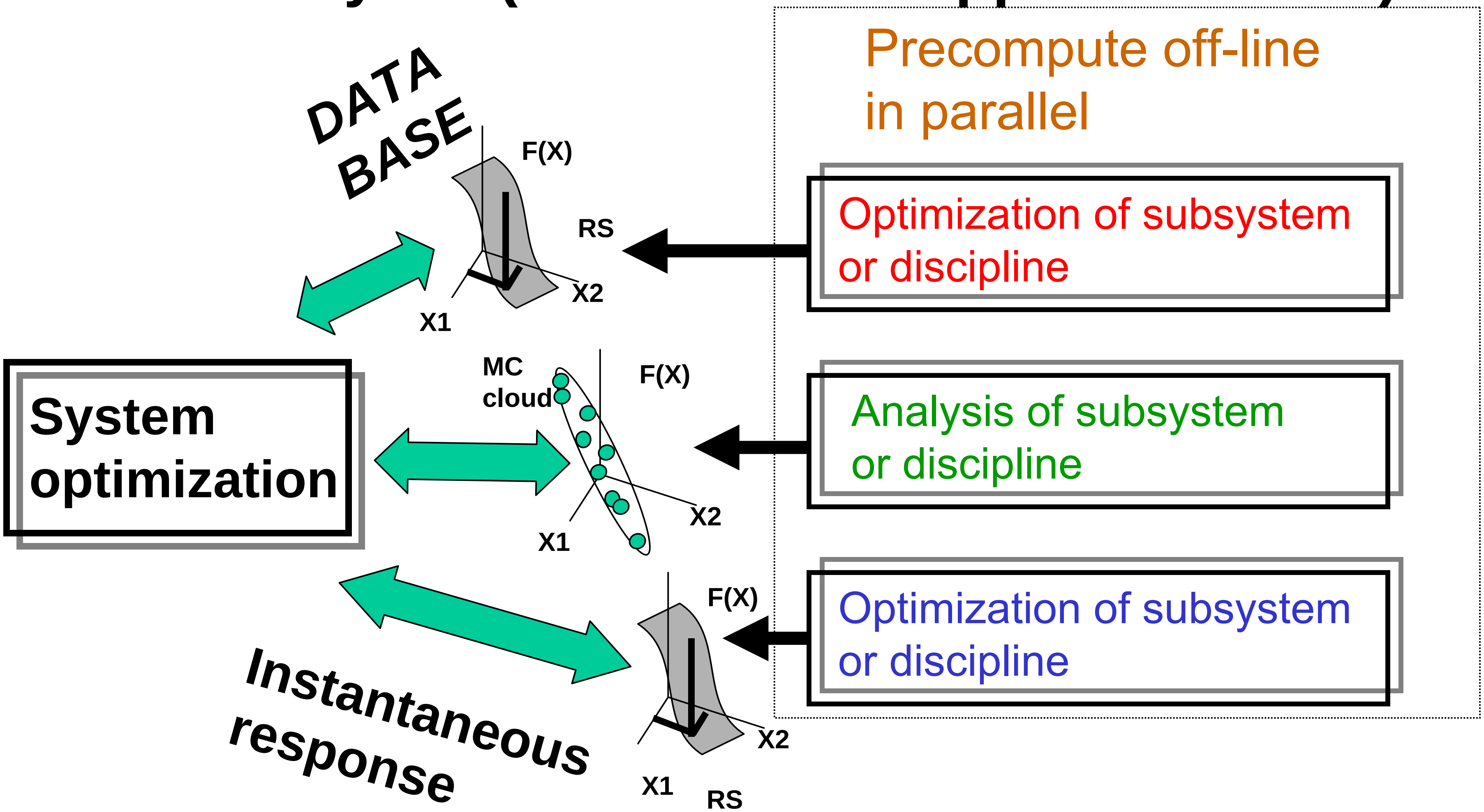
- A Response Surface is an *n*-dimensional hypersurface relating *n* inputs to a single response (output).

- Design of Experiments (DOE) methods used to disperse data points in design space.

- More detail on RS in section on Approximations



BLISS 2000: MDO Massive Computational Problem Solved by RS (or alternative approximations)



- **Radical conceptual simplification at the price of a lot more computing. Concurrent processing exploited.**

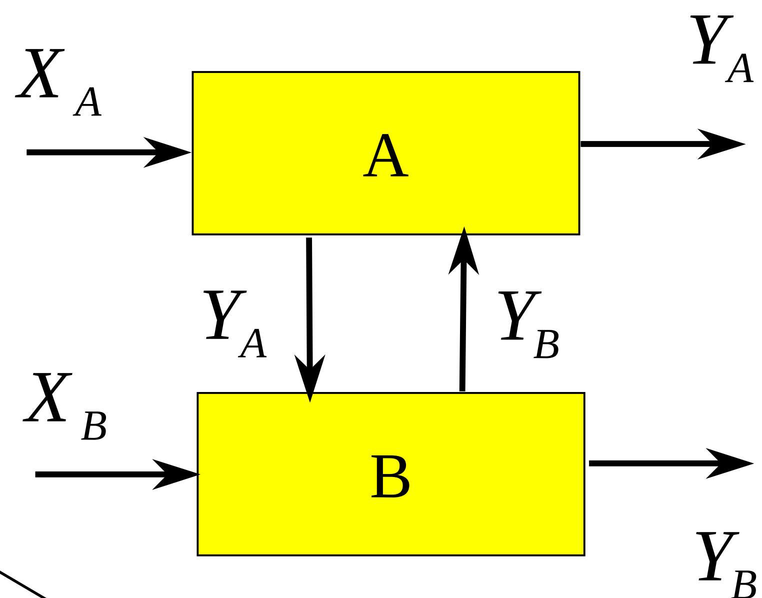
Coupled System Sensitivity

- Consider a multidisciplinary system with two subsystems A and B (e.g. Aero. & Struct.)
  - system equations can be written in symbolic form as

$$A[(X_A, Y_B), Y_A] = 0$$
$$B[(X_B, Y_A), Y_B] = 0$$

  - rewrite these as follows

$$Y_A = Y_A(X_A, Y_B)$$
$$Y_B = Y_B(X_B, Y_A)$$



these governing equations define

as implicit functions.
Implicit Function Theorem applies.

