Issues in Optimization

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How to know whether optimization is needed
How to recognize that the problem at hand needs optimization.

- General Rule of the Thumb: there must be at least two opposing trends as functions of a design variable.
Power Line Cable

- Given:
  - Ice load
  - Self-weight small
  - h/span small

Length(h)
A(h)
Volume(h)

tout cable
slack cable

• Ice load
• Self-weight small
• h/span small
Wing Thin-Walled Box

- Top cover panels are compressed
- Buckling stress = $f(t/b)^2$

![Graph](attachment:graph.png)

- Lift
- Thickness $t$
- Width $b$
- Wing box weight
- Cover weight
- Rib total weight

Horizontal axis:
- Many ribs
- Few ribs

Vertical axis:
- Wing box weight
- Cover weight
- Rib total weight
Multistage Rocket

- More segments (stages) = less weight to carry up = less fuel
- More segments = more junctions = more weight to carry up
- Typical optimum: 2 to 4.

Saturn V
Under-wing Nacelle Placement

- Inlet ahead of wing max. depth = shock wave impinges on forward slope = drag
- Nacelle moved aft = landing gear moves with it = larger tail (or longer body to rotate for take-off = more weight

Diagram showing:
- Wing underside
- Shock wave
- Nacelle
- Drag

Graph showing:
- Range max
- Weight
- Drag

Axes:
- Fore
- Nacelle
- Aft
National Taxation

• More tax/last $ = less reason to strive to earn
• More tax/$ = more $ collected per “unit of economic activity”
National Taxation

- More tax/last $ = less reason to strive to earn
- More tax/$ = more $ collected per “unit of economic activity”
- What to do:
  - If we are left of max = increase taxes
  - If we are right of max = cut taxes
Nothing to Optimize

- Monotonic trend
- No counter-trend
- Nothing to optimize

\[
\begin{align*}
\sigma &\quad \text{N/cm}^2 \\
A &\quad \text{cm}^2 \\
\sigma \text{ allowable} &
\end{align*}
\]
Various types of design optima
Design Definition: Sharp vs. Shallow

- Constraints - 0 contours
  - Bad side of

- Near-orthogonal intersection defines a design point
- Tangential definition identifies a band of designs

Constraints - 0 contours

Objective

Constraint

Descent
Multiobjective Optimization

\[ Q = \frac{1}{\text{quality} \& \text{performance} \& \text{comfort}} \]

1. pareto-optimum
2. pareto-frontier
3. V&W
4. R&R
A Few Pareto-Optimization Techniques

• Reduce to a single objective: \( F = \sum_i w_i f_i \)
where w’s are judgmental weighting factors

• Optimize for \( f_1 \); Get \( f^*_1 \);
  • Set a floor \( f_1 \geq f^*_1 \); Optimize for \( f_2 \); get \( f_2 \);
  • Keep floor \( f_1 \), add floor \( f_2 \); Optimize for \( f_3 \);
  • Repeat in this pattern to exhaust all f’s;
• The order of f’s matters and is judgmental

• Optimize for each \( f_i \) independently; Get n optimal designs;
Find a compromise design equidistant from all the above.

• Pareto-optimization intrinsically depends on judgmental preferences
Imparting Attributes by Optimization

• Changing $w_i$ in $F = \sum_i w_i f_i$
  modifies the design within broad range

• Example: Two objectives
  • setting $w_1 = 1$; $w_2 = 0$ produces design whose $F = f_1$
  • setting $w_1 = 0$; $w_2 = 1$ produces design whose $F = f_2$
  • setting $w_1 = 0.5$; $w_2 = 0.5$ produces design whose
    $F$ is in between.

• Using $w_i$ as control, optimization serves as a tool
  to “steer” the design toward a desired behavior or
  having pre-determined, desired attributes.
Optimum: Global vs. Local

Why the problem:

- Nonconvex objective or constraints (wiggly contours)
- Disjoint design space

Resonance

Local information, e.g., derivatives, does not distinguish local from global optima - the Grand Unsolved Problem in Analysis
What to do about it

A “shotgun” approach:

- Use a multiprocessor computer
- Start from many initial designs
- Execute multipath optimization
- Increase probability of locating global minimum
- Probability, no certainty

Multiprocessor computing = analyze many in time of one = new situation = can do what could not be done before.

