Trajectory Design For A Visible Geosynchronous Earth Imager

Edmund M. C. Kong
SSL Graduate Research Assistant

Prof David W. Miller
Director, MIT Space Systems Lab

Dr. Raymond J. Sedwick
Postdoctoral Associate, MIT Space Systems Lab

AIAA Space Technology Conference & Exposition
Albuquerque, New Mexico
30 September, 1999
Introduction

Objective: To compare the different imaging configurations for a Separated Spacecraft Interferometer operating from an Earth’s orbit

Outline:
- Interferometric requirements & Orbit Selection
- Equations of Motions (Hill’s Equations)
- Steered Planar Array
- Propellant Free Array: Collector S/C
- Results
- Summary
Interferometric Requirements:
Reqt 1. Equal science light pathlength for visible imaging
Reqt 2. Axi-symmetric angular resolution about LOS

Far-field assumption
- Array sees planar wavefronts from targets

Orbit Selection: Geosynchronous
- Higher altitude, lower perturbative effects (eg. $J_2$)
Equations of Motions

Assumption: First order perturbation about natural circular Keplerian orbit

Hill’s Equations:

\[
\begin{align*}
    a_x &= \ddot{x} - 3n^2 x - 2n \dot{y} \\
    a_y &= \ddot{y} + 2n \dot{x} \\
    a_z &= \ddot{z} + n^2 z
\end{align*}
\]

Total Spacecraft Velocity Increment:

\[
\Delta V = \int_0^{T_{life}} \sqrt{a_x^2 + a_y^2 + a_z^2} \, dt
\]

Example: \(\Delta V\) required to hold a spacecraft stationary at \((x,y,z)\)

Spacecraft instantaneous acceleration:

\[
\begin{align*}
    a_x &= -3n^2 x \\
    a_y &= 0 \\
    a_z &= n^2 z
\end{align*}
\]

\(\Delta V\) required:

\[
\Delta V = n^2 T_{life} \sqrt{9x^2 + z^2}
\]
Constraint collector spacecraft to a local horizontal circular trajectory with combiner spacecraft at the center (Reqs. 1 & 2)

\( \Delta V \) Requirement
- No \( \Delta V \) for stationary combiner spacecraft at \((0, y, 0)\)
- \( \Delta V \) for collector spacecraft

\[
\begin{bmatrix}
x \\
y \\
z_{Hill}
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \phi & -\sin \phi \\
0 & \sin \phi & \sin \phi
\end{bmatrix}
\begin{bmatrix}
\cos \psi & -\sin \psi & 0 \\
\sin \psi & \cos \psi & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x' \\
y' \\
z'
\end{bmatrix}
\]

Average collector s/c \( \Delta V \) at GEO altitude:

\[
\frac{\Delta V}{n^2 R_o T_{life}} = 1.55
\]
**DSS Architecture 2**

Constraint the projection of the collector spacecraft's trajectory to circular (Reqt. 2)

- Propellant free trajectories - (Project 2 x 1 ellipse in velocity plane)

\[
\begin{bmatrix}
  x \\
  y \\
  z_{Collector}
\end{bmatrix} = \begin{bmatrix}
  \pm \left(\frac{R_o}{2}\right) \cos nt \\
  \mp R_o \sin nt \\
  R_z \cos nt
\end{bmatrix}
\]

Intersection between a plane and a circular paraboloid results in an ellipse

- Placed combiner spacecraft placed at focus for equal pathlength (Reqt. 1)

- for \( R_z = R_o \)

\[
\begin{bmatrix}
  x \\
  y \\
  z_{Focus}
\end{bmatrix} = \begin{bmatrix}
  \left(16R_o^2 - 3p^2\right)/(16p) \\
  0 \\
  \pm p/4
\end{bmatrix}
\]

Vary \( R_z : (-\infty, \infty) \)
DSS Architecture 2 (cont.)