Coupled System Sensitivity - Equations

- These equations can be represented in matrix notation as

$$\begin{bmatrix} I & -\frac{\partial Y_A}{\partial Y_B} \\ -\frac{\partial Y_B}{\partial Y_A} & I \end{bmatrix} \begin{Bmatrix} \frac{dY_A}{dX_A} \\ \frac{dY_B}{dX_A} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial Y_A}{\partial X_A} \\ 0 \end{Bmatrix}$$

$$\begin{bmatrix} I & -\frac{\partial Y_A}{\partial Y_B} \\ -\frac{\partial Y_B}{\partial Y_A} & I \end{bmatrix} \begin{Bmatrix} \frac{dY_A}{dX_B} \\ \frac{dY_B}{dX_B} \end{Bmatrix} = \begin{Bmatrix} 0 \\ \frac{\partial Y_B}{\partial X_B} \end{Bmatrix}$$

same matrix

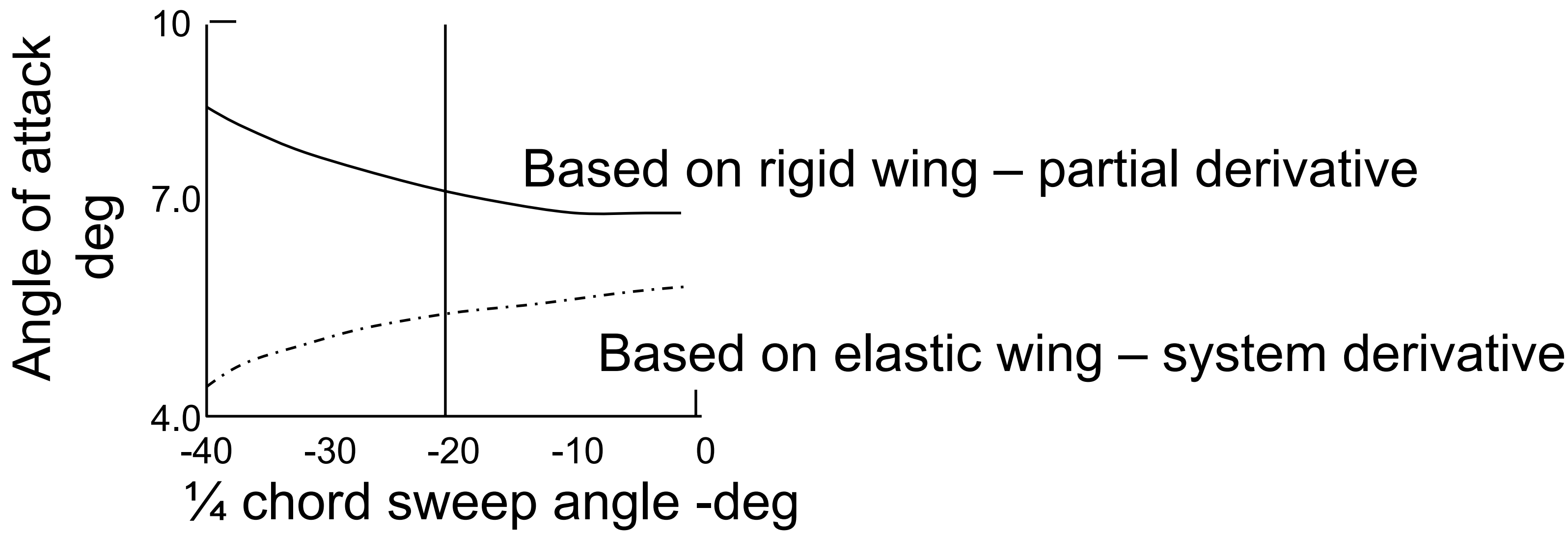
different Right Hand Sides

- Total derivatives can be computed if partial sensitivities computed in each subsystem are known

Linear, algebraical equations with multiple RHS

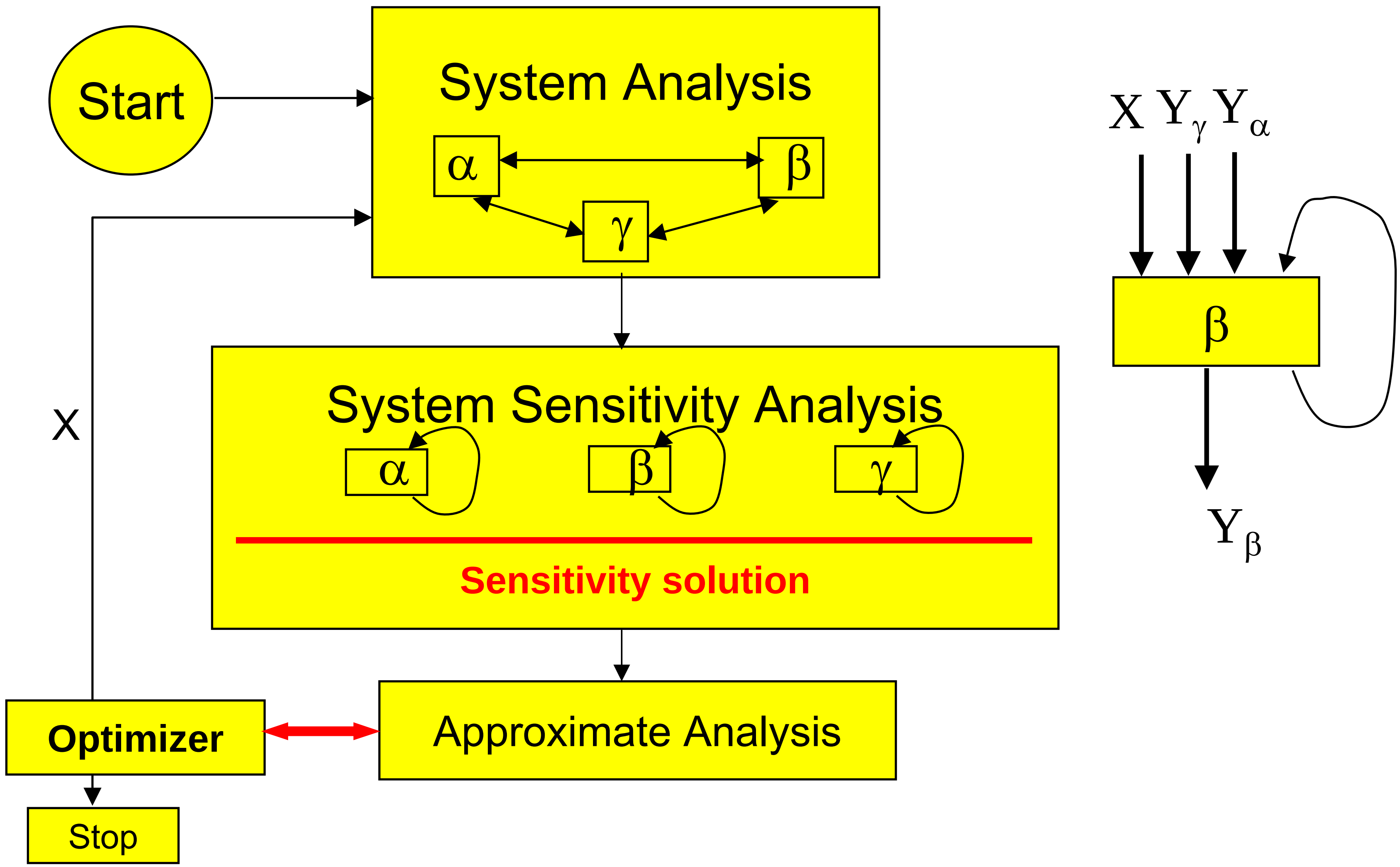
Example of System Derivative for Elastic Wing