A “shotgun” approach:

- “Tunneling” algorithm finds a better minimum
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Start
M1
Opt.
Tunnel
M2<M1

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• “Tunneling” algorithm finds a better minimum

• “Tunneling” algorithm

\[ \text{Start} \quad \text{Opt.} \quad \text{Tunnel} \quad \text{M1} \quad \text{M2} < \text{M1} \]
Using Optimization to Impart Desired Attributes
Larger scale example: \( \text{EDOF} = 11400; \)
Des. Var. = 126; Constraints = 24048;
Built-up, trapezoidal, slender transport aircraft wing

- Design variables: thicknesses of sheet metal, rod cross-sectional areas, inner volume (constant span and chord/depth ratio

- Constraints: equivalent stress and tip displacement

- Two loading cases: horizontal, 1 g flight with engine weight relief, and landing.

- Four attributes:
  - structural mass
  - 1st bending frequency
  - tip rotation
  - internal volume
Case: \[ F = w_1 \left( \frac{M}{M_0} \right) + w_2 \left( \frac{\text{Rotat}}{\text{Rotat}_0} \right) \]

\( \text{Rotat} = \text{wingtip twist angle} \)

- Broad variation: 52% to 180%
Optimization Crossing the Traditional Walls of Separation
Optimization Across Conventional Barriers

Vehicle design
- Focus on vehicle physics and variables directly related to it
  - E.g., range; wing aspect ratio

Fabrication
- Focus on manufacturing process and its variables
  - E.g., cost; riveting head speed
Two Loosely Connected Optimizations

• Seek design variables to maximize performance under constraints of:
  Physics
  Cost
  Manufacturing difficulty

• Seek process variables to reduce the fabrication cost.

The return on investment (ROI) is a unifying factor

\[ \text{ROI} = f(\text{Performance, Cost of Fabrication}) \]
Integrated Optimization

• Required: Sensitivity analysis on both sides

\[
\frac{\partial \text{Range}}{\partial \text{AspectRatio}} \quad \frac{\partial \text{Cost}}{\partial \text{Rivet head speed}}
\]

\[
\frac{\partial \text{(Rivet head speed)}}{\partial \text{AspectRatio}}
\]

\[
\text{ROI} = f(\text{Range, Cost of Fabrication})
\]

\[
\frac{\partial \text{ROI}}{\partial \text{AspectRatio}} = \frac{\partial \text{ROI}}{\partial \text{Cost}} \frac{\partial \text{Cost}}{\partial \text{Rivet h.s.}} \frac{\partial \text{(Rivet h.s.)}}{\partial \text{AspectRatio}} + \frac{\partial \text{(ROI)}}{\partial \text{Range}} \frac{\partial \text{Range}}{\partial \text{AspectRatio}}
\]
Integrated Optimization Design < --- > Fabrication

- Given the derivatives on both sides

Unified optimization may be constructed to seek vehicle design variable, e.g., AspectRatio, for maximum ROI incorporating AR effect on Range and on fabrication cost.
Optimization Applied to Complex Multidisciplinary Systems

Multidisciplinary Optimization
MDO
Coupling

Decomposition

What to optimize for at the discipline level

Approximations

Sensitivity
Wing drag and weight both influence the flight range $R$. 

$R$ is the system objective

- Structure influences $R$ by
  - directly by weight
  - indirectly by stiffness that affect displacements that affect drag

$$R = \left(\frac{k}{\text{Drag}}\right) \log \left[\frac{(W_o + W_s + W_f)}{(W_o + W_s)}\right]$$

- Dilemma: What to optimize the structure for? **Lightness**?
- Displacements = $1$/Stiffness?
- An optimal mix of the two?
**Trade-off between opposing objectives of lightness and stiffness**

- **What to optimize for?**
- **Answer:** minimum of $f = w_1 \text{ Weight} + w_2 \text{ Displacement}$
- vary $w_1$, $w_2$ to generate a population of wings of diverse Weight/Displacement ratios
  - Let system choose $w_1$, $w_2$. 