A family of paraboloids can fit onto the free elliptical trajectories
- Locus of foci maps out a hyperbola
- for \( R_z = R_o \)

\[
x = \frac{R_o^2 - 3z^2}{\pm 4z}
\]

Optimum focus:
\[
p = 2.2076R_o
\]
\[
\Delta V = 0.5642n^2R_oT_{life}
\]

\( \Delta V \) requirement:
- No \( \Delta V \) required for collector spacecraft
- Only need \( \Delta V \) to hold combiner spacecraft at paraboloid’s focus
Steering with optical delay lines

- Collector s/c follow $R_z = R_o$ elliptical trajectory from Architecture 2
- Delay lines to image off-nadir targets (Reqt. 1)

Collector-Combiner s/c distance:

$$D = R_o \sqrt{\frac{(\cos nt)^2}{4} + \left(\frac{1}{P_n} + \frac{5}{16} P_n\right) \cos nt + \frac{5}{8} \frac{25}{256} P_n^2 + \frac{1}{P_n^2}}$$

Collector s/c trajectory in propagation vector's $(x')$ frame:

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos \psi & \sin \psi & 0 \\ -\sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ \cos \phi & \sin \phi & 0 \\ 0 & -\sin \phi & \sin \phi \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\text{Hill}}$$

At GEO

- Maximum delay length from GEO $(x',D) = 0.310R_o$
- Minimum semi-minor axis projection $(y',z') = 0.914R_o$
Mission Parameters

<table>
<thead>
<tr>
<th>Components</th>
<th>Steered Planar</th>
<th>ODL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Combiner S/C</td>
<td>182.1 kg</td>
<td>182.1 kg</td>
</tr>
<tr>
<td>Combiner Propellant</td>
<td>-</td>
<td>$\Delta V/(n^2 R_0 T_{life}) = 0.56$</td>
</tr>
<tr>
<td>Collector S/C</td>
<td>87.1 kg</td>
<td>87.1 kg</td>
</tr>
<tr>
<td>Collector Delay Lines</td>
<td>-</td>
<td>0.34$R_0$</td>
</tr>
<tr>
<td>Collector Propellant</td>
<td>$\Delta V/(n^2 R_0 T_{life}) = 1.55$</td>
<td>-</td>
</tr>
</tbody>
</table>

Spacecraft Mass estimates from initial Deep Space 3 (DS3) design

- $T_{life} = 5$ years
- $R_0 = 500$ m (DS3 - 1000 m baseline)

Place ODL on Collector S/C

- Ease of operation
- Lower overall dry mass and therefore, lower system mass

For each spacecraft

- Determine $\Delta V$
- Propellant mass from Rocket equation

$$\frac{m_p}{m_d} = \exp\left(\frac{\Delta V}{I_{sp} g}\right) - 1$$

- $m_p$ - Propellant Mass (kg)
- $m_d$ - Spacecraft Dry Mass (kg)
- $I_{sp}$ - Specific impulse (sec)
- $g$ - Earth’s gravity (9.81 m/sec)
Impact of ODL

General Observations

- Relatively insensitive to the number of collector s/c (> 4 collector)
- Trading between propellant and ODL mass

$R_o = 500 \text{ m}$
- Break even point $I_{sp} = 250 \text{ s}$
  (DLC = 0.1 kg/m)
- Arch 1: $m_{comb} = 182.1$, $m_{coll} = 114.1$
- Arch 2: $m_{comb} = 200.4$, $m_{coll} = 104.1$

$R_o = 50 \text{ m}$
- Break even point $I_{sp} = 220 \text{ s}$
  (DLC = 0.1 kg/m)
- Arch 1: $m_{comb} = 182.1$, $m_{coll} = 89.7$
- Arch 2: $m_{comb} = 184.1$, $m_{coll} = 88.8$

\[ DLC \approx \frac{m_{coll}(\exp(DV/I_{sp}g) - 1)}{0.34R_o} \]
Summary (1)

- Interferometric Requirements

- Equations of Motions
  - Hill’s Equations
  - $\Delta V$ Calculation

- DSS Architecture 1
  - $\Delta V$ for collector spacecraft only
Summary (2)

- **DSS Architecture 2**
  - Free $\Delta V$ trajectories for collector spacecraft
  - Minimum $\Delta V$ combiner spacecraft location
  - Exploitation of conic sections

- **Results**
  - Delay Length vs Specific Impulse cross over point

- **Optical Delay Lines**
  - Delay lines to steer array’s LOS