- Example of partial and system sensitivities

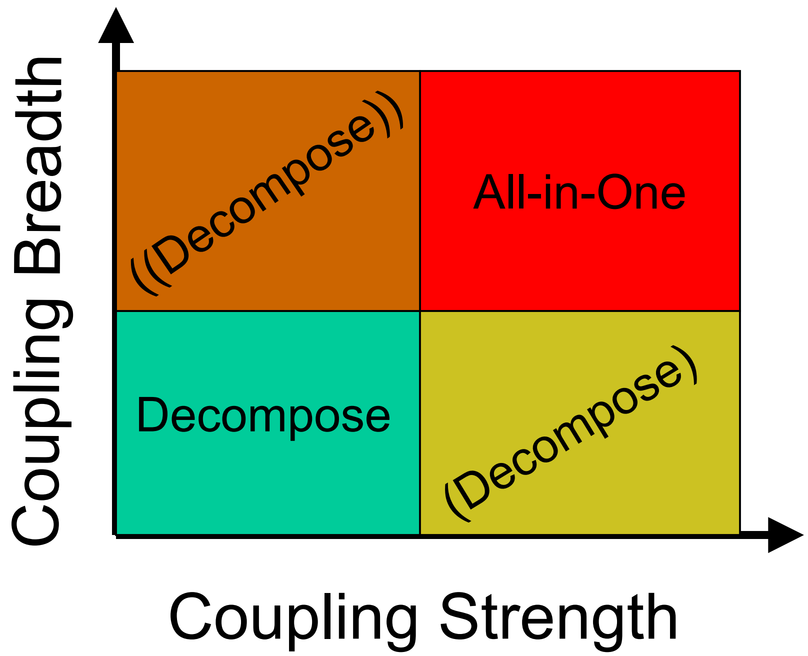


- In this example, the system coupling reverses the derivative sign

Flowchart of the System Optimization Process



System Internal Couplings Quantified



- Strength: relatively large

  ∂ YO/ ∂YI

- Breadth:

  {YO} and {YI} are long

  [∂ YO/ ∂YI] large and full

A Few Recent Application Examples

Multiprocessor Computers create
a new situation for MDO

Supersonic Business Jet Test Case

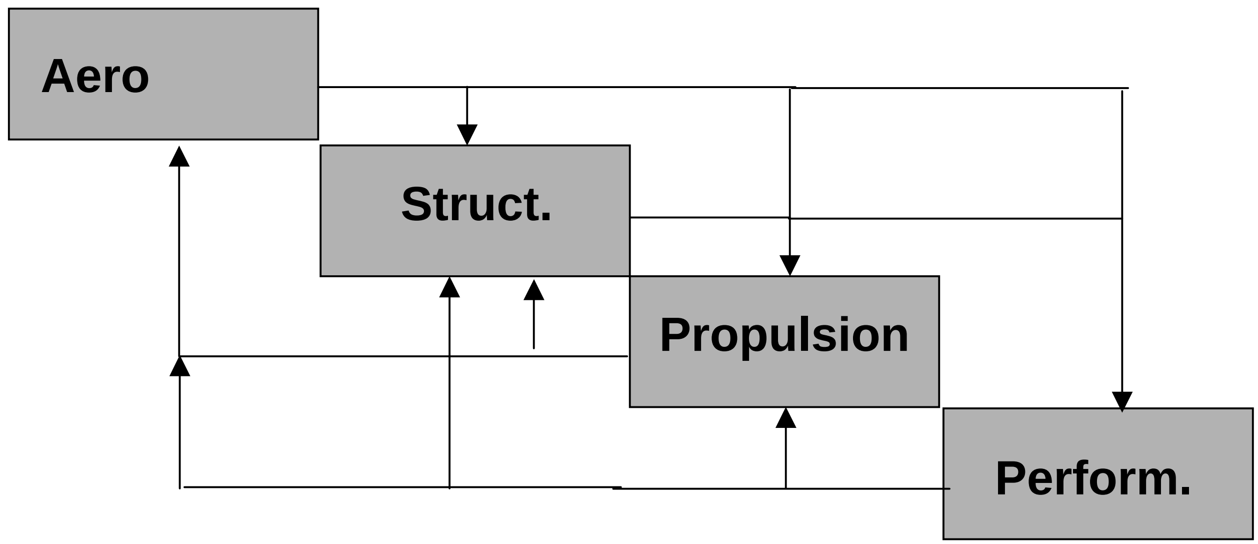
- **Structures (ELAPS)**
- **Aerodynamics (lift, drag, trim supersonic wave drag by A - Wave)**
- **Propulsion (look-up tables)**
- **Performance (Breguet equation for Range)**

Examples: **Xsh - wing aspect ratio, Engine scale factor**
Xloc - wing cover thickness, throttle setting
Y - aerodynamic loads, wing deformation.

Some stats:

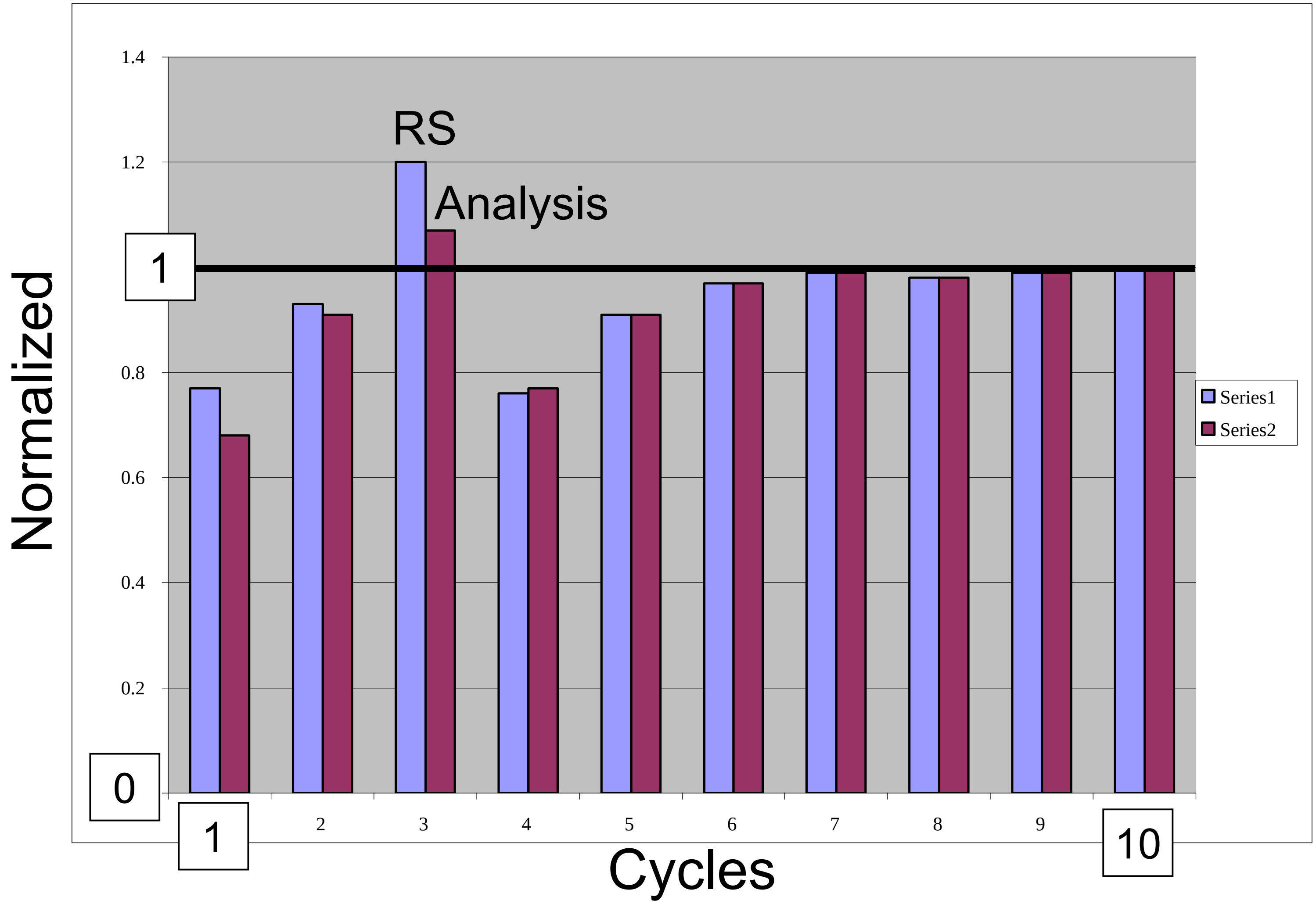
Xlocal: struct. 18
aero 3
propuls. 1
X shared: 9
Y coupl.: 9