**Diagram:**
- Weight and Displacement axes
- Thickness limited by stress
- Wing cover sheet thickness
- Lightness and Stiffness axes
- Displacement $\sim 1/$Stiffness

---

**Graph:**
- Linear relationship between Weight and Displacement
- Non-linear relationship between Thickness and Stress
- Optimal point at $f$ minimum
Approximations

- a.k.a. Surrogate Models

Why Approximations:

- OK for small problems
- Now-standard practice for large problems to reduce and control cost

$$\text{cents}$$
Design of Experiments (DOE) & Response Surfaces (RS)

- RS provides a “domain guidance”, rather than local guidance, to system optimizer

**DOE**

- Placing design points in design space in a pattern

  - Example: Star pattern (shown incomplete)

**RS**

\[ F(X) = a + \{b\}'X + \{X\}'[c]X \]

- Quadratic polynomial
- Hundreds of variables
Response Surface Approximation

- A Response Surface is an $n$-dimensional hypersurface relating $n$ inputs to a single response (output).

- Design of Experiments (DOE) methods used to disperse data points in design space.

- More detail on RS in section on Approximations.
BLISS 2000: MDO Massive Computational Problem Solved by RS (or alternative approximations)

- Radical conceptual simplification at the price of a lot more computing. Concurrent processing exploited.
Coupled System Sensitivity

- Consider a multidisciplinary system with two subsystems A and B (e.g. Aero. & Struct.)
  - System equations can be written in symbolic form as
    \[ A[(X_A, Y_B), Y_A] = 0 \]
    \[ B[(X_B, Y_A), Y_B] = 0 \]
  - Rewrite these as follows
    \[ Y_A = Y_A(X_A, Y_B) \]
    \[ Y_B = Y_B(X_B, Y_A) \]
  - These governing equations define as implicit functions.

Implicit Function Theorem applies.
Coupled System Sensitivity - Equations

- These equations can be represented in matrix notation as

\[
\begin{bmatrix}
I & -\frac{\partial Y_A}{\partial Y_B} \\
-\frac{\partial Y_B}{\partial Y_A} & I
\end{bmatrix}
\begin{bmatrix}
\frac{dY_A}{dX_A} \\
\frac{dY_B}{dX_B}
\end{bmatrix}
= 
\begin{bmatrix}
\frac{\partial Y_A}{\partial X_A} \\
0
\end{bmatrix}
\]

- Total derivatives can be computed if partial sensitivities computed in each subsystem are known.

Linear, algebraical equations with multiple RHS.
Example of System Derivative for Elastic Wing

- Example of partial and system sensitivities

- In this example, the system coupling reverses the derivative sign
Flowchart of the System Optimization Process

Start → System Analysis → System Sensitivity Analysis → Sensitivity solution → Approximate Analysis → Stop

Optimizer
System Internal Couplings Quantified

- **Strength:** relatively large \( \frac{\partial Y_O}{\partial Y_I} \)

- **Breadth:**
  - \{Y_O\} and \{Y_I\} are long
  - \[\frac{\partial Y_O}{\partial Y_I}\] large and full
A Few Recent Application Examples

Multiprocessor Computers create a new situation for MDO
Supersonic Business Jet Test Case

- Structures (ELAPS)
- Aerodynamics (lift, drag, trim supersonic wave drag by A - Wave)
- Propulsion (look-up tables)
- Performance (Breguet equation for Range)

Examples: Xsh - wing aspect ratio, Engine scale factor
Xloc - wing cover thickness, throttle setting
Y - aerodynamic loads, wing deformation.