System of Modules (Black Boxes) for
Supersonic Business Jet Test Case



- Data Dependence Graph
- RS - quadratic polynomials, adjusted for error control

Flight Range as the Objective



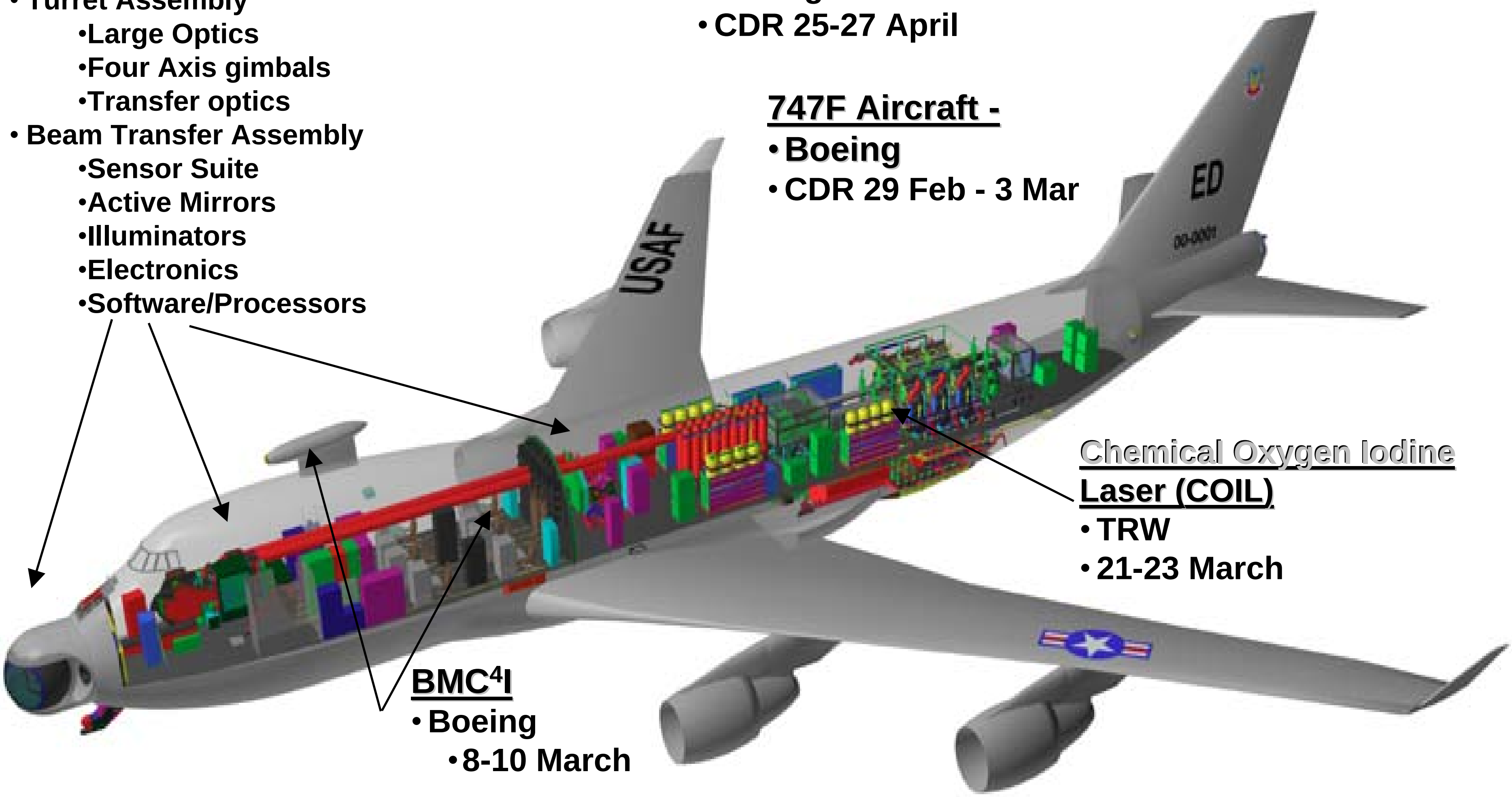
- Histogram of RS predictions and actual analysis for Range

Air Borne Laser System Design: another application of the similar scheme

Beam Control System
- Turret Assembly
  - Large Optics
  - Four Axis gimbals
  - Transfer optics
- Beam Transfer Assembly
  - Sensor Suite
  - Active Mirrors
  - Illuminators
  - Electronics
  - Software/Processors

System Level Design
- Boeing
- CDR 25-27 April

747F Aircraft -
- Boeing
- CDR 29 Feb - 3 Mar

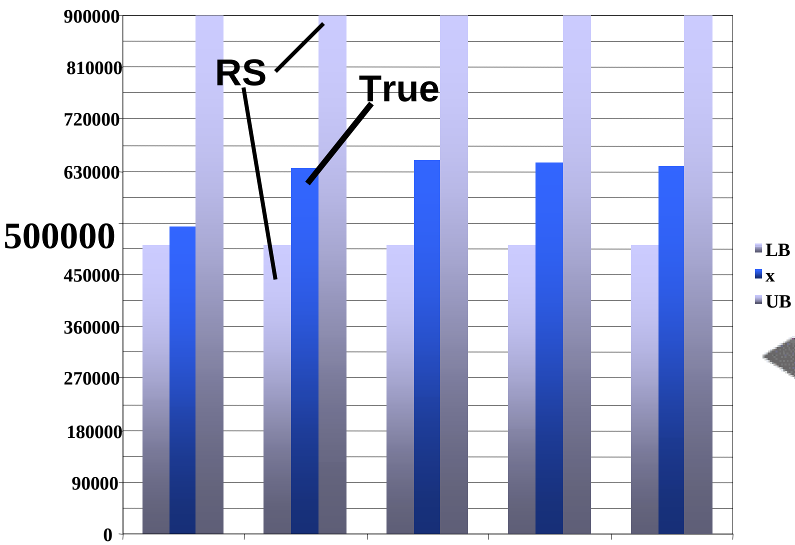


Chemical Oxygen Iodine Laser (COIL)
- TRW
- 21-23 March

BMC4I
- Boeing
  - 8-10 March

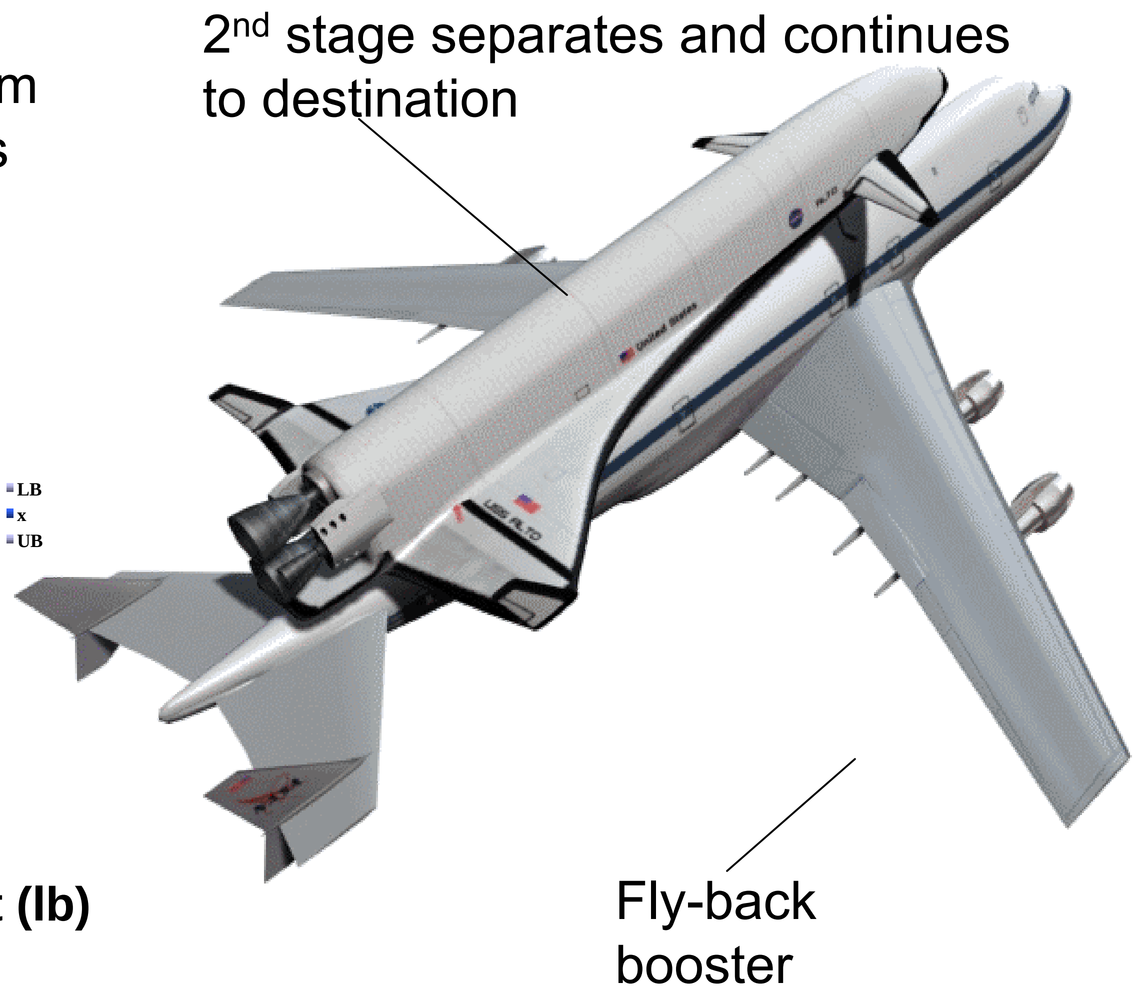
A Candidate for Shuttle Replacement: Two-stage Orbital Transport

• Collaborated with GWU, and ASCAC Branches: System Analysis and Vehicle Analysis



• **Result sample: System Weight (lb) Variance over MDO iterations**.

• **Initial design was infeasible**



NVH Model

- A Body-In-Prime (BIP) Model - Trimmed Body Structure without the powertrain and suspension subsystems

- MSC/NASTRAN Finite Element Model of 350,000+ edof;
- Normal Modes, Static Stress, & Design Sensitivity analysis using Solution Sequence 200;
- 29 design variables (sizing, spring stiffness);

Computational Performance

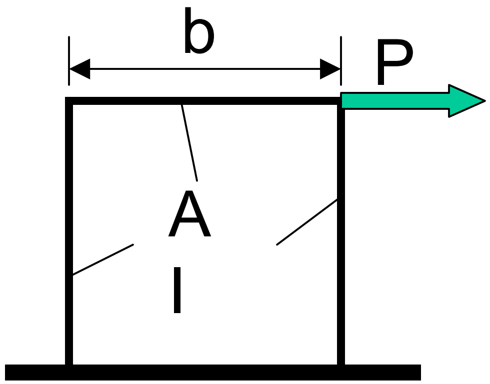
- Fine grain parallelism of Crash Code was an important factor in reducing the optimization procedure total elapsed time: *291 hours cut to 24 hours for a single analysis using 12 processors*.
- Response Surface Approximation for crash responses that enabled coarse grain parallel computing provided significant reduction in total elapsed time: *21 concurrent crash analysis using 12 processors each over 24 hours (252 processors total).*
- For effective utilization of a multiprocessor computer, user has to become acquainted with the machine architecture.

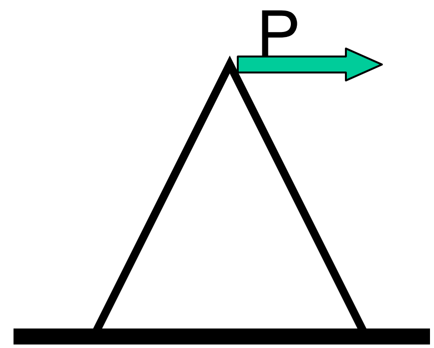
255 days of elapsed computing time cut to 1 day

Computer Power vs. Mental Power

Quantity vs Quality

Invention by Optimization?