Some stats:

Xlocal:  struct. 18
        aero 3
        propuls. 1
X shared: 9
Y coupl.: 9
System of Modules (Black Boxes) for Supersonic Business Jet Test Case

- Data Dependence Graph
- RS - quadratic polynomials, adjusted for error control
Flight **Range** as the Objective

- Histogram of RS predictions and actual analysis for Range
Air Borne Laser System Design: another application of the similar scheme

**Beam Control System**
- Turret Assembly
  - Large Optics
  - Four Axis gimbals
  - Transfer optics
- Beam Transfer Assembly
  - Sensor Suite
  - Active Mirrors
  - Illuminators
  - Electronics
  - Software/Processors

**System Level Design**
- Boeing
- CDR 25-27 April

**747F Aircraft**
- Boeing
- CDR 29 Feb - 3 Mar

**BMC4I**
- Boeing
- 8-10 March

**Chemical Oxygen Iodine Laser (COIL)**
- TRW
- 21-23 March
A Candidate for Shuttle Replacement: Two-stage Orbital Transport

- Collaborated with GWU, and ASCAC Branches: System Analysis and Vehicle Analysis

- Result sample: System Weight (lb) Variance over MDO iterations.
- Initial design was infeasible

2nd stage separates and continues to destination

Fly-back booster
NVH Model

• A Body-In-Prime (BIP) Model - Trimmed Body Structure without the powertrain and suspension subsystems

• MSC/NASTRAN Finite Element Model of 350,000+ edof;

• Normal Modes, Static Stress, & Design Sensitivity analysis using Solution Sequence 200;

• 29 design variables (sizing, spring stiffness);
Computational Performance

• Fine grain parallelism of Crash Code was an important factor in reducing the optimization procedure total elapsed time: 
  291 hours cut to 24 hours for a single analysis using 12 processors.

• Response Surface Approximation for crash responses that enabled coarse grain parallel computing provided significant reduction in total elapsed time: 
  21 concurrent crash analysis using 12 processors each over 24 hours (252 processors total).

• For effective utilization of a multiprocessor computer, user has to become acquainted with the machine architecture.

255 days of elapsed computing time cut to 1 day
Computer Power vs. Mental Power

Quantity vs. Quality
Invention by Optimization?

\{X\} = \{A, I, b\}; Minimize weight; See \(b \rightarrow\) Zero

- Optimization transformed frame into truss
- A qualitative change
- Why:
  - Structural efficiency is ranked:
    - Tension best
    - Compression
    - Bending worst
- If one did not know this, and would not know the concept of a truss, this transformation would look as invention of truss.
Optimizing Minimum Drag/Constant Lift Airfoil for Transonic Regime

• Drag minimized while holding constant lift by geometrically adding the base airfoils.
• Each base airfoil had some aerodynamic merit
• Result: a new type, flat-top “Whitcomb airfoil”.

• If this was done before Whitcomb invented the flat-top airfoil (he use a file & wind tunnel), this would look like an invention.
Continuous quantitative transformation vs. conceptual quantum jump

• Common feature in both previous examples:
  • Variable(s) existed whose continuous change enabled transformation to qualitatively new design

• Counter-example:
  • Optimization may reduce but cannot grow what is not there, at least implicitly, in the initial design.
Technology Progress: Sigmoidal Staircase

- Optimization assists in rapid advance phase
- Human creativity "shifts gears" to next step
Augmenting number crunching power of computer with “good practice” rules
Topology Optimization

• Modern version of what Michelangelo said 500 years ago: (paraphrased) 
  “to create a sculpture just remove the unnecessary material”

Topology optimization removes “pixels” from base material
• This optimization cannot include buckling constraints because the slender members do not emerge as such until the end.

• Subtle point: it is difficult to keep the analysis valid when the imparted change requires new constraints.
Design by Rules

Structural weight

Problem

Solution

String

Problem

Solution

Truss

Problem

Solution

narrow

Problem

obstacle
Complications…

Solution 1

Solution 2

….things are getting too complicated

• Human eye-brain apparatus excels in handling geometrical complexities amplified by abundance of choices

• By some evidence, eye-brain apparatus may process 250 MB data in a fraction of a second.
• Optimization most useful where quantitative content is high
Closure

• Optimization became an engineer’s partner in design

• It excels at handling the quantitative side of design

• It’s applications range from component to systems

• It’s utility is dramatically increasing with the advent of massively concurrent computing

• Current trend: extend optimization to entire life cycle with emphasis on economics, include uncertainties.

• Engineer remains the principal creator, data interpreter, and design decision maker.
Dr. Jaroslaw Sobieski