{X} = {A, I, b}; Minimize weight; See b → Zero

- Optimization transformed frame into truss
- A qualitative change
- Why:
    - structural efficiency is ranked:

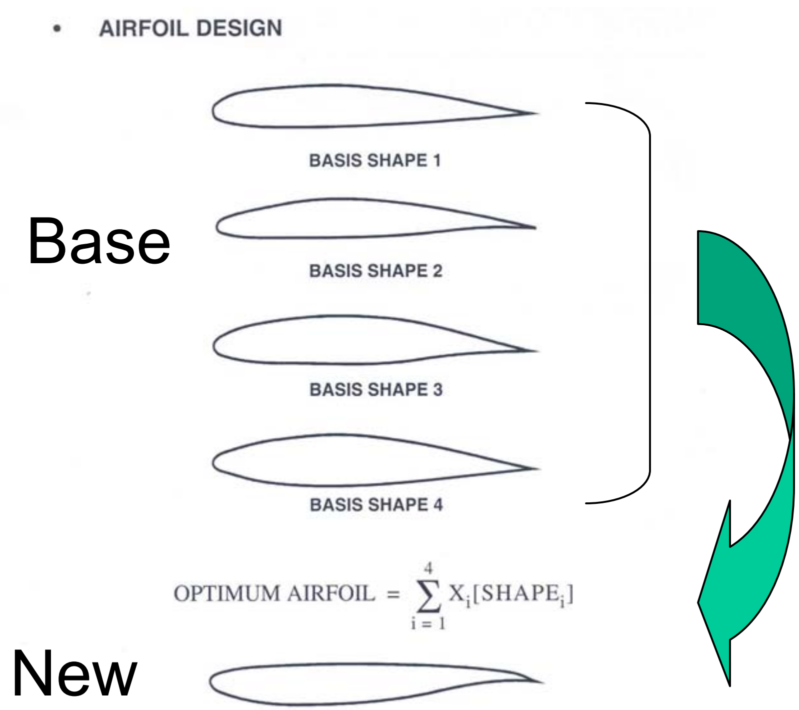
Tension best

Compression

Bending worst

- If one did not know this, and would not know the concept of a truss, this transformation would look as invention of truss.

Optimizing Minimum Drag/Constant Lift Airfoil for Transonic Regime



- Drag minimized while holding constant lift by geometrically adding the base airfoils.
- Each base airfoil had some aerodynamic merit
- Result: a new type, flat-top "Whitcomb airfoil".

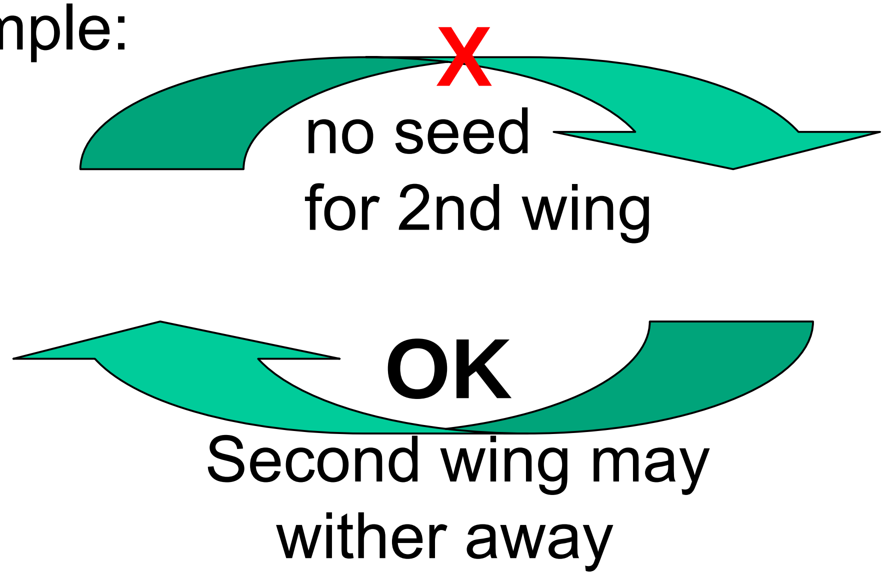
- If this was done before Whitcomb invented the flat-top airfoil (he use a file & wind tunnel), this would look like an invention.

Continuous quantitative transformation vs. conceptual quantum jump

•Common feature in both previous examples:

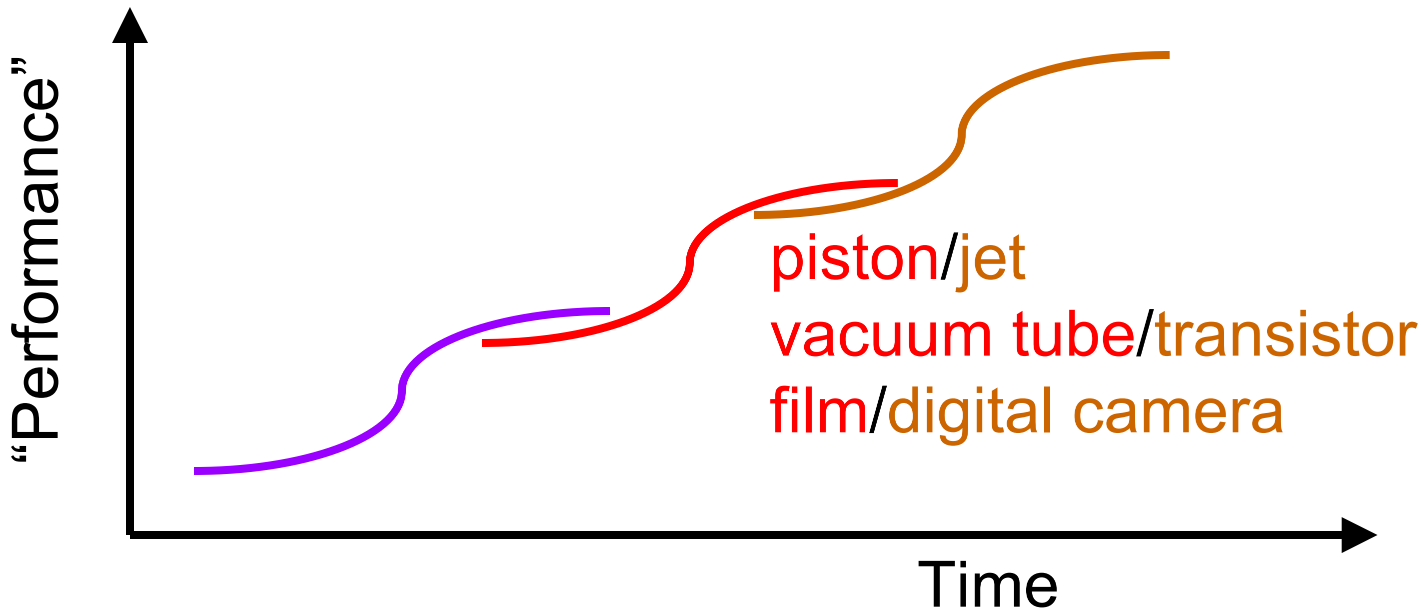
•Variable(s) existed whose continuous change enabled transformation to qualitatively new design

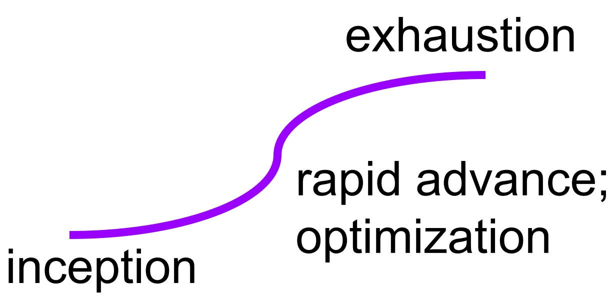
• Counter-example:



• Optimization may reduce but cannot grow what is not there, at least implicitly, in the initial design.

Technology Progress: Sigmoidal Staircase



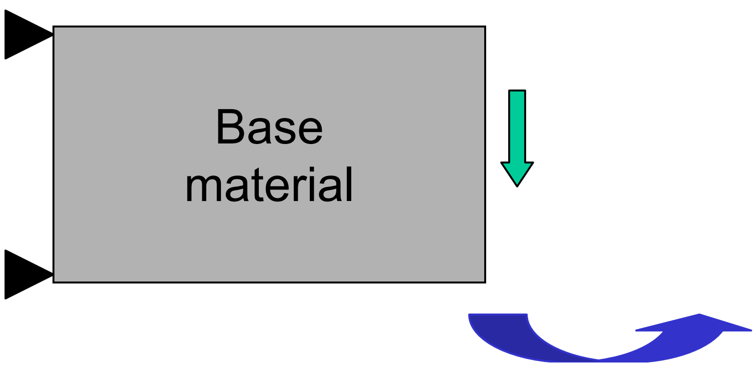


- Optimization assists in rapid advance phase
- Human creativity "shifts gears" to next step

Augmenting number crunching power of computer with “good practice” rules

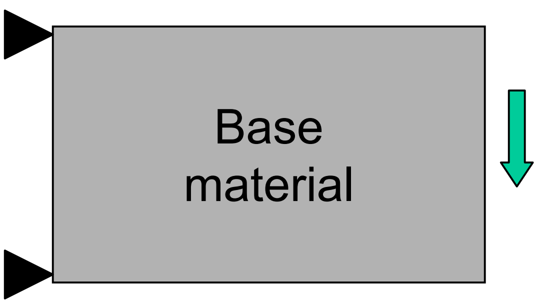
Topology Optimization

• Modern version of what Michelangelo said 500 years ago:
(paraphrased)
"to create a sculpture just remove the unnecessary material"



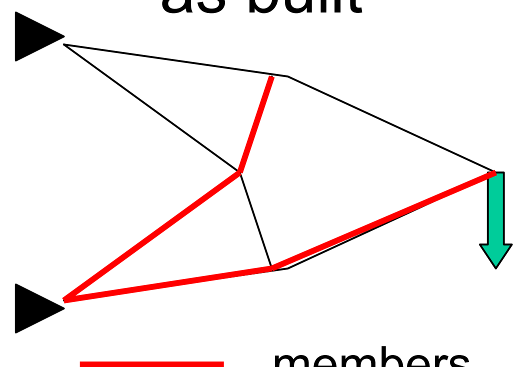
Topology optimization removes "pixels" from base material

Topology Optimization - 2



theoretical

as built

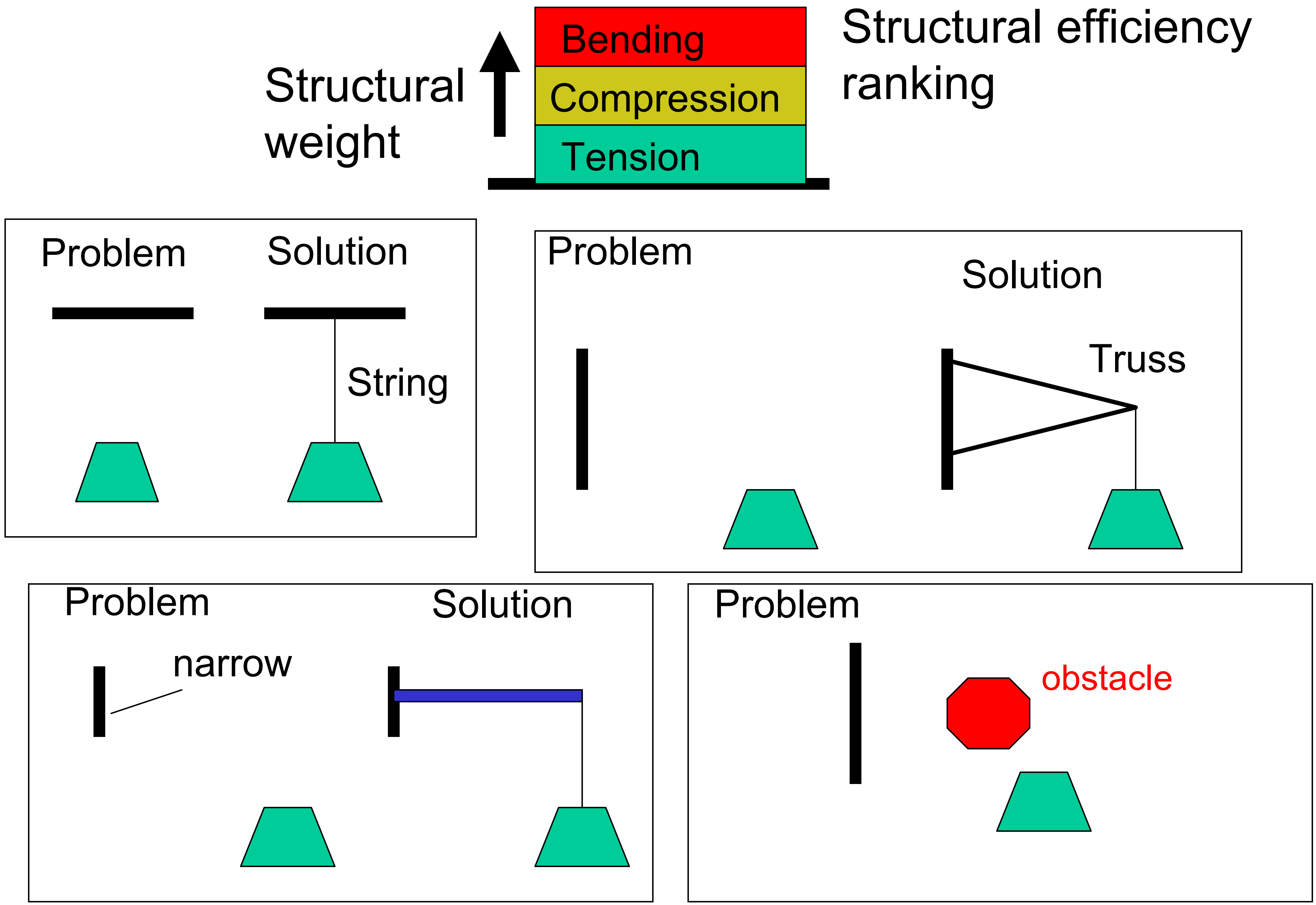


members
In compression

•This optimization can not include buckling constraints because the slender members do not emerge as such until the end.

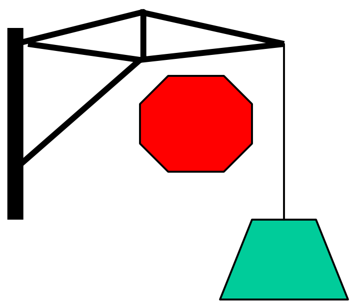
• Subtle point: it is difficult to keep the analysis valid when the imparted change requires new constraints.

Design by Rules

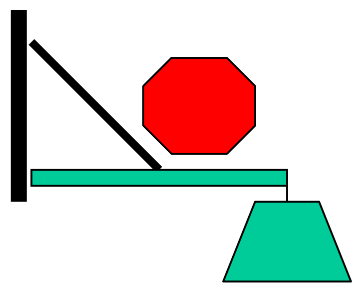


Complications…

Solution 1



Solution 2

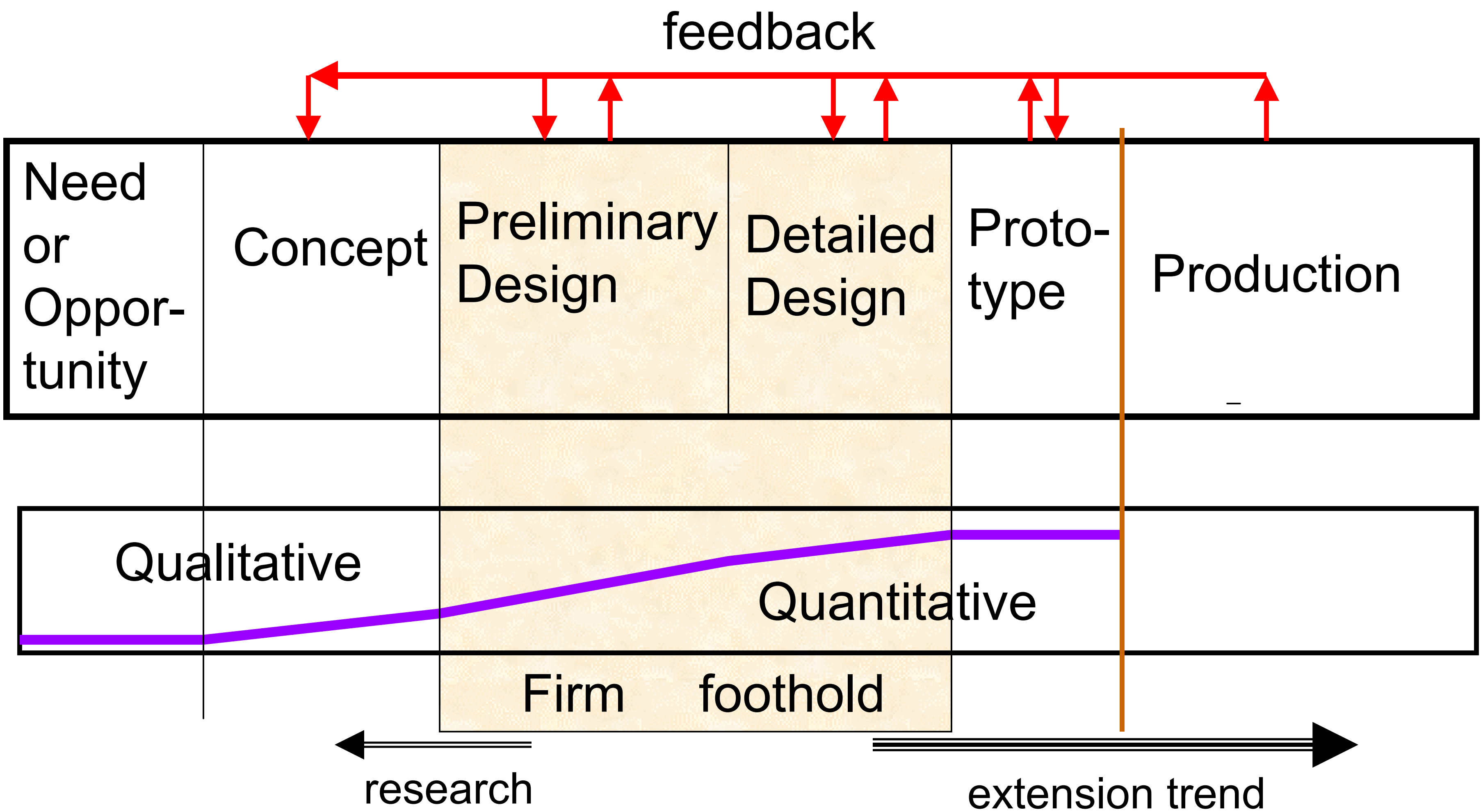


….things are getting too complicated

• Human eye-brain apparatus excels in handling geometrical complexities amplified by abundance of choices

• By some evidence, eye-brain apparatus may process 250 MB data in a fraction of a second.

Optimization in Design Process



- Optimization most useful where quantitative content is high

Closure

- Optimization became an engineer's partner in design
- It excels at handling the quantitative side of design
- It's applications range from component to systems
- It's utility is dramatically increasing with the advent of massively concurrent computing
- Current trend: extend optimization to entire life cycle with emphasis on economics, include uncertainties.
- Engineer remains the principal creator, data interpreter, and design decision maker.

Dr. Jaroslaw Sobieski

Degrees through doctorate in technical sciences from the Technical University of Warsaw (TUW), Poland. Concurrently with industry design and consulting, faculty positions at TUW 1955-66, St. Louis University 1966-71; George Washington University 1972-91; University of Virginia 1991- present; and post-doctoral research at the Technical University of Norway, Trondheim, 1964-65 and summer 1966. On the staff of NASA Langley since 1971, several research and supervisory position in structural and in multidisciplinary analysis & design optimization, and design studies of aerospace systems. Manager of the Langley's portion of the Computational Aerospace Sciences Project under the High Performance Computing and Communication Program HPCCP, 1996-2000. Currently Sr. Res. Scientist in Analytical and Computational Methods. AIAA Fellow and the Founding Chairman of the AIAA Technical Committee for Multidisciplinary Design Optimization. Recipient of: the NASA Medal for Exceptional Engineering Achievement, and the AIAA National Award for Multidisciplinary Analysis and Optimization in 1996. Co-Recipient of the SAE Wright Brothers Medal 1999. Several technical publications in professional journals and books. Co-Editor of international journal Structural and Multidisciplinary Optimization. Listed in the Marquis' Who is Who in America and Who is Who in the World.